C. CORDES,
B. USMONOV

# BASICS OF PERFORMANCE

Ks-6

**Kitob quyida ko`rsatilgan muddatga topshirilishi shart**

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THE MINISTRY OF HIGHER AND SECONDARY SPECIAL
EDUCATION OF THE REPUBLIC OF UZBEKISTAN

CLAUS CORDES,
BOTIR USMONOV

# BASICS OF PERFORMANCE

*The teaching aid is intended for students of the aviation in statute trained in area 5310400 – «Aircraft engineering and technical operation of aircrafts»*

TASHKENT – 2015

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The teaching aid is intended for students of the aviation in statute trained in area 5310400 – «Aircraft engineering and technical operation of aircrafts».

The teaching aid consists of introduction, an airplane design, dynamics, the forces operating on an airplane, take-off and landing speeds, horizontal flight, cruiser speed, operation and other themes. The teaching aid can be used for students of area of 5310400 - «Aircraft engineering and technical operation of aircrafts» as the teaching aid on disciplines «The theory of aviation engines of air vessels», «Operational reliability of air vessels», «The equipment of systems of air vessels».

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*Reviewers:*

**N.Abdujabarov** – Head of chair «Aircraft engineering and technical operation of aircrafts», PhD;
**T.Sagdiyev** – Dean of Aviation faculty, PhD.

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## Foreword

This script shall be used at the Tashkent State technical University to follow through the course of “introduction to flight mechanics – performance”.

The course will strictly follow the script and intends to present the basics of the matter, accompanied by frequent calculation examples. Therefore, the content should be easy to follow and to understand.

We have used all our experience in teaching, to complete the script, and intention is a gentle and stepwise introduction. For that reason, not to deal with fast and high flying aeroplanes, but with slow and low flying ones to avoid the effect of compressibility and Mach number effects. Any student with a basic knowledge of higher mathematics should be capable to follow through all applied mathematical applications.

We also tried to present all necessary knowledge and facts from other disciplines, so that no other literature is needed to understand.

A sample aeroplane is defined at the beginning, which will be used repeatedly for the computations. By referring to one aeroplane model, it should become clear, which design criteria’s have a dominating influence on performance, and which “spots” of the flight envelope are important.

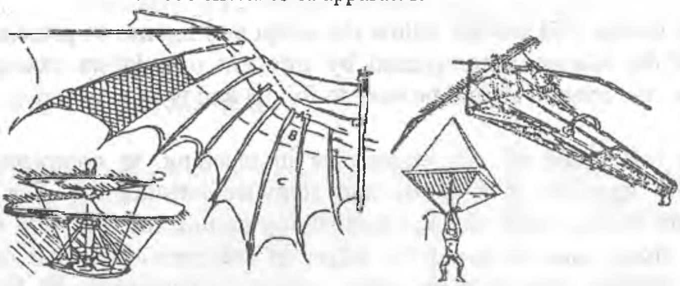
This script is also intended to be used by students to extend their knowledge about aeroplanes and other fly vehicles in general or to use the presented calculation methods, to learn more about their aeroplane, which they use in their training.

Students are strongly encouraged, to criticize the script, its intention, content and presentation. Especially those, who cannot follow it during classes, but use it for home learning are asked to do so.

## History of Aircraft

***Concept of Flight***

Leonardo da Vinci (1452-1519): He who was a brilliant scientific genius as well as a great artist, was a pioneer of aeronautics. He carefully studied the flight of birds and drew various sketches of apparatus.



**[Figure 1-5]** Sketches of Leonardo da Vinci

George Cayley (1773-1857): In 1810, he formulated the basic principle of aerodynamics. He is the one who suggested the possibility of principle of flight using fixed wing aircraft; if the wing overcomes the drag force with enough power, the lift force will be bigger than the gravity force. So, the energetic experiments about gliders are issued in 1800s.

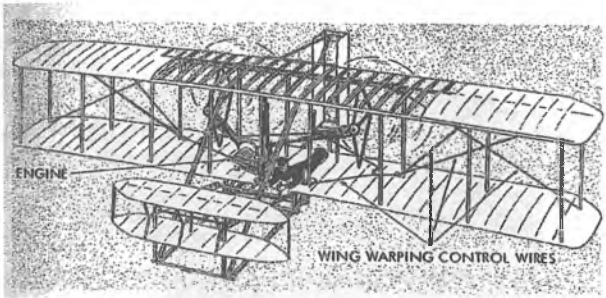
***Balloons and Airship***

Balloons enabled men to stay in the air. The first practical balloon was invented by the two Montgolfier brothers of France in 1783. Before the end of that year, several persons made ascensions in free balloons.

For more than a century after the first ascent of the Montgolfier brothers' balloon, aeronauts struggled with the problem of directional control. The initial approach to controlled flight was made in 1852 by Henri Giffard when he successfully flew a cigar-shaped balloon, driven by a 3 HP steam engine and propeller. Giffard's balloon was the forerunner of the dirigible which means that it can be directed, controlled and steered.

***The Wright Brothers***

December 17, 1903 is the one of the best known date in aviation history. On that day Orville and Wilbur Wright flew the first powered heavier-than-air craft at Kitty Hawk, North Carolina. They succeeded because they persevered in analyzing and testing the data gathered by their predecessors; theory of George Cayley and glider of Otto Lilienthal (1848-1896, who died because of fall accident which happened during his glider experiment). The significant discovery that led to their success was a system of maintaining wing stability. The first flight of The Flyer in 1903 lasted only 12 sec and flew 37m with engine which had 4 cylinders and developed 12 HP.




[Figure 1-6] The Wright Brothers, "Flyer"

The success of the Wrights was rapidly followed by further gains. In 1906 the first airplane was flown in Europe. In 1911 C. P. Rodgers managed to fly a plane from New York to Pasadena, California in 49 days; his actual time in the air was 82 hrs.

***World War I***

At the war's opening, airplanes were used mainly as scouts to search out the enemy's position, and no armament was carried. Then Roland Garros, a veteran French pilot, used a machine gun which he shot past an armor-plated propeller. The Germans, realizing what an immense advantage the use of the machine gun in an airplane gave the opposition, retaliated with a device that synchronized the gun with the propeller, so that the pilot could "shoot through" his propeller. Thereafter the airplane became a fighting machine rather than merely an observer.

One of the greatest innovations to come out of war was the all-metal monoplane produced by Hugo Junkers, a German engineer. This aircraft had a cantilever wing, that is, a wing with no outside supporting struts or bracing.

***After World War I***

Flying had proved its potential during the war. But immediately after the war, the aircraft industry faltered because the person in the street showed little inclination to risk one's life by flying. As the twenties wore on, however, aviation began slowly to adapt itself to peacetime conditions. Before the decade was over non-stop trans-continental, trans-oceanic, and polar flights began to keep aviation in the headlines.

The man who was to become the most famous flier of the era was an ex-airmail pilot, Charles A. Lindbergh. He took off on May 20, 1927 from New York City. When this first nonstop solo flight across the Atlantic ended just 33 hours and half later with his landing at Le Bourget airfield near Paris, he was the hero of millions. This flight assured people that the airplane was no longer a toy, and the public at last became eager to learn about aviation and its future.

[Figure 1-7] Douglas Company "DC-3"

Great technological advances were made in the 1930s. The retractable landing gear was developed, engines were made more efficient, and a new supercharger enabled planes to fly at higher altitudes. Radio and flight instruments were perfected so that pilots were able to "fly blind." With the invention of the variable-pitch propeller, better speed control was made possible.

In July 1936 the Douglas DC-3 made its first flight from Chicago to New York. The DC-3 combined all the important features that had been introduced during the previous years of research and development in an airplane. Because of its great speed, safety, and economy of operation it became the chief plane used by the airlines. With DC-3, America came to dominate the commercial air routes of the world.

***World War II***

The famous fighters in the early World War II are Messerschmitt of Germany, Spitfire of the United Kingdom, Hellcat of the United States and Zero of Japan. And in the latter of the war, P-38 Lightning, F-51 Mustang and Corsair of Lockheed, La-5 of the Soviet Union and Bf-109 Messerschmitt and Ju-87 Junkers of German are famous.

Long before World War II, the possibilities of the jet-propelled airplane had become evident to some far-sighted designers. In England Frank Whittle who contrived the concept of Jet engine when he was student took out his first patents of a jet engine in 1930. In German Hans von Ohain had patents as early as 1935. Jets were not ready, however, for much wartime use. Germany built over 1,000 jets, some of which were put into service in the last months of the war.

Radar had been developed in the U.K. and U.S.A. made that fit for practical use. It made the U.K. get a victory against German in an aerial battle. Radar also contributed to the flight operation in all-weather conditions, so that B-29 could bomb over the clouds in the last of the war.

***After World War II***

After the war, lots of scientists of German were moved to the United States or Soviet Union and they kept on studying with finance support. This situation contributed to make the Cold war between these two nations about weapon competition after the war.

The aircraft industry in America directed most of its efforts toward the development of jet propulsion for military planes. The first battles between jet fighters took place over Korea in June 1950. Representative fighters were F-86, MIG-15 and MIG-17.

[Figure 1-8] Bell "X-1"

Captain Charles E. Yeager flew the first supersonic rocket-powered airplane, the Bell X-1, in 1947. Airplane engineers and designers were again faced with new problems brought about by supersonic flight. They had to build stringer and more powerful airplanes to withstand high velocities at great heights. New materials had to be developed to resist the tremendous temperatures created by jet speeds.

In 1948, the jet age really came into being when a British jet flew faster than the speed of sound. Great Britain produced the first jet-propelled civilian transports, and the de Havilland Comet was used in the first jet air service between London and Africa in May 1952. Boeing Company developed the passenger plane B-707 which 100 passengers can board in 1957. DC-8 of Douglas Company and TU-104 of the Soviet Union also had an inaugural flight in 1958. In 1970s B-747, extra-large plane, started to fly and got the high possession rate in long-range airway. And Concord made by British-French joint started to fly at the speed of M 2.05 in 1971, however it failed to be commercial. A300 Airbus of European joint had the first flight in 1972 and succeeded to be commercial business to get 30% of possession rate in world market of the passenger flight which had no rivals of United States.

***Current Flight***

Nowadays, airplanes have mobility, simplification of cockpit and low-cognitive condition with developing of computer technology, engine, material and avionics. It makes the pilot have safe and easy flying. In military use, HDU(Head Up Display), HMD(Helmet Mounted Display), MFD(Multi-Function Display) make the pilot have high-convenience and speedy recognition. Progressed engine makes the flight supersonic cruise without After Burner, and Thrust Vectoring increase the mobility of the flight. B-2 bomber, F-117A and F-22 of United States apply Stealth technology which make the opposition be difficult to recognize. These all technologies are the result of developing of computer technology and control concept such like FBW(Fly by Wire).

## I Introduction performance in flight mechanics

The discipline of flight mechanics is divided into three main matters: performance, flight handling characteristics (stability and control) and aero elasticity.

The performance chapter deals with the capabilities of an aeroplane regarding speed, altitude, range, endurance, turns, take-off and landing. An aeroplane has six degrees of freedom, three in translator motion (forward, sideward and vertical) and three in rotation around its axes (rolling, pitching and yawing). Performance only deals with the translatory freedom and therefore an aeroplane for simplification can be considered as a mass point in the three-dimensional space.

Flight handling characteristics deal with stability and control, and their discussion is based on a rigid, three-dimensional model of an aeroplane that can execute all six degrees of freedom. The rigid aeroplane is a permissible simplification of the real aeroplane that experiences very little deformation from the forces acting on it.

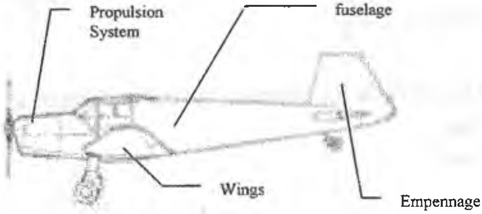
Aero elasticity, finally, considers this elastic behaviour, which causes recoupling effects on forces and moments.

| Discipline | Aeroplane model | Freedom of motion |
|---|---|---|
| **Performance** | mass point | translatory (3) |
| **Stability and control** | rigid three dimensional | translatory (3), rotation (3) |
| **Aero elasticity** | elastic three dimensional | translatory (3), rotation (3) elastic within the structure |

This book will discuss mainly performance matters in the low speed and low altitude regime aircraft and briefly in addition to the main text introduce students with helicopter and space vehicles performances!

### *I.1.1 Elements of an aeroplane*

For the requirements of this book only those elements of an aeroplane shall be mentioned, that are in relation to the performance discussions.




The **fuselage** houses the payload of the aeroplane, which may be passengers, cargo or mail as well as the crew required to operate the aeroplane, which may be a single pilot or an airline crew with pilots and flight attendants.

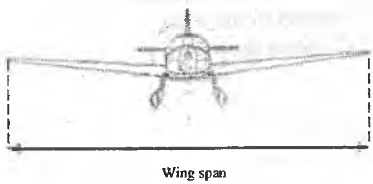
The **wings** primarily produce the lift that is required to overcome the weight. They also house parts of the flight controls may bear the landing gear and fuel tanks as well as other system components.

The **empennage** provides stability and control. Stability is provided by the vertical and horizontal stabilizer and control is executed via the rudder and the elevator(s).

The **propulsion** system produces the thrust, that is required to overcome the drag in forward motion and can be a piston engine or gas turbine driving a propeller or a jet engine.

### *I.1.2 dimensions of aeroplanes*

The dimensions of an aeroplane, that are required for performance discussions are the **wingspan** b [m] , which is the distance from the left wingtip to the right wingtip



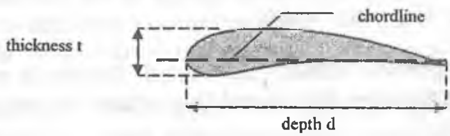

the **wing area** or surface S [$m^2$] , which includes the area in the fuselage

the **aspect ratio** $\Lambda$ [-] , which prescribes the ratio of wingspan and wing area by the formula : $\Lambda = \frac{b^2}{S}$

the **taper ratio** [-], which is the ratio of depth at wingtip and depth at wing root $\lambda = \frac{d_{outer}}{d_{inner}}$

$d_{inner}$ $d_{outer}$

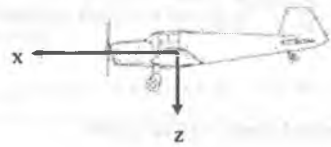
and the wing parameters depth d [m] , thickness t [m] as well as the chord line

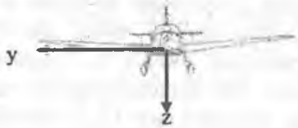



### *I.1.3 axes and coordinates of aeroplanes*

The naming of the axes follows the "three finger rule" of the right hand.
Three systems are established: - the body axes system
the aerodynamic axes system
the geostationary axes system

The **body axes system** is related to the aeroplane itself and has its origin in the centre of gravity.
The longitudinal axes "x" shows forward.
The lateral axes "y" shows to the right.
The vertical axes "z" shows down.




The **aerodynamic axes system** shows to the direction, the airflow comes from.

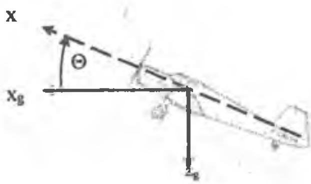


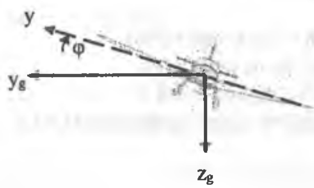
α - angle of attack [ °]                    β - side slip angle [°]

The angle of attack is the angle between the chord line and the airflow direction ($-x_a$).

The side slip angle is the angle between the x-axes and the airflow direction ($-x_a$).

The **geostationary axes system** is used to describe the attitude (position in space) of the airplane.



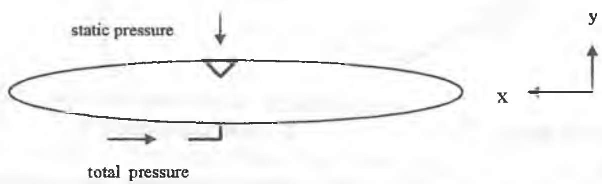
$\Theta$ = pitch angle [ °]
angle of x-axes against the horizon ($x_g$)

$\varphi$ = bank angle [ °]
angle of y – axes against the horizon ($y_g$)

### *I.1.4 Measurement of altitude, vertical speed and airspeed*

It will be shown later, that the air pressure in the atmosphere is a function of altitude (see I.2.1). This allows measurement of the air pressure in flight and indication of the corresponding altitude instead. The vertical speed can be indicated, when the rate of air pressure change is measured. Finally, the dynamic air pressure (see I.2.2) is used to compute the airspeed. The dynamic pressure can be measured as the difference between the total air pressure (see I.2.2) and the static air pressure.

The static air pressure is measured perpendicular to the airflow around the aeroplane and the total air pressure by using a tube with its opening directly into the airflow.




## *I.2 Basics of fluid dynamics*

The basics of fluid dynamics will be introduced and discussed so far, that an understanding of basic flight mechanics can be achieved.

### *I.2.1 Air and its variables of state*

Air is a gas mixture that contains

**78 % nitrogen ($N_2$)**
**21 % oxygen ($O_2$)**
**1 % Argon (Ar)**
**0,03 % carbon dioxide ($CO_2$)**

as well as hydrogen ($H_2$) and the noble gases Neon (Ne), Helium (He), Krypton (Kr) and Xenon (Xe). In addition, air in nature can contain up to 5 Vol % of water vapour.

The variables of state of air, that are of importance in this script, are density, pressure, temperature, speed of sound, viscosity and compressibility.

**Density**

The volume V is the three dimensional space, which is filled with a substance of the mass m. If the variables of state of this substance are constant, its volume is direct proportional to its mass.

The specific volume is the volume, related to a unit of mass of this substance.

$$v = \frac{V}{m} \qquad \left[\frac{m^3}{kg}\right]$$

and indicates, how much space is required for the substance per that unit of mass.

The reciprocal of the specific volume is the density ρ,

$$\rho = \frac{m}{V} \qquad \left[\frac{kg}{m^3}\right]$$

which indicates, how much mass of a substance is contained in a unit of volume.

The density has a dominating influence on aeroplane performance; however, it cannot be measured directly. This must be done using the general gas equation, which will be introduced shortly.

**Pressure**

Pressure is the force, that acts perpendicular to any surface.

$$pressure = \frac{force}{area} \qquad p = \frac{F}{A} \qquad \left[\frac{N}{m^2}\right]$$

In case of air pressure, this is the weight of an air mass over a reference surface.

$$p = \frac{W}{A}$$

**W** : weight [N]

with $W = m \cdot g$

**m** : mass of the substance [kg]

**g** : gravitational acceleration [m/s$^2$]

The formula for the atmospheric pressure can be derived as follows:

$$p = \frac{W}{A} = \frac{m \cdot g}{A} = m \cdot \frac{g}{A} = \rho \cdot V \cdot \frac{g}{A} = \rho \cdot g \cdot \frac{V}{A} = \rho \cdot g \cdot h$$

(basic equation of hydrostatics)

This equation, however, cannot be used to determine the static pressure in the free atmosphere, as its upper limit, from where the measurement would have to be taken, cannot be defined clearly due to the transition from the atmosphere to the "empty" space. It would also be impracticable, as in aviation height means the distance over the earth or sea. In addition, the density is not constant and varies with height.

Nevertheless, this equation demonstrates the origin for air pressure in the atmosphere clearly and explains its qualitative trend.

A formula, which is useful for aviation purposes, will be introduced under I.3.3.

**Temperature**

The temperature of a substance indicates the kinetic energy of its molecules. If, with decreasing temperature, they do no longer possess any kinetic energy, the absolute zero point is reached. Referred to the ice point of water, the absolute zero point is -273,15°C. This is the zero point of the absolute temperature scale, which is scaled with the unit Kelvin K. Celsius temperature t and absolute temperature T use identical spread. With $T_0$ = 273 K, the readings can be transferred using

$t = T - T_0$.

**Speed of sound**

Propagation of sound occurs in longitudinal waves with increasing and decreasing pressure. The speed of propagation solely depends on the characteristics of the transport medium. The relevant characteristics in a gas are:

isentropic exponent **κ**
the specific gas constant **R**
the absolute temperature **T**

The speed of sound a is calculated with the formula:

$$a = \sqrt{\kappa \cdot R \cdot T} \qquad \left[\frac{m}{s}\right]$$

The Mach number is the ratio between the actual speed of an object and the actual speed of sound:

$$M = \frac{v}{a} \qquad [-]$$

An aeroplane, that is operated with the Mach number technique has a speed of

$$v = M \cdot a \qquad \left[\frac{m}{s}\right].$$

For an aeroplane operated in the low speed regime, Mach number related effects need not be considered and therefore are not presented here.

**Viscosity**

The viscosity of a fluid indicates its capability to resist deformation. The most important factor influencing viscosity is temperature. At higher temperatures the exchange of impulses perpendicular to the direction of main flow increases energy exchange and thus viscosity. In fluid dynamics the viscosity at a certain density of the fluid is used and called kinematic viscosity

$$\nu = \frac{\eta}{\rho} \quad \left[\frac{m^2}{s}\right]$$

**Compressibility**

Any gas, that is exposed to an increase in pressure and cannot divert, is compressed. Its volume decreases. This compression does not influence its variables of state, as long as its velocity is low against the speed of sound. For flight mechanical matters one has agreed to consider air as incompressible up to a Mach number of M = 0,3.

**Specific gas constant**

For all homogenous substances, a certain relation for the variables of state pressure p, temperature T and density ρ exists. Their mathematical linkage is called thermal equation of state. For gases at moderate pressures below 100 bar the relation of $\frac{p}{\rho \cdot T}$ is constant for different values and called specific gas constant **R**. A gas, for which this relation applies, is called ideal gas. For air, that can be considered as an ideal gas in flight mechanics, the specific gas constant is **R** = 287 J/kg K.

**Isentropic and polytrophic exponent**

The change of the variables of state of a gas during a thermal or mechanic process is called a change of state, which may happen with or without dissipation. For example, a process of compression or decompression of a gas can be described by the law of Poisson:

$$p \cdot v^n = const.$$

If there are no losses due to thermal transfer of energy over the system boundary, **n** is called the isentropic exponent κ, which stands for a fully reversible process. In real systems, there are always losses, and n, the poly-tropic exponent, will be smaller than κ, meaning a gas, that is compressed and expanded to the former volume will have a lower pressure after the process, due to thermal losses and friction in the apparatus.

For air the following values apply: **κ** = 1,4 **n** = 1,235

### I.2.2 Dynamic and total pressure

If any object is exposed to an airflow, this airflow will come to a complete stop at the very forward edge of that structure, before it diverts downstream. This point, where the surface is exposed perpendicular to the streamlines, is called stagnation point. In the stagnation point the velocity of the flow is zero. The kinetic energy of the molecules, however, cannot dissipate and increases the pressure in the stagnation point. The amount of pressure increase caused by that deceleration depends on the velocity of the undisturbed airflow and its density and is called dynamic pressure q.

$$q = \frac{\rho}{2} v^2$$

As static pressure p always acts into all directions and is present anywhere at any time, it must be added to the dynamic pressure even in the stagnation point. The resulting pressure is the total pressure $p_t$.

$$p_t = q + p$$

As it is possible to measure the total and the static pressure by probes in an aeroplane (see I.1.4), the dynamic pressure q can be found through:

$$q = p_t - p$$

With the dynamic pressure the airspeed then can be computed as:

$$v = \sqrt{\frac{2q}{\rho}}$$

### I.2.3 Equations of continuity and Bernoulli

One of the basic equations of fluid dynamics is the equation of continuity. It states, that in any mass flow, no substance can appear or disappear.

For a tube, in which a fluid moves, it means, that the mass flow is the same at any point. Consequently, the velocity has to change, when the diameter changes. In a large diameter, the velocity is low and vice versa.

$\dot{m} = A \cdot v \cdot \rho = const$ (mass flow)

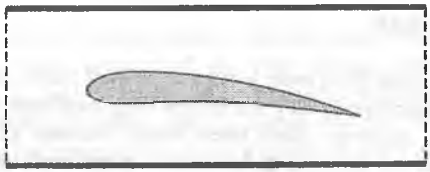
If the density can be regarded as constant, even the volume flow remains constant.

$\dot{V} = A \cdot v = const$ (volume flow)

For any narrowing or expanding tube the relationship of velocities and diameters is:

$$A_1 \cdot v_1 = A_2 \cdot v_2 = A_n \cdot v_n \quad \Rightarrow \quad \frac{v_2}{v_1} = \frac{A_1}{A_2} \quad \Rightarrow \quad v_2 = \frac{A_1}{A_2} \cdot v_1$$

A contraction of a cross section can also be achieved by bringing an obstacle into the flow. Then the fluid has to flow around this object at a higher velocity, following the equation of continuity. An aeroplane or a wing can be such an obstacle in a control room. The air, flowing around such an obstacle or a wing, must increase its velocity.



If an air mass is accelerated, there must be an "energy-store", that can execute the required physical work to accelerate it. In a mechanical pendulum, potential energy is used to accelerate it towards the lowest point of its track, where the kinetic energy reaches a maximum, but the potential energy is zero. In the turning points (the highest points), all energy of the pendulum is potential and there is no kinetics energy in it. The sum of potential and kinetic energy, however, always is constant.

$\mathbf{E_{pot} + E_{kin}}$ = constant

In a moving fluid without friction, the analogy to potential energy is static pressure and to kinetic energy is dynamic pressure. The sum of static and dynamic pressure also is constant at any time:

$\mathbf{p_{stat} + p_{dyn}}$ = constant (equation of Bernoulli)

Whenever the cross section for a moving fluid changes, the fluid – following the equation of continuity – has to change its velocity.

$$p_1 + q_1 = p_2 + q_2 \qquad p_1 + \frac{\rho}{2} v_1^2 = p_2 + \frac{\rho}{2} v_2^2 \qquad p_2 = p_1 + \frac{\rho}{2}\left(v_1^2 - v_2^2\right)$$

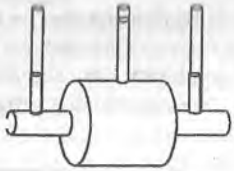
$$v_1^2 - v_2^2 = \frac{2(p_2 - p_1)}{\rho}$$

When the cross section decreases, the velocity must increase. $v_2 = \frac{A_1}{A_2} \cdot v_1$

(continuity)

With $\mathbf{v_2} > \mathbf{v_1}$ the term $(\mathbf{v_1^2 - v_2^2})$ will be negative, and then $p_2$ will be lower than $\mathbf{p_1}$. (Bernoulli)

If a fluid is forced to change its velocity, the static pressure will decrease and the dynamic pressure will increase in an acceleration and vice versa. (continuity plus Bernoulli)



### I.2.4 *Real fluids*

Until here, fluids have been looked at as ideal fluids. Ideal fluids can transfer pressure but no shear forces, they are free of outer and inner shear stress.

Real fluids, however, show shear stress between all molecules moving at different velocities and between these molecules and surfaces.

Shear stress between different molecules, that move at different velocities is called inner friction. Shear stress between moving molecules and a surface is called outer friction. The amount of friction depends on the viscosity of the fluid and the velocity difference in between the molecules or against the surface.

### I.2.5 *Reynolds's number*

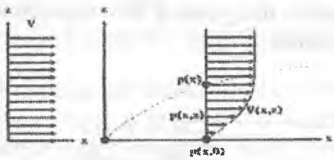
The effect of friction on a fluid moving over a surface depends on the velocity and the travelled distance over the surface. A higher velocity increases the shear gradient close to the surface and a longer distance travelled executes more "stopping force" on the fluid next to it. The transformation of friction within the fluid depends on its viscosity. These different influences are expressed by the "Reynolds's number", which indicates a relationship between inertial and friction effects on a fluid in motion.

$$Re = \frac{v \cdot l}{\nu} \quad [-]$$

The Re-number can be used to compare the status of different flow patterns. One particular fluid with a certain viscosity has the same state, whether it has travelled a short distance over a surface at a high velocity or a long distance at a low velocity.

### I.2.6 *Boundary layer*

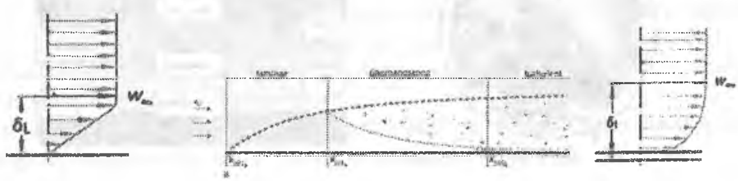
Due to the adherence of fluids to surfaces, the velocity directly on the surface is zero. But at a certain distance from the surface it reaches the velocity of the free and undisturbed flow. The layer, in which this velocity gradient occurs, is called the boundary layer. It is defined to reach from the surface to that distance, where 99 % of the velocity of the free flow are reached.



Due to the a.m. effects of inner friction, the thickness of the boundary layer increases downstream. Some amount of the total energy (static and dynamic pressure) is transferred to thermal energy. As the continuity equation is valid, the velocity and so the dynamic pressure cannot change and thus the static pressure drops independent from the pressure change as a function of the profile thickness.

### *I.2.7 Laminar and turbulent airflow*

Within the boundary layer, the streamlines initially follow a parallel pattern, which is called a laminar flow. As the inner friction adds up with increasing distance travelled, the thickness of the laminar boundary layer increases. Within the layer a discrete particle receives accelerating forces from the next faster layer (outward) and decelerating forces from the next slower layer (inward) and tends to begin a rotating motion. Thus little local turbulences are initiated, by which the particle experiences alternating accelerating and decelerating impulses. Finally the boundary layer destabilizes, changes its state and becomes turbulent. The turbulent flow shows a sharper gradient close to the surface, which causes higher friction drag, it has however, the capability to transfer kinetic energy from the outer and faster zone to the inner and slower zone. The changeover from laminar to turbulent airflow occurs at a certain Reynolds's number, the critical Reynolds's number.



### *I.2.8 Separation of airflow*

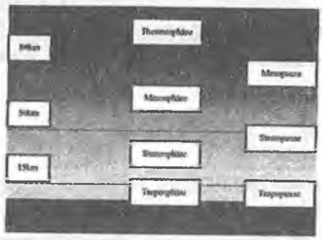
As long as the thickness of an aerofoil increases, the velocity of the fluid has to increase according to the equation of continuity and its static pressure decreases. After the point of maximum thickness, this process is reversal. The velocity decreases and the static pressure increases. As the fluid has to flow "upstream" into an area of rising static pressure, it experiences additional deceleration; depending on the pressure, rise gradient may come to a stop directly on the surface, and then separates from it. Behind this point of flow separation, an area of "dead waters" or even a reversal flow exists.

As the particles in the turbulent boundary layer continuously experience impulses from particles with higher velocity, they tend to maintain their velocity over a longer distance and therefor separate later than those in a laminar boundary layer do.

## I.3 The atmosphere

The earth is surrounded by the atmosphere, that enables life on earth and that can be described by its current status, the weather.

It is permanently out of balance, but tries to regain that status and never reaches it. Therefore a chain reaction is initiated, that never ends. The atmosphere is in permanent motion.



### *I.3.1 Spheres for aviation*

**<u>Vertical structure</u>**

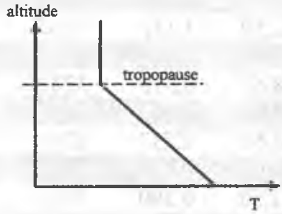
The atmosphere is divided into different spheres, that are characterized by different parameters. For aviation only the lower sphere, the troposphere and the lower part of 2nd lowest sphere, the stratosphere are of concern. The frontier between the troposphere and the stratosphere is called tropopause.

The troposphere reaches from the surface to an average height of **11.000** metres. It is characterised by decreasing temperature with increasing height.

The lower stratosphere is characterised by a temperature constant with height (isotherm ).

Both circumstances affect airplane performance significantly.

The height of the tropopause varies with geographical position and time of the year. It can be as low as **5.000** m over the poles in winter and as high as **18.000** m over the equator.




### *I.3.2 Pressure, temperature, density, speed of sound and viscosity as function of altitude in the troposphere*

Due to the influence of radiation from the sun the temperature and the pressure on the surface of the earth vary as a function of location (latitude, altitude) and the time (daytime, season). Pressure and temperature then have influence on the other already mentioned variables of state: density, speed of sound and viscosity.

### *I.3.3 the International Standard Atmosphere*

In order to create an atmosphere, one can refer to (e.g. to have a basis for actual performance calculations in aeroplane operation or to compare the performance of different aeroplane models), the ISA (international standard atmosphere) was defined.

It names a state for mean sea level and certain gradients for pressure, temperature, density and speed of sound as a function of altitude.

**Data for ISA :**

Pressure in MSL : $p_0 = 1013{,}25$ hPa
Temperature in MSL : $T_0 = 15°$ C = 288,15 K
Density in MSL : $\rho_0 = 1{,}225$ kg / m$^3$
Speed of sound in MSL: $a_0 = 340$ m/s

Overview of ISA from sea level to **11.000** m:

| H[m] | p [hPa] | T [K] | ρ [kg/m³] | a [m/s] | a [km/h] | t [°C] |
|---|---|---|---|---|---|---|
| 0 | 1013,25 | 288,00 | 1,225 | 340,2 | 1225 | 15 |
| 100 | 1001,29 | 287,35 | 1,214 | 339,8 | 1223 | 14 |
| 200 | 989,45 | 286,70 | 1,203 | 339,4 | 1222 | 14 |
| 300 | 977,73 | 286,05 | 1,191 | 339,0 | 1220 | 13 |
| 400 | 966,11 | 285,40 | 1,180 | 338,6 | 1219 | 13 |
| 500 | 954,61 | 284,75 | 1,168 | 338,2 | 1218 | 12 |
| 1000 | 898,75 | 281,50 | 1,112 | 336,3 | 1211 | 9 |
| 2000 | 794,95 | 275,00 | 1,007 | 332,4 | 1197 | 2 |
| 3000 | 701,09 | 268,50 | 0,910 | 328,5 | 1183 | -4 |
| 4000 | 616,40 | 262,00 | 0,820 | 324,5 | 1168 | -11 |
| 5000 | 540,20 | 255,50 | 0,737 | 320,4 | 1153 | -17 |
| 6000 | 471,81 | 249,00 | 0,660 | 316,3 | 1139 | -24 |
| 7000 | 410,61 | 242,50 | 0,590 | 312,1 | 1124 | -30 |
| 8000 | 356,00 | 236,00 | 0,526 | 307,9 | 1108 | -37 |
| 9000 | 307,42 | 229,50 | 0,467 | 303,7 | 1093 | -43 |
| 10 000 | 264,36 | 223,00 | 0,413 | 299,3 | 1077 | -50 |
| 11 000 | 226,32 | 216,50 | 0,364 | 294,9 | 1060 | -56 |

Data for any altitude in the troposphere according to ISA can be found with the following formulas:

$$p = p_0 \cdot (1 - 0{,}02256 \cdot H)^{5{,}256}$$

$$T = T_0 - (6{,}5 \cdot H)$$

$$\rho = \rho_0 \cdot (1 - 0{,}2256 \cdot H)^{4{,}256}$$

$$[H] = km$$

All calculations in this book refer to ISA data, if not stated otherwise!

### I.4 The sample aeroplane for the examples

The sample aeroplane used in this book is a low wing cantilever monoplane with a piston engine, that could look like the one depicted below. It is a tail wheel design with a flapless wing.

| | |
|---|---|
| Maximum mass $\mathbf{m_{max}}$: | 1.000 kg |
| Wing area **S**: | 15 $m^2$ |
| Wingspan **b**: | 10,95 m |
| Aspect ratio **Λ**: | 8 |
| Taper ratio : | not of concern |
| Oswald efficiency factor **e**: | 0,8 |
| Coefficient drag for zero lift $\mathbf{c_{D0}}$: | 0,022 |
| Max coefficient lift $\mathbf{c_L}$: | 1,2/1,5/1,7 (flaps up/take-off, /landing) |
| Engine $\mathbf{P_{max}}$: | 100 kW |
| Engine $\mathbf{n_{max}}$: | 2500 $min^{-1}$ |
| Engine $\mathbf{n_{max\,v=0}}$: | 2200 $min^{-1}$ |
| Engine $\mathbf{n_{max} = f(v)}$ : | $n(v) = (4{,}2857 \cdot v + 2200)\ min^{-1}$ |
| $\mathbf{P_{eng}(n)}$ : | $P(n) = (0{,}1 \cdot n - 150)$ kW |
| Propeller efficiency $\boldsymbol{\eta_{prop} = f(\lambda)}$: | $\eta(\lambda) = -60 \cdot \lambda^2 + 16 \cdot \lambda - 0{,}3$ |
| Propeller diameter: | 2,15 m |
| Altitude factor for engine power $\mathbf{\nu = f(H)}$ : | $\nu(H) = 0{,}000186 \cdot \frac{p_H}{\sqrt{T_H}} - 0{,}11$ (Wedrow) |
| Power specific fuel consumption : | $\mathbf{b_P} = 0{,}000075$ kg / kW / s |

Only the first five factors have been explained until now, but the remaining ones will be discussed in detail in the respective chapter.

## II Performance

### II.1 Forces on the aeroplane

As mentioned earlier in the introduction an aeroplane can be regarded as a mass point for performance considerations. Consequently, all forces, to be discussed hereafter, act on the centre of gravity **c.g.** of the aeroplane.

#### II.1.1 *Weight*

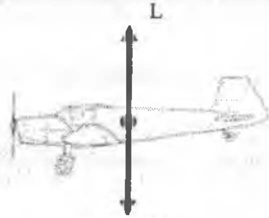
The weight W of any object and so of an aeroplane, whether in flight or on the ground, is resulting from the gravitational acceleration acting on its mass. According to Newton's first axiom (force equals mass times acceleration) the formula for weight reads:

$$W = m \cdot g \qquad \left[kg \cdot \frac{m}{s^2}\right] = [N]$$

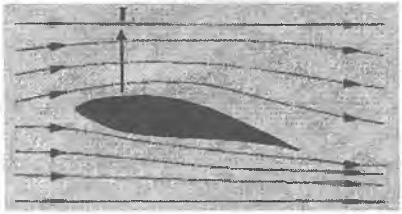
Weight is a vector, acting on the **c.g.**, pointing towards the centre of earth along the $\mathbf{z_g}$-axes and with the length according to the formula. The notation in this book, as for all forces to be introduced, will be in the scalar form, however.


#### *II.1.2 Lift*

To keep an aeroplane in balance along the vertical axes $\mathbf{z_g}$ a force or a component of a force must counteract the weight **W**. This force is the lift **L** [N].



The lift **L** is created by the airflow around the profile of the wing, the aerofoil.




## Generation of lift:

When the airflow passes around the aerofoil, its velocity has to increase according to the equation of continuity. At the increased velocity, the dynamic pressure increases and with the total pressure remaining constant according to the equation of Bernoulli the static pressure has to decrease. This pressure drop generates a difference in pressure between the upper and the lower side of the aerofoil, and a pressure difference acting on a surface causes a force perpendicular to it, which in case of the wing of an aeroplane is lift

$$p = \frac{F}{A} \quad \Rightarrow \quad F = p \cdot A \quad \left[\frac{N}{m^2} \cdot m^2\right] = [N]$$

The pressure drop as a function of velocity increase shall be demonstrated and a formula shall be developed now.

To be independent of specific wing parameters like wingspan and depth the pressure change shall be expressed as a relative change against a reference pressure. As it is undisturbed by the aerofoil and easy to determine, the dynamic pressure of the free airflow is selected as the reference pressure:

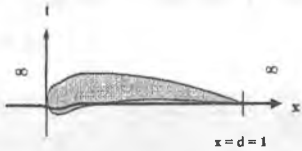
$$\frac{\textit{static pressure change}}{\textit{free dynamic pressure}}$$

As that static pressure change is a pressure drop ($p_x < p_\infty$), it must have a negative prefix.

$$p_x - p_\infty < 0$$

Index x is indicating a position between the leading and the trailing edge of the aerofoil

Index $\infty$ is indicating a position in the free airflow before or behind the aerofoil




$$\Rightarrow \quad \frac{static\ pressure\ change}{free\ dynamic\ pressure} = \frac{p_x - p_\infty}{q_\infty}$$

The total pressure is constant along the depth of the profile:

$$p_{tx} = p_{t\infty}$$

$$q_x + p_x = q_\infty + p_\infty$$

$$q_x = \frac{\rho}{2} v_x^2 \qquad q_\infty = \frac{\rho}{2} v_\infty^2$$

$$\frac{q_x}{q_\infty} = \frac{v_x^2}{v_\infty^2}$$

$$q_x = q_\infty \cdot \frac{v_x^2}{v_\infty^2}$$

$$p_x - p_\infty = q_\infty - q_x$$

$$p_x - p_\infty = q_\infty - q_\infty \cdot \frac{v_x^2}{v_\infty^2}$$

$$p_x - p_\infty = q_\infty \cdot \left(1 - \frac{v_x^2}{v_\infty^2}\right)$$

$$\frac{p_x - p_\infty}{q_\infty} = 1 - \left(\frac{v_x^2}{v_\infty^2}\right)$$

The a.m. ratio $\frac{static\ pressure\ change}{free\ dynamic\ pressure}$ then is: $\frac{p_x - p_\infty}{q_\infty} = 1 - \frac{v_x^2}{v_\infty^2}$

As aimed for, this relationship is independent of any specific profile data except **t/d** (thickness over depth) which causes $\mathbf{v_x}$, and it is called "pressure coefficient" $\mathbf{c_p}$.

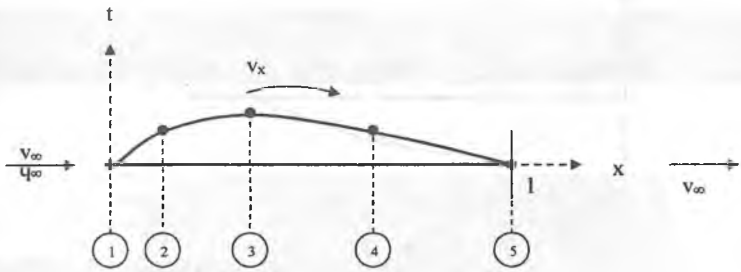
$$c_p = 1 - \frac{v_x^2}{v_\infty^2}$$

With $\mathbf{v_x}$ as the only variable, that changes permanently over the aerofoil ($\mathbf{v_x = f(x)}$), $\mathbf{c_p}$ can be either found for any position x over the aerofoil ($\mathbf{0 \le x \le d=1}$) or the average $\mathbf{c_p}$ ($\bar{c}_p$) for the entire aerofoil can be calculated by determining $\mathbf{c_p}$s in infinite small steps, add them up and divide the sum by the profile depth d (mathematical integration).

$$\bar{c}_p = \frac{1}{d}\int_0^d c_p \, dx$$

To demonstrate this calculation method, a simplified example shall be used initially.

The sample aerofoil is curved on its upper side only and very flat on its lower side, the angle of attack **α** shall be 0°.




1 : at the leading edge (**x** = **0**) the velocity decreases to zero in the stagnation point

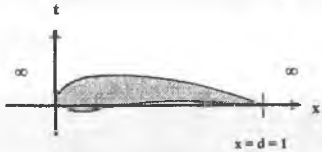
2 : behind the stagnation point, with increasing thickness **t** the velocity increases rapidly and reaches $\mathbf{v_x}$ only after a short distance

3 : the velocity further increases and reaches its maximum at the point of maximum profile thickness

4 : after the point of maximum thickness the velocity decreases with the decrease of profile thickness

5 : at the trailing edge the velocity reaches the velocity of the undisturbed airflow $\mathbf{v_\infty}$

$\mathbf{c_p}$ over the aerofoil consequently varies as follows :




$$\Rightarrow \quad \frac{\textit{static pressure change}}{\textit{free dynamic pressure}} = \frac{p_x - p_\infty}{q_\infty}$$

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$$\frac{q_x}{q_\infty} = \frac{v_x^{\ 2}}{v_\infty^{\ 2}}$$

$$q_x = q_\infty \cdot \frac{v_x^{\ 2}}{v_\infty^{\ 2}}$$

$$p_x - p_\infty = q_\infty - q_x$$

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$$p_x - p_\infty = q_\infty \cdot (1 - \frac{v_x^{\ 2}}{v_\infty^{\ 2}})$$

$$\frac{p_x - p_\infty}{q_\infty} = 1 - \left( \frac{v_x^{\ 2}}{v_\infty^{\ 2}} \right)$$

The a.m. ratio $\dfrac{\textit{static pressure change}}{\textit{free dynamic pressure}}$ then is: $\dfrac{p_x - p_\infty}{q_\infty} = 1 - \dfrac{v_x^{\ 2}}{v_\infty^{\ 2}}$

As aimed for, this relationship is independent of any specific profile data except **t/d** (thickness over depth) which causes $\mathbf{v_x}$, and it is called "pressure coefficient" $\mathbf{c_p}$.

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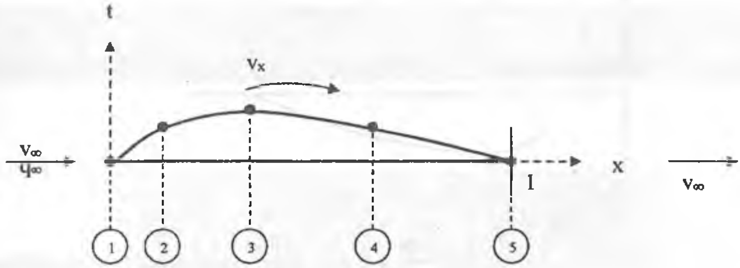


1 : at the leading edge (**x** = **0**) the velocity decreases to zero in the stagnation point

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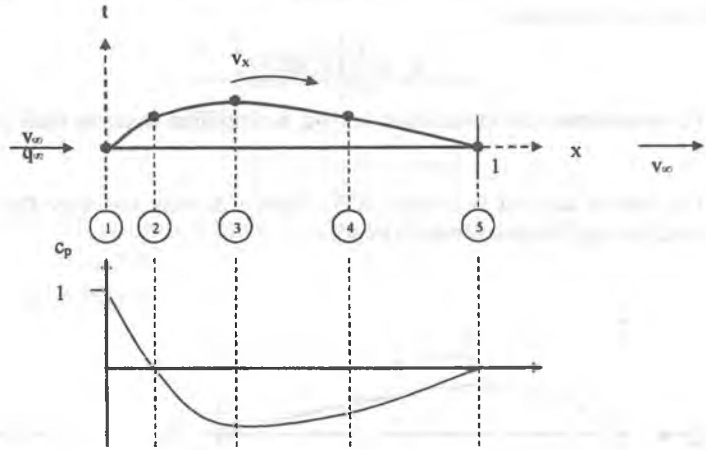
3 : the velocity further increases and reaches its maximum at the point of maximum profile thickness

4 : after the point of maximum thickness the velocity decreases with the decrease of profile thickness

5 : at the trailing edge the velocity reaches the velocity of the undisturbed airflow $\mathbf{v_\infty}$

$\mathbf{c_p}$ over the aerofoil consequently varies as follows :




1 : $v_x = 0 \quad \Rightarrow \quad c_p = 1 - \frac{0}{v_\infty^2} = 1$ (static pressure equals total pressure)

2 : $v_x = v_\infty \quad \Rightarrow \quad c_p = 1 - \frac{v_\infty^2}{v_\infty^2} = 0$ (static pressure like in free airflow)

3 : $v_x = N \cdot v_\infty \quad \Rightarrow \quad c_p = 1 - \frac{N \cdot v_\infty^2}{v_\infty^2} = 1 - N$ (static pressure reaches lowest value)

4 : $v_x = n \cdot v_\infty \quad \Rightarrow \quad c_p = 1 - \frac{n \cdot v_\infty^2}{v_\infty^2} = 1 - n$ (static pressure gradually increases)

5 : $v_x = v_\infty \quad \Rightarrow \quad c_p = 1 - \frac{v_\infty^2}{v_\infty^2} = 1 - 1 = 0$ (static pressure like in free airflow)

Between **1** and **2** the $\mathbf{c_p}$ is greater than zero, which according to $c_p = \frac{p_x - p_\infty}{q_\infty}$ means, that the static pressure $\mathbf{p_x}$ is higher than $\mathbf{p_\infty}$. After passing **2**, however, $\mathbf{p_x}$ is lower than $\mathbf{p_\infty}$ and reaches its lowest value at **3**, before it slowly increases via **4** to the static pressure of the free airflow at the trailing edge in **5**.

So, between **1** and **2**, there is an overpressure condition, creating a down force, whereas between **2** and **5** there is under pressure creating an up force.

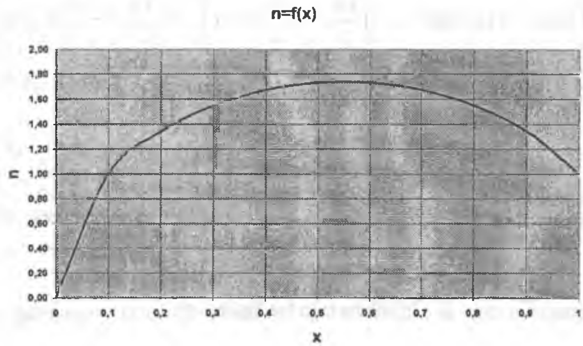
The coefficient pressure $\mathbf{c_p}$ can be computed if $\mathbf{v_x = f(x)}$ is known.

In this example the following simplified equation for $\mathbf{v_x}$ shall be assumed:

$$v_x = \sqrt{v_\infty^{\,2}(-10x^2 + 11x)} \qquad \text{with } \mathbf{0 \le x \le d=1}$$

The speed distribution over the aerofoil in terms of $n \cdot \mathbf{v_\infty}$ then is:

| x [-] | 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n [-] | 0,000 | 1,000 | 1,342 | 1,549 | 1,673 | 1,732 | 1,732 | 1,673 | 1,549 | 1,342 | 1,000 |




With $v_x = \sqrt{v_\infty^{\,2}(-10x^2 + 11x)}$ $\mathbf{c_p}$ is:

$$v_x^{\,2} = v_\infty^{\,2} \cdot (-10x^2 + 11x)$$

$$c_p = 1 - \frac{v_x^{\,2}}{v_\infty^{\,2}} = 1 - \frac{v_\infty^{\,2} \cdot (-10x^2 + 11x)}{v_\infty^{\,2}}$$

$$c_p = 1 - (-10x^2 + 11x)$$

$$c_p = 10x^2 - 11x + 1$$

$\mathbf{c_p = f(x)}$ can be computed with the above equation.

The average $\mathbf{c_p}$ over the entire depth of the aerofoil can be found through:

$$\bar{c}_p = \frac{1}{d} \cdot \int_0^d \left(10x^2 - 11x + 1\right) dx$$

It could be seen, that $c_p = f(x)$ changes its prefix in point 2, so the integration has to be split, from $x = 0$ to $x_2$ ($c_p = 0$) and from $x_2$ to $x_5 = 1 = d$.

To find $x_{2/5}$, the $p - q$ – formula can be applied to the equation of $c_p = f(x)$:

$$c_p = 0 = 10x^2 - 11x + 1$$

$$x^2 - \frac{11}{10}x + \frac{1}{10} = 0$$

$$x_{2/5} = \frac{11}{20} \pm \sqrt{\frac{121}{400} - \frac{40}{400}} = \frac{11}{20} \pm \sqrt{\frac{81}{400}} = \frac{11}{20} \pm \frac{9}{20}$$

$$x_2 = 0{,}1 \qquad x_5 = 1$$

The prefix changes at $x = 0{,}1$.

$$\bar{c}_p = \frac{1}{1} \cdot \int_0^1 \left(10x^2 - 11x + 1\right) dx = \left[\frac{10}{3}x^3 - \frac{11}{2}x^2 + x\right]_0^{0,1} + \left[\frac{10}{3}x^3 - \frac{11}{2}x^2 + x\right]_{0,1}^{1} = -1{,}167$$

The lift itself can now be found as follows:

$$\bar{c}_p = 1 - \frac{v_x^2}{v_\infty^2} = \frac{p_x - p_\infty}{q_\infty} = \frac{\Delta p}{q_\infty} \qquad \Rightarrow \qquad \Delta p = \bar{c}_p \cdot q_\infty$$

The average coefficient pressure is called "coefficient lift" $c_L$ [-] $\Delta p = c_L \cdot q_\infty$

With force $F = \Delta p \cdot S$ lift then can be found as:

$$L = \Delta p \cdot S = c_L \cdot q_\infty \cdot S$$

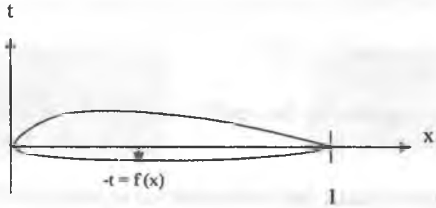
$$L = c_L \cdot \frac{\rho}{2} v_\infty^2 \cdot S \quad [N]$$

The lift can be calculated, if the wing area S and the density ρ are known. S is taken from the sample aeroplane with $S = 15\ m^2$ and the density from mean sea level, which is $\rho = 1{,}225\ kg/m^3$. Any speed can be taken, for example $v_\infty = 100$ m/s.

$$L = 1{,}167 \cdot \frac{1{,}225}{2} \cdot 100^2 \cdot 15 \left[\frac{kg\ m^2\ m^2}{m^3\ s^2}\right] \qquad L = 107\,218\ N$$

Remember: the details of the prefix for $c_L$ will be discussed shortly. For uplift is has been chosen positive here!

In the next step, the entire profile including upper and lower side will be considered.




All the statements, that have been made for the upper side also apply for the lower side, except $\mathbf{v_x = f\ (x)}$ is adhering to a different formula due to the different contour on the lower side $\mathbf{-t = f(x)}$.

For the lower side the following sample formula applies:

$$v_x = \sqrt{v_\infty{}^2 \cdot (-5x^2 + 6x)}$$

With it $\mathbf{c_p}$ reads:

$$v_x{}^2 = v_\infty{}^2 \cdot \left(-5x^2 + 6x\right)$$

$$c_p = 1 - \frac{v_\infty{}^2 \cdot \left(-5x^2 + 6x\right)}{v_\infty{}^2} = 1 - \left(-5x^2 + 6x\right)$$

$$c_p = 5x^2 - 6x + 1$$

Using the same calculation method as before the $\mathbf{c_L}$ can be found:

$$c_L = \bar{c}_p = \frac{1}{d} \cdot \int_0^{d=1} \left(5x^2 - 6x + 1\right) dx$$

$\mathbf{c_p}$ is zero at $\mathbf{x_2} = 0{,}2$ and $\mathbf{x_5} = 1$.

$$c_L = \bar{c}_p = \left[\frac{5}{3}x^3 - 3x^2 + x\right]_0^{0,2} + \left[\frac{5}{3}x^3 - 3x^2 + x\right]_{0,2}^{1} = -0{,}146$$

To find the $\mathbf{c_L}$ (which is the average $\mathbf{c_p}$) for the entire aerofoil, and thus representing the pressure difference between upper and lower side, the $\mathbf{c_p}$s for both sides are added as follows:

$$c_L = \frac{1}{d} \cdot \int_0^d \left(c_{p\ lower\ side} - c_{p\ upper\ side}\right) dx$$

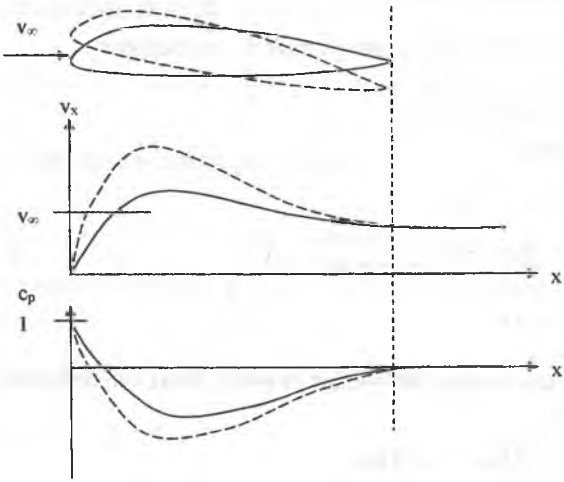
which in the example is :

$$c_L = -0{,}146 - (-1{,}167) = 1{,}021$$

Remember: by sequencing the coefficients in this equation the problem with prefixes can be overcome!

So far the angle of attack has been assumed as zero, $\alpha = 0°$.

If the angle of attack is increased ($\alpha > 0°$) , the velocity- and $c_p$ - patterns the over the aerofoil change (only shown for the upper side here).

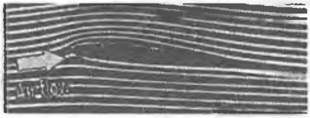



With a higher $\alpha$ the velocity increase after the stagnation point is sharper and the velocity reaches a higher value, thus producing a larger drop of static pressure.

The reverse happens on the lower side. The velocity increase after the stagnation point flattens and the maximum velocity, that is reached, decreases – the drop in static pressure decreases.

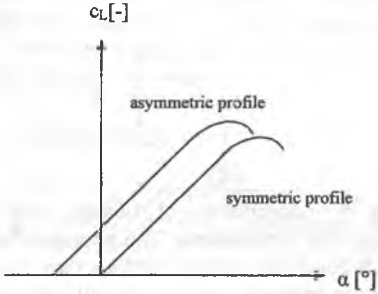
Summing up both phenomena, the $c_L$ increases with angle of attack:
$c_L = f(\alpha)$

It has, however, to be noticed, that the airflow over the aerofoil enters into a region of steadily increasing static pressure behind the point of maximum thickness. At a certain high angle of attack this positive pressure gradient is so steep, that the airflow can no longer overcome it, and the airflow separates from the wing surface, causing a sharp velocity drop and $c_p$ decrease.



There is one certain angle of attack for every profile that allows a maximum $\mathbf{c_L}$. Any further increase in $\boldsymbol{\alpha}$ sharply reduces $\mathbf{c_L}$.

On a symmetric profile, there is no pressure difference between the upper and the lower side with $\boldsymbol{\alpha} = 0°$. On a non-symmetric profile (convex to the upper side), there is already a pressure difference for $\boldsymbol{\alpha} = 0°$, the $c_L$ follows these conditions accordingly.




Another meaning of the coefficient lift can be found with the following transformation of the lift formula:

$$L = c_L \cdot \frac{\rho}{2} v^2 \cdot S = c_L \cdot q \cdot S$$

+

$$c_L = \frac{L}{q \cdot S}$$

The coefficient lift is the ratio of the created lift versus the generating factors dynamic pressure and wing surface and thus is showing the effectiveness of an aerofoil in producing lift.

### II.1.3 *drag*

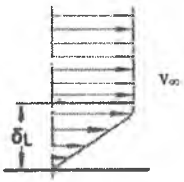
An aeroplane in its forward motion experiences drag. Drag is a force acting against its cause of origin and tends to stop the object in motion. Referring to a flying aeroplane it directs along the negative (backward) $x_a$ – axes.

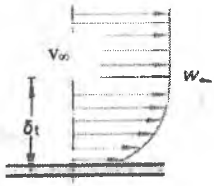

The total drag, that an aeroplane experiences, is the sum of various drag components.

profile drag

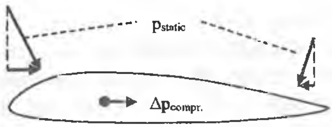
In this immediate context, anybody, that is exposed to an airflow, is considered a profile. Any profile in an airflow experiences friction and compressive drag.

The friction drag results from the shear stress in the boundary layer and is low in a laminar boundary layer and high in a turbulent layer due to the velocity gradients. At high velocities, the shear gradient and so the friction increases.



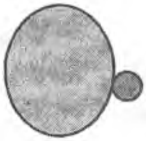

Compressive drag is a consequence of friction in the boundary layer, which shows a downstream increase in thickness. The boundary layer then displaces the streamlines of the flow around the profile, and thus they have to maintain a higher velocity with a lower static pressure. The sum of all local pressures normal to the surface then has a component showing downstream with the airflow, and this pressure component multiplied with the projection surface facing opposite the flow creates a force, that is also directed downstream, the compressive drag.




The sum of friction drag and compressive drag is the profile drag.

**<u>Interference drag:</u>**

Different components of an aeroplane influence the airflow individually. Superposition of these individual flow patterns leads to local areas of additionally increased velocity and pressure distributions, which increase friction and profile drag.

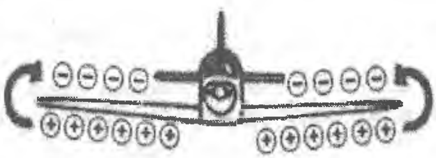



Friction, profile and interference drag exist, whenever an aeroplane is in motion, whether lift is created or not. For that reason, their sum is called "parasite drag" :

Profile drag + interference drag = parasite drag

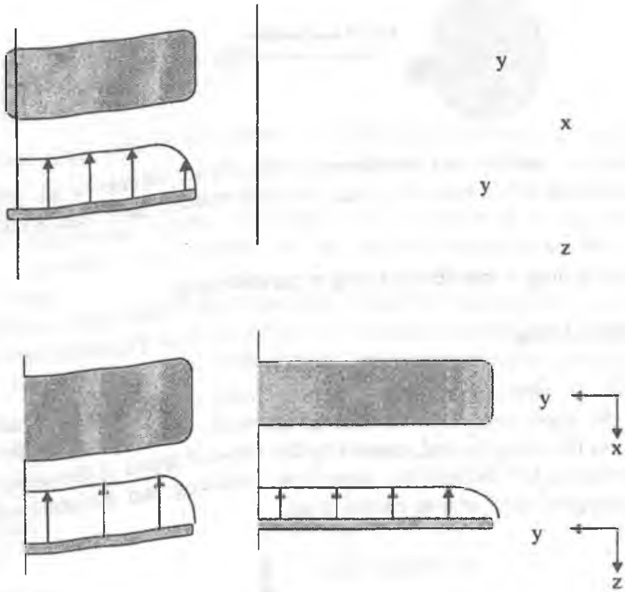
Induced drag:

Lift, as already explained, results from a difference in static pressure between the upper and the lower side of the wing. This pressure difference tries to equalize via the wing tip and, caused by the forward speed of the aeroplane, creates a vortex that is left behind the wing tips. Initiating and maintaining this vortex requires physical work and so causes drag.

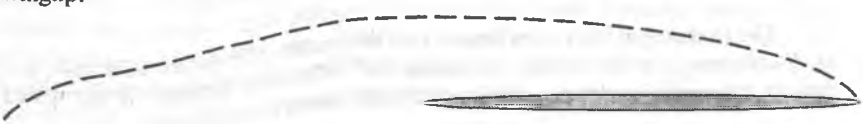


The intensity of the vortex depends on the amount of pressure difference and its distribution over the wing. The amount of differential pressure is expressed through the coefficient lift $c_L$ , a high coefficient causes a strong vortex.

The distribution of the pressure difference over the wing surface can be indicated through the local lift **L = f (x,y)**. If the local lift is low at the wingtip, the vortex is of lower intensity. This can be achieved by either reducing the surface S towards the wingtip with a high taper ratio, changing the aerofoil to a lower chamber outwards or by increasing the aspect ratio.




An ideal lift distribution over the wingspan would be of elliptic shape with the maximum lift at the centre of the wing decreasing to zero exactly at the wingtip.



This ideal, however, can only be approached and not be reached. The "Oswald efficiency factor" e [-] indicates, to which degree this ideal pressure distribution has been achieved in an actual design. For the sample aeroplane of this book, **e** reads 0.8, which is in the typical range.

As a result, the induced drag is low with a low $c_L$ and a high **e** and $\Lambda$, induced drag $D_i = f(c_L, e, \Lambda)$.

The total drag of an aeroplane then is:

$$\left.\left.\left.\begin{array}{c}\text{friction drag}\\+\\\text{compressive drag}\end{array}\right\}\begin{array}{c}\text{profile drag}\\+\\\text{interference drag}\end{array}\right\}\begin{array}{c}\text{parasite drag}\\+\\\text{induced drag}\end{array}\right\}\text{total drag}$$

To be independent of absolute aeroplane data and to be able to compare different designs, the coefficient drag **$c_D$** [-] has to be introduced according to the $c_L$. It is used in the same manner to calculate the drag.

$$D = c_D \cdot \frac{\rho}{2} v_\infty^{\ 2} \cdot S$$

The **$c_D$** is split to represent the parts of parasite and induced drag. These components are called "coefficient drag zero lift" **$c_{D0}$** and "coefficient induced drag" **$c_{Di}$**.

$$c_D = c_{D0} + c_{Di}$$

The formula for **$c_{Di}$** reads: $\quad c_{Di} = \dfrac{c_L^{\ 2}}{e \cdot \pi \cdot \Lambda} \quad$ (with $\pi$ for geometric reasons)

$$\Rightarrow \quad D = \left( c_{D0} + \frac{c_L^{\ 2}}{e \cdot \pi \cdot \Lambda} \right) \cdot \frac{\rho}{2} v_\infty^{\ 2} \cdot S$$

According to the **$c_L$** the **$c_D$** also indicates, how much drag a profile generates in relation to its surface and the existing dynamic pressure.

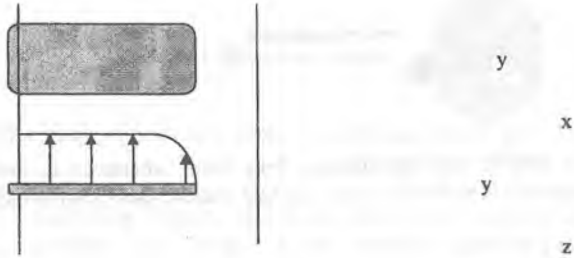
$$D = c_D \cdot q \cdot S$$

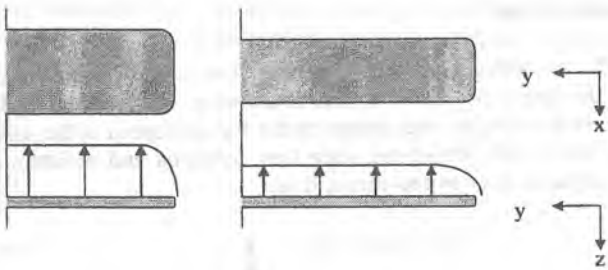
$$c_D = \frac{D}{q \cdot S}$$

### II.1.4 *Thrust*

The force driving the aeroplane forward into the positive **$x_a$** – direction and thus overcomes drag is thrust **T** [N].

Thrust is a force and generated by acceleration of an air mass through a propulsion system, which either may be a propeller, driven by a piston engine or gas turbine or a jet engine.

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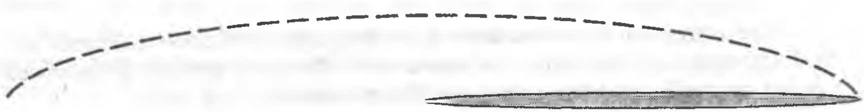

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induced drag $\mathbf{D_i = f\ (c_L, e, \Lambda)}$.

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### 11.1.4 *Thrust*

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Thrust is a force and generated by acceleration of an air mass through a propulsion system, which either may be a propeller, driven by a piston engine or gas turbine or a jet engine.

The acceleration of an air mass corresponds to a change of its impulse.

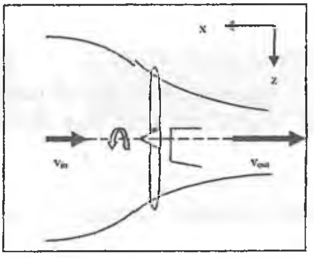
*impuls*: $I = m \cdot v$

*change of impuls*: $\Delta I = m \cdot \Delta v$

*Newton's $2^{nd}$ axiom*: $F = m \cdot a = m \cdot \frac{\Delta v}{\Delta t} = \frac{\Delta I}{\Delta t}$

*thrust T*: $T = \frac{\Delta I}{\Delta t} = \frac{m \cdot \Delta v}{\Delta t} = \frac{m}{\Delta t} \cdot \Delta v = \dot{m} \cdot (v_{out} - v_{in})$

The thrust is the air mass, that flows through the propulsion system per time multiplied with its acceleration within the system.




According to the equation of continuity the diameter of the propeller stream tightens up with increasing velocity.

For an aeroplane, that does not move (**$v_{in}$** = 0), the "static thrust" equals the impulse of the airstream out.

$$T_0 = \dot{m} \cdot v_{out}$$

**$v_{out}$** depends on the propeller load (thrust/area) [N/m$^2$] and the density of the air [kg m$^3$].

$$v_{out} = \sqrt{\frac{T_0 / S_P}{\rho}}$$

As there is no heating of the airflow in a propeller propulsion system, the density of the air remains constant and equals the density outside the system: $\rho_{in} = \rho_{out} = \rho_{\infty}$

also the mass flow is constant : $\dot{m}_{out} = \dot{m}_{in}$

mass flow : $\dot{m} = A \cdot v \cdot \rho \quad \left[\frac{kg}{s}\right]$

$\mathbf{v_{in}}$ equals the flight speed : $\mathbf{v} = \mathbf{v}_{\mathbf{in}}$

$$\Rightarrow \quad T = \dot{m} \cdot (v_{out} - v)$$

It is typical for propeller propulsion systems to work with a comparable high mass flow and a comparable low $\mathbf{v_{out}}$. The power transferred from an engine to the airflow via the propeller corresponds to the increased kinetic energy of the airflow per time.

$$P = \frac{W}{t} = \frac{m}{2}v^2 \cdot \frac{1}{t} = \frac{m}{2t} \cdot v^2$$

As mass in that equation is a factor with exponent **1** and velocity a factor with exponent **2**, propeller systems using high mass flow rather than high velocities can work with comparable low power. The comparable low velocity of the airstream as compared to a jet engine, however, enables comparable low speeds of the aeroplane only.

## II.2 Lift and drag as function of speed

### *II.2.1 Lift as a function of speed*

For an aeroplane in flight the lift L always counteracts weight W and consequently changes, when this weight changes, which normally happens slowly by burning fuel or suddenly by dropping loads of any kind. For performance considerations, however, the lift can be considered as constant for a limited period of time.

The parameters, that are elements of the lift formula $L = c_L \cdot \frac{\rho}{2} v^2 \cdot S$, may change, however.

Here the surface S and the altitude (density) shall be considered as constant ($S, \rho$ = const.).

If the airspeed of the aeroplane is changed, the last remaining factor $\mathbf{c_L}$ has to be changed as well. Increasing speed allows a decreasing coefficient lift and vice versa. As the coefficient lift also is a function of the angle of attack $\boldsymbol{\alpha}$, it also can be stated, that a slowing aeroplane has to increase its angle of attack and an accelerating may lower it. It has already been mentioned, that for any aerofoil a maximum $\alpha$ exists, generating a maximum $\mathbf{c_L}$, $\mathbf{c_{Lmax}}$. Hence, for any aeroplane there is a minimum possible airspeed, where it reaches its maximum $\boldsymbol{\alpha}$ and $\mathbf{c_L}$.

The lift formula can be transferred to indicate the required or possible airspeed in horizontal flight as a function of $c_L$.

$$L = c_L \cdot \frac{\rho}{2} v^2 \cdot S$$

$$L = W = m \cdot g$$

$$m \cdot g = c_L \cdot \frac{\rho}{2} v^2 \cdot S$$

$$v = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot c_L \cdot S}}$$

At this point, the term wing load can be easily introduced. Wing load is the mass of the aeroplane per unit of wing surface.

$$wingload = \frac{mass}{unit\ wingsurface} \quad \left[\frac{kg}{m^2}\right]$$

With the wing load introduced, the a.m. formula can be written:

$$v = \sqrt{\frac{2 \cdot g}{\rho \cdot c_L} \cdot \frac{m}{S}}$$

The wing load directly indicates of what kind an aeroplane is concerning its operating speeds. The required airspeeds increase with the factor $\sqrt{m/S}$, so a heavy aeroplane or one with a small wing surface is a high speed aeroplane and vice versa.

The sample aeroplane will be used in the following calculation examples of speeds as a function of lift coefficient **v = f ($c_L$).**

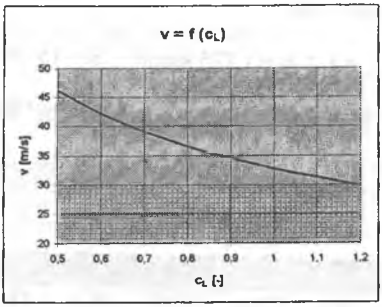
With a constant mass at a constant altitude (density) various angles of attack and so lift coefficients will be assumed and the speeds that belong to these flight conditions will be looked at.

conditions: **m** = 1000 kg **ρ** = 1,225 kg/m³ **S** = 15 m²

$c_L$ = 0,5 / 0,6 / 0,7 / 0,8 / 0,9 / 1,0 / 1,1 / 1,2

formula: $v = \sqrt{\frac{2 \cdot g \cdot m}{\rho \cdot S}} \cdot \frac{1}{\sqrt{c_L}} \, m/s = \sqrt{\frac{2 \cdot 9{,}81 \cdot 1000}{1{,}225 \cdot 15}} \cdot \sqrt{\frac{1}{c_L}} \, m/s$

| $c_L$ [-] | 0,5 | 0,6 | 0,7 | 0,8 | 0.9 | 1 | 1,1 | 1,2 |
|---|---|---|---|---|---|---|---|---|
| v [m/s] | 46,2 | 42,2 | 39,1 | 36,5 | 34,4 | 32,7 | 31,2 | 29,8 |





high speed low $\alpha$,$c_L$ | medium speed medium $\alpha$, $c_L$ | minimum speed maximum $\alpha$, $c_L$

The above image shows a slowing aeroplane in horizontal flight at constant altitude ( density) (left to right), that has to increase its $c_L$ ($\alpha$) continuously with decreasing speed until the minimum speed is reached at the maximum possible angle of attack.

The speed $v$ = 29,8 m/s at $c_{Lmax}$ = 1,2 is the lowest, this aeroplane can fly with flaps up. So, the minimum speed an aeroplane is capable to fly is achieved, when it is operated at the maximum possible angle of attack $\alpha_{max}$, which results in $c_{Lmax}$.

$$\Rightarrow \quad v_{min} = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot S} \cdot \frac{1}{c_{L\,max}}}$$

The formula and the graph show, that the relationship between $c_L$ and $v$ is nonlinear :

$$v = \sqrt{\frac{2 \cdot g \cdot m}{\rho \cdot S}} \cdot \sqrt{\frac{1}{c_L}} = K \cdot \sqrt{\frac{1}{c_L}}$$

Another decrease in required/possible speeds, however, is possible, when the mass of the aeroplane is reduced, which can be achieved during the design process or in operation by reducing the load and/or fuel.

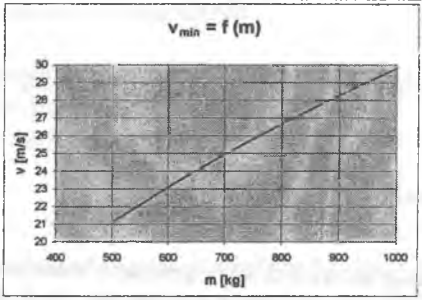
The example is continued:

conditions : $c_{L\,max} = 1{,}2$  $\rho = 1{,}225\ kg/m^3$    $S = 15\ m^2$

$m$ = 500 kg / 600 kg / 700 kg / 800 kg / 900 kg / 1000 kg

formula : $$v_{min} = \sqrt{\frac{2 \cdot g}{\rho \cdot c_{L\,max} \cdot S}} \cdot \sqrt{m}\ \ {}^{m}\!/_{s}$$

| m [kg] | 500 | 600 | 700 | 800 | 900 | 1000 |
|---|---|---|---|---|---|---|
| v min [m/s] | 21,1 | 23,1 | 25,0 | 26,7 | 28,3 | 29,8 |




The below image shows an aeroplane at different masses in horizontal flight with constant $c_L$ (or $\alpha$), that has to vary its speed.


high mass
high speed

medium mass
medium speed

low mass
low speed

### *II.2.2 Drag as function of speed*

The relationship of drag and speed is more complicated and will be demonstrated now.

It has been shown, that the coefficient drag $c_D$ must be divided into "coefficient drag zero lift" $c_{D0}$ and the "coefficient induced drag" $c_{Di}$. The "coefficient drag zero lift" can be considered as constant here, and consequently the "drag at zero lift" rises with speed in the 2nd exponent.

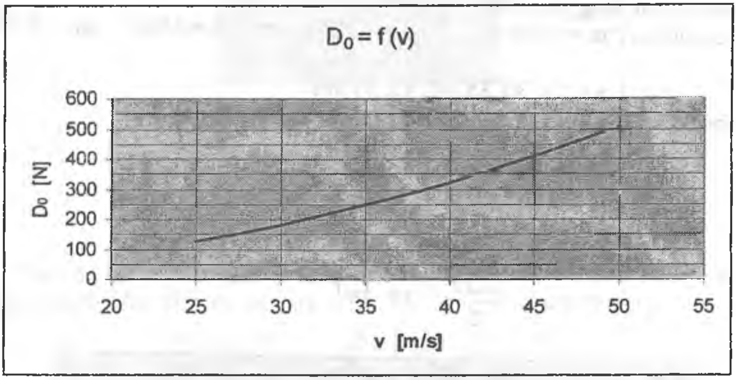
conditions : m = 1000 kg   $\rho$ = 1,225 kg/m³   S = 15 m²   $c_{D0}$ = 0,022

v = 25, 30, 35, 40, 45, 50 m/s

formula : $$D_0 = 0{,}022 \cdot \frac{1{,}225}{2} \cdot 15 \cdot v^2 \quad [N]$$

| v [m/s] | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|
| D [N] | 126 | 182 | 248 | 323 | 409 | 505 |

The double speed results in a four times higher drag, caused by $v^2$!




The rise of drag zero lift as a function of speed can be clearly seen. To achieve a better imagination of these numbers, it can be said, that the drag zero lift for the sample single engine aeroplane at 50 m/s, which equals 180 km/h, is 500 N, which is pretty exactly the force, that is needed to lift and hold a mass of 50 kg. At 108 km/h (30 m/s) it is only a little less than 200 N, which equals a force required to lift and hold nearly 20 kg, which can be easily done by any person for some time!

The "coefficient induced drag" $c_{Di}$ is not constant with speed. An earlier presented formula has explained its dependence on coefficient lift, and the interaction of $c_L$ with v has been shown in the foregoing chapter. A high speed means a low coefficient lift and so a low $c_{Di}$ , a low speed means a high coefficient lift and a high $c_{Di}$.

$$c_{Di} = \frac{c_L^{\,2}}{K}$$


The depicted aeroplane at a low airspeed and high angle of attack requires a high pressure difference between upper and lower side of the wing, which causes a strong vortex and high induced drag.

The induced drag of the sample aeroplane shall be computed under the same conditions as the drag zero lift:

conditions : **m** = 1000 kg  $\rho$ = 1,225 kg/m³  S = 15 m²  $c_{D0}$ = 0,022

v = 25, 30, 35, 40, 45, 50 m/s

formula :

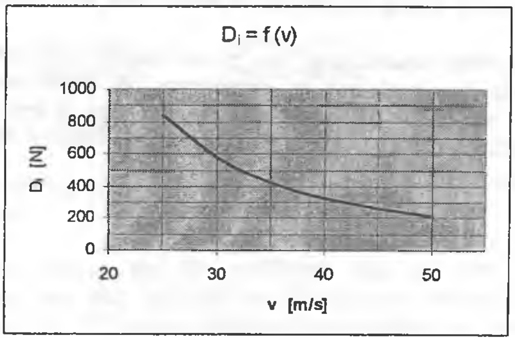
$$c_{Di} = \frac{c_L^{\,2}}{e \cdot \pi \cdot \Lambda}$$

$$c_L = \frac{2 \cdot g}{\rho \cdot v^2} \cdot \frac{m}{S}$$

$$D_i = c_{Di} \cdot \frac{1{,}225}{2} v^2 \cdot 15 \quad [N]$$

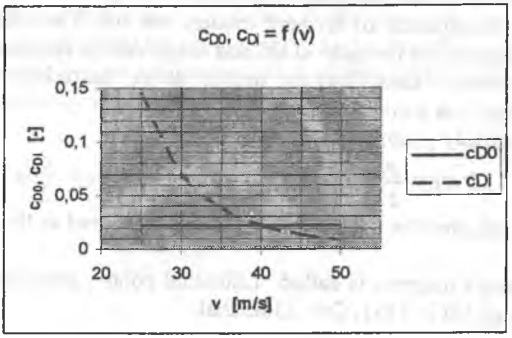
| v [m/s] | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|
| $c_L$ [-] | 1,708 | 1,186 | 0,872 | 0,667 | 0,527 | 0,427 |
| $c_{Di}$ [-] | 0,1452 | 0,0700 | 0,0378 | 0,0222 | 0,0138 | 0,0091 |
| $D_i$ | 834 | 579 | 425 | 326 | 257 | 208 |

$c_L$ > 1,2 are only theoretic values, as $c_{Lmax}$ = 1,2 for the sample aeroplane !



The decrease of induced drag can be clearly seen. The decrease of the coefficient induced drag $c_{Di}$ is even much sharper. The drag decrease is slower, as it is dependent on $v^2$.

To illustrate this, $c_{D0}$ and $c_{Di}$ shall be shown in one diagram as function of speed.

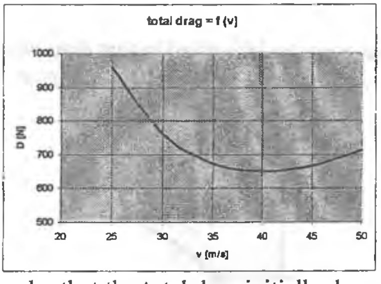



$c_{D0}$ and $c_{Di}$ are equal at about 40 m/s speed.

Finally, the total drag as a function of speed can be calculated. It is the sum of drag at zero lift and induced drag.

$$D = D_0 + D_i \qquad D = (c_{D0} + c_{Di}) \cdot \frac{\rho}{2} v^2 \cdot S$$

| v [m/s] | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|
| D [N] | 960 | 761 | 673 | 649 | 667 | 714 |




It can be seen clearly, that the total drag initially decreases with speed due to the strongly decreasing induced drag. Then a minimum is reached in a moderate speed regime, before the drag rises due to the swallowing further decrease of induced drag and the stronger rise of parasite drag.

**This dependence of drag from speed has a significant influence on the entire performance of any aeroplane!**

### *II.2.3 Lift and drag*

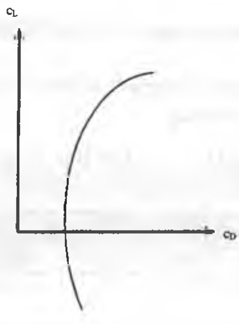
Lift is required for an aeroplane to fulfil the transport task. Drag, however, is an unavoidable consequence of forward motion and lift. The effectiveness of an aeroplane then depends on the ratio of lift and drag. As the required lift is constant over a limited period of time (fuel use neglect able), the ratio of lift and drag is best, when the drag is at a minimum.

From the already given formulas for lift and drag

$$L = c_L \cdot \frac{\rho}{2} \cdot v^2 \cdot S \qquad \textit{and} \qquad D = c_D \cdot \frac{\rho}{2} \cdot v^2 \cdot S$$

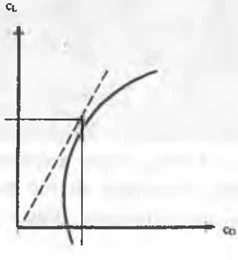
it can be seen, that the ratio **L/D** can also be expressed as the ratio $\mathbf{c_L / c_D}$.

The respective diagram is called "Lilienthal polar", after the first man, who flew using dynamic lift in 1891, Otto Lilienthal.



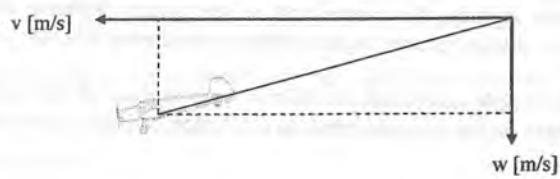

One important point can be identified in that diagram:

The best ratio $c_L$ / $c_D$ is achieved, where the tangent from the origin touches the graph.



The increase in $c_D$ at very low $c_L$s is caused by slightly negative angles of attack on cambered profiles. So, the a.m. formula $c_D = f(c_L)$ should not be used for very low $c_L$s ($\leq 0{,}2$) at very high speeds of an particular aeroplane.

The speed, that must be flown to achieve this best ratio can be calculated from the $c_L$. The ratio $c_L / c_D$ is also called the gliding number **E**. It can be easily imagined for an unpowered aeroplane, that has to give up altitude (potential energy) to overcome the drag in a gliding flight. Then the lost potential energy, that was used to maintain speed, corresponds to the drag, while the travelled distance reflects the lift. An aeroplane with a gliding number of e.g. **E** = 20 so can travel 20 m distance by losing 1 m altitude.




$$\frac{v}{w} = \frac{L}{D} = E$$

## II.3 Propulsion system

### *II.3.1 Non charged piston engine*

A non-charged piston engine shall be assumed for the sample aeroplane of this book. In such an engine the output power depends on the torque [Nm] and the speed [$min^{-1}$]. The torque depends on the average pressure in the cylinders during the working stroke, which acts on the surface of the piston $(pressure \cdot area = force)$ and this force acts via the arm of the crankshaft$(force \cdot arm = torque)$. The average pressure in the cylinder for a given engine and a given throttle position solely depends on the density of the air, that is sucked in. As the density of the air is decreasing with altitude, the torque and so the power decrease with increasing altitude. A non-charged piston engine cannot compensate this loss of density with altitude.

For the sample aeroplane the following formulas will be used to calculate the output power on the propeller shaft as a function of engine speed and altitude.

$$P(n) = (0{,}1 \cdot n - 150)kW$$

$$P(H) = P(n) \cdot \nu$$

$$\nu = 0{,}000186 \cdot \frac{p}{\sqrt{T}} - 0{,}11$$

**v** as a factor clearly reflects the trends of static pressure **p** and temperature **T** with altitude. A decreasing pressure reduces the density and the available power, a decreasing temperature, however, increases the density and the available power.

A sample calculation shall be executed:

Engine speed = 2300 $min^{-1}$

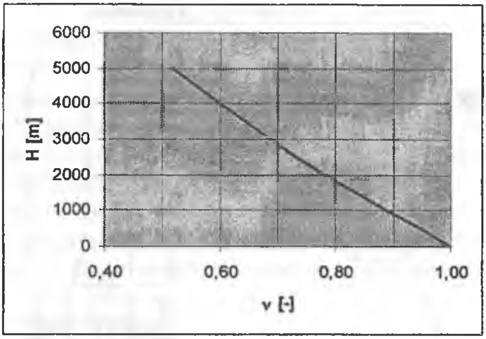
$P = (0{,}1 \cdot 2300 - 150)\ kW = 80\ kW$

**H** = 1000 m **p** = 89875 Pa **T** = 281,5 K

$$\nu = 0{,}000186 \cdot \frac{89875}{\sqrt{281{,}5}} - 0{,}11 = 0{,}886$$

$$P = 80\ kW \cdot 0{,}886 = 70{,}9\ kW$$

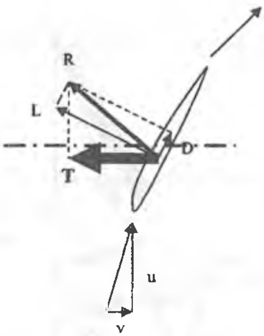
The sample engine delivers 70,9 kW power at 2300 $min^{-1}$ and 1000 m altitude.




The diagram shows **v** as a function of altitude from mean sea level to 5000 m.

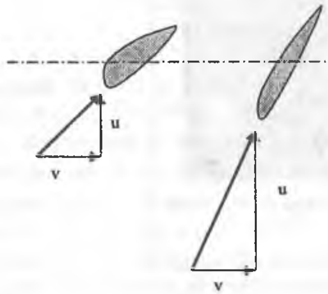
*II.3.2 Propeller*

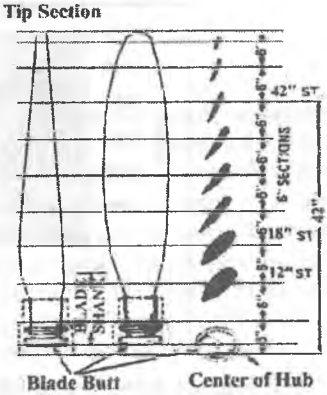
A propeller can be compared with a wing, that is rotating around the x-axes of the aeroplane or the propeller's own axes. The propeller blade, like the wing, develops lift and experiences drag. The lift acts as forward thrust, and the drag must be overcome by the engine torque.




The downwash angle behind the propeller blade increases the velocity component along the aeroplane longitudinal axes, and thus the airflow experiences an acceleration in negative x direction.

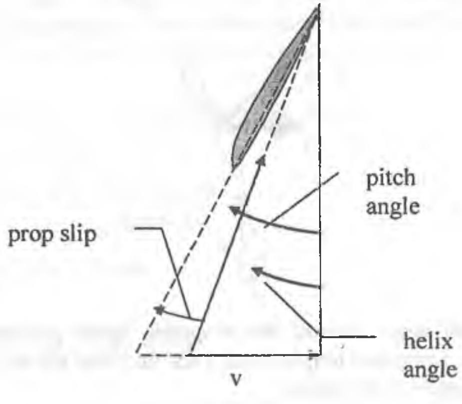
The airflow direction and velocity ahead of the propeller blade is vectorial added from the flight speed of the aeroplane and the circumferal speed of the propeller, the latter of which increases with distance from the propeller hub.




To achieve an uniform blade angle of attack against the airflow, the blade is twisted, the angle of incidence is lowered from the hub to the tip. The angle of incident at r = 0,7 R has been chosen to be representative for propellers.

If a propeller could be screwed through a solid, the distance, that it would travel forward, is called the propeller pitch. But as the propeller needs an angle of attack, the helix angle, which is the angle it travels forward against the air, is smaller than the pitch angle. The difference between the pitch- and the helix – angle is called the propeller slip.




The actual forward distance per propeller revolution is called the progress rate λ.

$$\lambda = \frac{flight\ speed}{circumferal\ speed} = \frac{v}{u}\quad [-]$$

As the progress rate also differs with distance from the hub, the value at 0,7 R is considered to be representative for any propeller.

As a wing has an optimum angle of attack, that produces maximum lift with minimum drag, a propeller blade has a certain progress rate, which gives most thrust with least drag, which has to overcome by engine torque. Consequently, the power given from the propeller to the aeroplane in relation to that given from the engine to the propeller, the propeller efficiency $\eta_P$ [-], is a function of the progress rate.

$$\eta_P = \frac{thrust \cdot flight\ speed}{torque \cdot engine\ speed} = \frac{power\ given\ to\ aeroplane}{engine\ power\ to\ propeller}$$

For the sample aeroplane in this book the propeller efficiency as a function of progress rate can be calculated as:

$$\eta_P(\lambda) = -60 \cdot \lambda^2 + 16\lambda - 0{,}3$$

The progress rate can be found as follows:

$$\lambda = \frac{v}{u} \qquad u = \frac{U \cdot n}{60} = \frac{2{,}15m \cdot 2 \cdot \pi \cdot 0{,}7 \cdot n}{60 \cdot s} = 0{,}158 \cdot n \frac{m}{s}$$

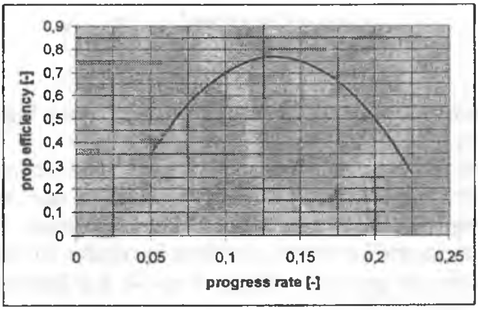
$$\lambda = \frac{v}{0{,}158 \cdot n} [-]$$

The range of the progress rate reaches from max engine speed static to 2500 $min^{-1}$ at top speed.

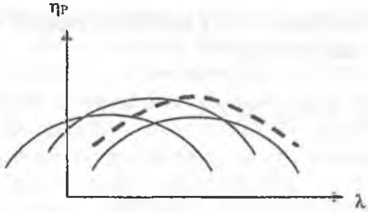
$$\lambda = \frac{0}{0{,}158 \cdot 2200} = 0 \qquad \lambda = \frac{100}{0{,}158 \cdot 2500} = 0{,}2532$$

The top speed of 100 m/s is an estimate only at this time and will be verified later!

With the given formula the propeller efficiency as a function of progress rate is:




With a variable pitch propeller, a better range of propeller efficiency could be achieved, because the variable angle of incidence could be used to stay closer to the optimum progress rate over a wider speed range.




In this book, however, only a fixed (pitch) propeller will be assumed for the sample aeroplane.

### *II.3.3 Thrust*

With the a.m. mentioned formula the thrust from the propulsion system can be calculated by multiplying the output engine power, which is a function of engine speed and density with the propeller efficiency as a function of progress rate and dividing the product with flight speed.

$$thrust = \frac{engine\ power \cdot propeller\ efficiency}{flight\ speed}$$

*e.g.*

$$T = \frac{80kW \cdot 0{,}7}{56\frac{m}{s}} = \frac{56000\ Nm \cdot s}{56\ m \cdot s} = 1000\ N$$

$$P(n,H) = (0{,}1 \cdot n - 150)kW \cdot \left(0{,}000186 \cdot \frac{p_H}{\sqrt{T_H}} - 0{,}11\right)$$

### *II.3.4 Specific fuel consumption*

The specific fuel consumption ***SFC*** is defined as the mass of fuel that is needed by an engine to develop 1 kW of output power for the time span of 1 second.

$$SFC = \frac{fuel\ mass}{power\ unit \cdot time\ unit} = \frac{kg}{kW \cdot s}$$

The ***SFC*** is needed for range and time calculations, where the required power in a certain flight condition can be calculated. With that result and the ***SFC*** the distance or the time, that can be flown with a given mass of fuel can be computed.

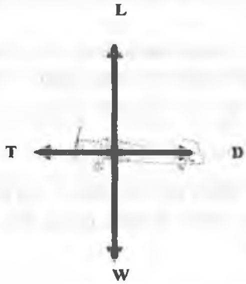
Although the ***SFC*** at the same power output varies with the required throttle setting, a standard ***SFC*** will be used in this book for simplification, which only depends on the delivered power.

A typical piston engine consumes 200 g/ hp / h, which is a ***SFC*** of 0,075 g / kW / s.

## II.4 Horizontal flight

### *II.4.1 Forces in uncelebrated horizontal flight*

Horizontal flight means, that an aeroplane maintains its altitude on a straight flight path, which requires lift and weight to be equal in size and opposite in direction. If a horizontal flight in addition is uncelebrated, thrust and drag also must be of the same amount and as well opposite in direction.




As in horizontal flight lift and weight always have the same amount, they will not be discussed in this chapter. The drag of an aeroplane, however, is subject of various parameters that will be examined on their influence now.

These parameters can be easily identified by looking at the drag formula.

$$D = c_D \cdot \frac{\rho}{2} v^2 \cdot S$$

$$D = (c_{D,0} + c_{D,i}) \cdot \frac{\rho}{2} v^2 \cdot S$$

$$D = \left( c_{D,0} + \frac{c_L^{\ 2}}{e \cdot \pi \cdot \Lambda} \right) \cdot \frac{\rho}{2} v^2 \cdot S$$

$$D = \left( c_{D,0} + \left( \frac{2 \cdot m \cdot g}{\rho \cdot v^2 \cdot S} \right)^2 \cdot \frac{1}{e \cdot \pi \cdot \Lambda} \right) \cdot \frac{\rho}{2} v^2 \cdot S$$

The operational variables in that formula are speed v, density ρ (representing altitude) and mass m. The influence of speed on drag has already been shown in II.2.2

The following will examine the influence of density and mass on the drag of an aeroplane.

### *II.4.2 Drag as a function of mass*

On the previous side it has been shown, that the coefficient lift rises linear with mass, if the speed is unchanged. The $c_L$ itself is an element of $c_{Di}$ and acting in the second potency. The other factor $c_{D0}$, however, is not depending on mass.

Calculation method:

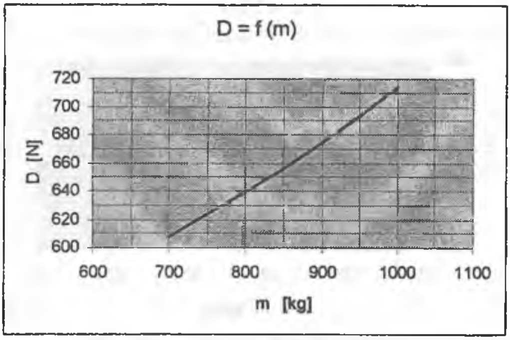
**m** = 700, 800, 900, 1000 kg

**v** = const = 50 m/s, **ρ** = const. = 1,225 kg/m³, **S** = 15 m², **e** = 0,8, **Λ** = 8, $c_{D0}$ = 0,022

(from sample aeroplane)

$$c_L = \frac{2 \cdot m \cdot g}{\rho \cdot v^2 \cdot S} \qquad c_D = c_{D,0} + \frac{c_L^{\;2}}{e \cdot \pi \cdot \Lambda} \qquad D = c_D \cdot \frac{\rho}{2} v^2 \cdot S$$

| m [kg] | 700 | 800 | 900 | 1000 |
|---|---|---|---|---|
| D [N] | 607 | 639 | 674 | 714 |




As a result it can be seen, that the drag at a constant speed and altitude rises with mass.

The next question is, whether there exists a certain speed for minimum drag at differing masses at a constant altitude.

Calculation method:

**m** = 700, 800, 900, 1000 kg

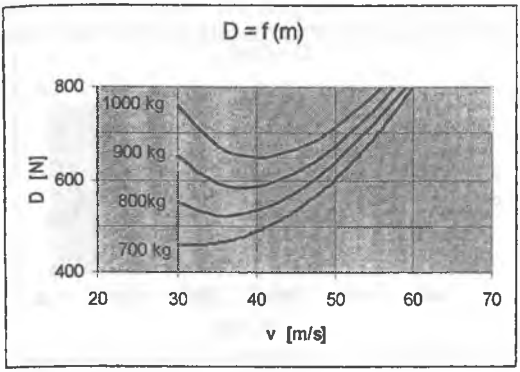
**v** = 30, 40, 50, 60, 70, 80, 90, 100 m/s

$\rho = const. = 1{,}225\ kg/m^3$, $S = 15\ m^2$, $e = 0{,}8$, $A = 8$, $c_{D0} = 0{,}022$
(from sample aeroplane)

$$D = (c_{D,0} + \frac{c_L^{\,2}}{e \cdot \pi \cdot \Lambda}) \cdot \frac{\rho}{2} v^2 \cdot S$$

The change of mass is reflected by the change of $c_L$, so $c_L$ and **v** are the variables in this consideration. As $c_L$ is decreasing with increasing **v**, there must be a minimum for the product of $c_L^2$ and $v^2$.

| **v [m/s]** | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---|---|---|---|---|---|---|---|
| **D 700 kg [N]** | 466 | 456 | 483 | 535 | 607 | 696 | 799 |
| **D 800 kg [N]** | 553 | 520 | 532 | 574 | 639 | 722 | 820 |
| **D 900 kg [N]** | 651 | 592 | 587 | 618 | 674 | 751 | 845 |
| **D1000 kg [N]** | 761 | 673 | 649 | 667 | 714 | 784 | 872 |




The table and the diagram show, that the drag rises with mass as well as the speed to achieve a minimum drag for a given mass.

The next question is how to find that speed, which causes a minimum drag for any actual mass.

In horizontal flight, the lift remains constant for a given mass at any speed. As the speed for minimum drag is searched, the ratio of lift over drag **L/D** must be at a maximum.

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{c_L \cdot \frac{\rho}{2} v^2 \cdot S}{c_D \cdot \frac{\rho}{2} v^2 \cdot S}\right)_{max} \quad \Rightarrow \quad \left(\frac{L}{D}\right)_{max} = \left(\frac{c_L}{c_D}\right)_{max}$$

In II.2.3 it has been shown in the "Lilienthal polar", that there is one $c_L$ (where the tangent from the origin touches the polar), that grants maximum coefficient lift over coefficient drag. Consequently, there is one special $c_L$, that, at constant lift, gives the least drag in horizontal flight for all masses. It can be simply found with a table of $c_L$ and $c_D$. From experience, this $c_L$ should be found in the range between 0,5 and 0,8.

The ratio $c_L / c_D$ is called the gliding number **E**.

$$c_D = c_{D,0} + \frac{c_L^2}{e \cdot \pi \cdot \Lambda} = 0{,}022 + \frac{c_L^2}{e \cdot \pi \cdot \Lambda}$$

| $c_L$ [-] | 0,5 | 0,55 | 0,6 | 0,65 | 0,7 | 0,75 | 0,8 |
|---|---|---|---|---|---|---|---|
| $c_D$ [-] | 0,0344 | 0,0371 | 0,0399 | 0,0430 | 0,0464 | 0,0500 | 0,0538 |
| **E** | 14,52 | 14,84 | 15,03 | 15,11 | 15,09 | 15,00 | 14,86 |

The table shows, that the best ratio $c_L$ over $c_D$ is achieved with an $c_L$ of 0,65 (the exact value may differ slightly).

Now this $c_L$ can be used to find the speeds, that for different masses give the speeds for minimum drag.

$$v = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot 0{,}65 \cdot S}} \qquad \rho = 1{,}225 \text{ kg m}^{-3}, \quad S = 15 \text{ m}^2$$

$$\frac{v_1}{v_2} = \sqrt{\frac{\frac{2 \cdot m_1 \cdot g}{\rho \cdot c_L \cdot S}}{\frac{2 \cdot m_2 \cdot g}{\rho \cdot c_L \cdot S}}} = \sqrt{\frac{m_1}{m_2}} \quad \Rightarrow \quad v_2 = v_1 \cdot \sqrt{\frac{m_2}{m_1}}$$

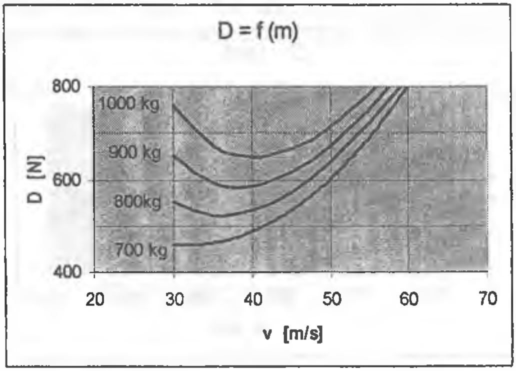
Due to the direct relationship between angle of attack and $c_L$ a pilot can use one certain angle of attack, which in level flight corresponds to one pitch angle **Θ**, to achieve flight with minimum drag.

$\rho$ = const. = 1,225 kg/m³, $S = 15$ m², $e = 0{,}8$, $\Lambda = 8$, $c_{D0} = 0{,}022$
(from sample aeroplane)

$$D = (c_{D,0} + \frac{c_L^{\,2}}{e \cdot \pi \cdot \Lambda}) \cdot \frac{\rho}{2} v^2 \cdot S$$

The change of mass is reflected by the change of $c_L$, so $c_L$ and **v** are the variables in this consideration. As $c_L$ is decreasing with increasing **v**, there must be a minimum for the product of $c_L^2$ and $v^2$.

| v [m/s] | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---|---|---|---|---|---|---|---|
| **D 700 kg [N]** | 466 | 456 | 483 | 535 | 607 | 696 | 799 |
| **D 800 kg [N]** | 553 | 520 | 532 | 574 | 639 | 722 | 820 |
| **D 900 kg [N]** | 651 | 592 | 587 | 618 | 674 | 751 | 845 |
| **D1000 kg [N]** | 761 | 673 | 649 | 667 | 714 | 784 | 872 |




The table and the diagram show, that the drag rises with mass as well as the speed to achieve a minimum drag for a given mass.

The next question is how to find that speed, which causes a minimum drag for any actual mass.

In horizontal flight, the lift remains constant for a given mass at any speed. As the speed for minimum drag is searched, the ratio of lift over drag **L/D** must be at a maximum.

$$\left(\frac{L}{D}\right)_{max} = \left(\frac{c_L \cdot \frac{\rho}{2} v^2 \cdot S}{c_D \cdot \frac{\rho}{2} v^2 \cdot S}\right)_{max} \quad \Rightarrow \quad \left(\frac{L}{D}\right)_{max} = \left(\frac{c_L}{c_D}\right)_{max}$$

In II.2.3 it has been shown in the "Lilienthal polar", that there is one $c_L$ (where the tangent from the origin touches the polar), that grants maximum coefficient lift over coefficient drag. Consequently, there is one special $c_L$, that, at constant lift, gives the least drag in horizontal flight for all masses. It can be simply found with a table of $c_L$ and $c_D$. From experience, this $c_L$ should be found in the range between 0,5 and 0,8.

The ratio $c_L / c_D$ is called the gliding number **E**.

$$c_D = c_{D,0} + \frac{c_L^2}{e \cdot \pi \cdot \Lambda} = 0{,}022 + \frac{c_L^2}{e \cdot \pi \cdot \Lambda}$$

| $c_L$ [-] | 0,5 | 0,55 | 0,6 | 0,65 | 0,7 | 0,75 | 0,8 |
|---|---|---|---|---|---|---|---|
| $c_D$ [-] | 0,0344 | 0,0371 | 0,0399 | 0,0430 | 0,0464 | 0,0500 | 0,0538 |
| E | 14,52 | 14,84 | 15,03 | 15,11 | 15,09 | 15,00 | 14,86 |

The table shows, that the best ratio $c_L$ over $c_D$ is achieved with an $c_L$ of 0,65 (the exact value may differ slightly).

Now this $c_L$ can be used to find the speeds, that for different masses give the speeds for minimum drag.

$$v = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot 0{,}65 \cdot S}} \qquad \rho = 1{,}225 \text{ kg m}^{-3}, \quad S = 15 \text{ m}^2$$

$$\frac{v_1}{v_2} = \sqrt{\frac{\frac{2 \cdot m_1 \cdot g}{\rho \cdot c_L \cdot S}}{\frac{2 \cdot m_2 \cdot g}{\rho \cdot c_L \cdot S}}} = \sqrt{\frac{m_1}{m_2}} \quad \Rightarrow \quad v_2 = v_1 \cdot \sqrt{\frac{m_2}{m_1}}$$

Due to the direct relationship between angle of attack and $c_L$ a pilot can use one certain angle of attack, which in level flight corresponds to one pitch angle $\Theta$, to achieve flight with minimum drag.

### *II.4.3 Drag as a function of altitude*

Here it shall be examined how the drag varies as a function of speed and altitude, respectively, how the minimum drag and the speed to achieve minimum drag vary with altitude.

Altitude is not an explicit factor in the calculations and will be represented by density of air.

Calculation method:

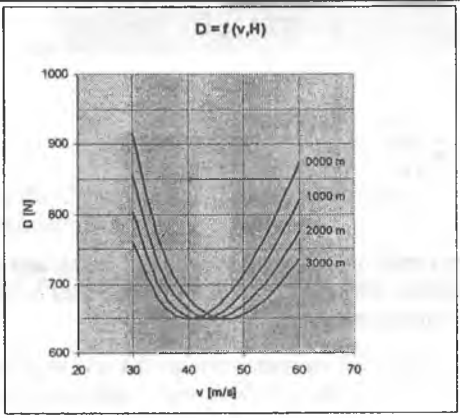
Altitudes: 0 / 1000 / 2000 / 3000 m resp. densities 1,225 / 1,112 / 1,007 / 0,910 $kgm^{-3}$

**v** = 30 ÷ 60 m/s **S** = 15 $m^2$, **e** = 0,8, **Λ** = 8, $c_{D0}$ = 0,022 **m** = 1000 kg

$$D = (c_{D,0} + \frac{c_L^{\,2}}{e \cdot \pi \cdot \Lambda}) \cdot \frac{\rho}{2} v^2 \cdot S$$

The mass is “hidden” in the $c_L$, the density stands for the altitude.

| v [m/s] | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---|---|---|---|---|---|---|---|
| $D_{0\,m}$ | 761 | 673 | 649 | 667 | 714 | 784 | 872 |
| $D_{1000\,m}$ | 803 | 693 | 652 | 655 | 688 | 745 | 820 |
| $D_{2000\,m}$ | 854 | 721 | 662 | 650 | 669 | 712 | 774 |
| $D_{3000\,m}$ | 915 | 757 | 679 | 651 | 656 | 686 | 735 |



The diagram shows clearly, that the minimum drag is the same at all altitudes, however, the speed to achieve a flight with minimum drag, increases with altitude.

$$D_1 = c_{D1} \cdot \frac{\rho_1}{2} v_1^2 \cdot S = c_{D2} \cdot \frac{\rho_2}{2} v_2^2 \cdot S = D_2 \quad \Rightarrow \quad \rho_1 \cdot v_1^2 = \rho_2 \cdot v_2^2$$

$$v_2 = v_1 \cdot \sqrt{\frac{\rho_1}{\rho_2}}$$

### *II.4.4 Thrust as a function of speed and altitude*

In the following, only the maximum available thrust will be discussed. If required for a certain flight condition any lower thrust can be achieved by reducing the throttle setting.

Under II.1.4 the formula for thrust has been given as: $T = \dot{m} \cdot (v_{out} - v_{in})$

This equation is good for a qualitative assessment of thrust over speed and altitude. The altitude again is represented by density, and density is hidden in the mass flow: $\dot{m} = \rho \cdot v \cdot A$

With an increasing altitude the density and so the thrust is decreasing.

The airflow velocity into the propeller area is identical to the flight speed: $v_{in} = v$

The airflow velocity behind the propeller depends on many variables and is difficult to determine. Therefore, another method shall be developed to calculate the available thrust in one-step from the flight speed, the pressure and the temperature.

Thrust is power given from the propeller divided by speed: $T = \frac{P}{v}$

the power given from the propeller is shaft power multiplied with efficiency: $P = P_{eng} \cdot \eta_P$

Shaft power is dependent on engine speed: $P_{eng\,0} = (0{,}1 \cdot n - 150)\,kW$

Engine speed at full throttle depends on flight speed : $n = 4{,}2857 \cdot v + 2200$

Shaft power at any altitude depends on density: $P_{eng,altitude} = P_{eng,0} \cdot \nu$

progress rate depends on flight- and engine speed :

$$\lambda = \frac{v}{u} = \frac{v}{2 \cdot \pi \cdot r \cdot \frac{n}{60}} = \frac{v}{0{,}158 \cdot n}$$

Propeller efficiency depends on progress rate: $\eta_P = \left(-60 \cdot \lambda^2 + 16 \cdot \lambda - 0{,}3\right)$

Now the thrust can be calculated as a function of flight speed, engine speed, air pressure and temperature:

$$T = \frac{P}{v} = \frac{P_{eng,alt} \cdot \eta_P}{v} = \frac{P_{eng,0} \cdot \nu \cdot \eta_P}{v}$$

$$T = \frac{0{,}1 \cdot n - 150}{v} \cdot \frac{0{,}000186 \cdot \frac{p_{alt}}{\sqrt{T_{alt}}} - 0{,}11}{v} \cdot \frac{-60 \cdot \lambda^2 + 16 \cdot \lambda - 0{,}3}{v}$$

$$T = \frac{0{,}1 \cdot (4{,}2857 \cdot v + 2200) - 150}{v} \cdot \frac{0{,}000186 \cdot \frac{p_{alt}}{\sqrt{T_{alt}}} - 0{,}11}{v} \cdot \frac{-60 \cdot (\frac{v}{u})^2 + 16 \cdot (\frac{v}{u}) - 0{,}3}{v}$$

$$T = \frac{(0{,}42857 \cdot v + 220) - 150}{v} \cdot \frac{0{,}000186 \cdot \frac{p_{alt}}{\sqrt{T_{alt}}} - 0{,}11}{v} \cdot \frac{-60 \cdot (\frac{v}{0{,}158 \cdot n})^2 + 16 \cdot (\frac{v}{0{,}158 \cdot n}) - 0{,}3}{v}$$

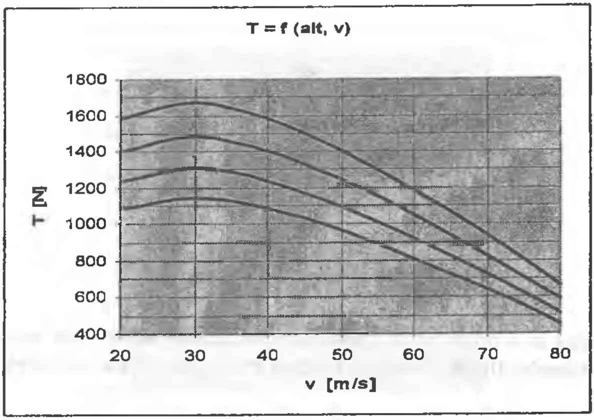
$$T = \frac{(0{,}42857 \cdot v + 220) - 150}{v} \cdot \frac{0{,}000186 \cdot \frac{p_{alt}}{\sqrt{T_{alt}}} - 0{,}11}{v} \cdot$$

$$\frac{-60 \cdot (\frac{v}{0{,}158 \cdot (4{,}2857 \cdot v + 2200)})^2 + 16 \cdot (\frac{v}{0{,}158 \cdot (4{,}2857 \cdot v + 2200)}) - 0{,}3}{v}$$

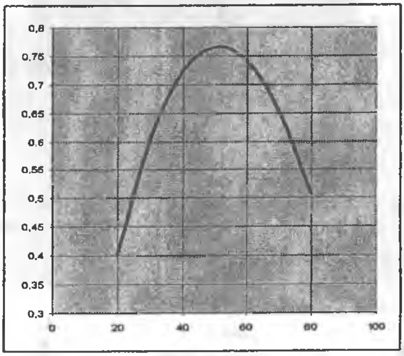
$$T = \frac{(0{,}42857 \cdot v + 220) - 150}{v} \cdot \frac{0{,}000186 \cdot \frac{p_{alt}}{\sqrt{T_{alt}}} - 0{,}11}{v} \cdot \frac{-60 \cdot (\frac{v}{0{,}677 \cdot v + 347{,}6})^2 + 16 \cdot (\frac{v}{0{,}677 \cdot v + 347{,}6}) - 0{,}3}{v}$$

The only unknown variables in this (long) formula are the aeroplane speed and the atmospheric pressure and temperature.

Now with a programmable computer a diagram, that indicates thrust as function of altitude and speed can be completed.



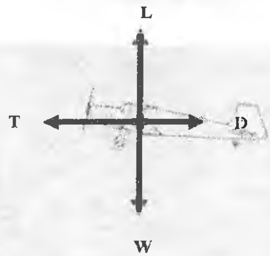

On a first glance the initial increase of thrust towards the maximum at around 30 m/s does not seem to be correct when the formula $T = \dot{m} \cdot (v_{out} - v)$ is taken as reference. The propeller efficiency, however, which depends on the progress rate, looks as depicted in the next diagram for the sea level condition and maximum achievable engine speed.



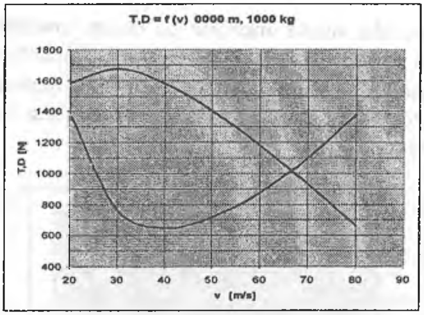

As long as the efficiency as function of speed increases faster than the speed factor decreases, the total thrust increases.

### *II.4.5 Thrust and drag*

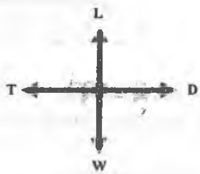
The ratio of thrust and drag is the key to the performance of an aeroplane. It can be easily recognized from the balanced condition in horizontal flight, that any thrust, that is higher than drag can make the aeroplane accelerate or, as it will be shown later, can be used to climb.




But as drag is a function of speed, the maximum achievable speed, that can be flown in horizontal flight, is reached, when maximum thrust and drag are equal.

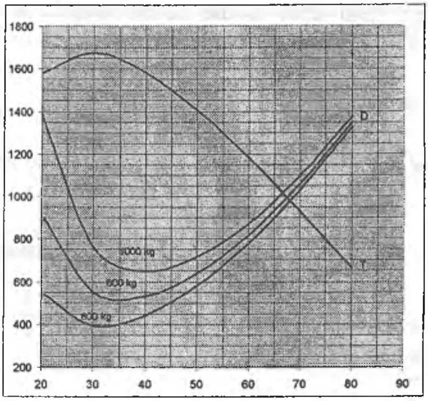



The shown diagram has been calculated for the sample aeroplane at mean sea level with a mass of 1000 kg. The maximum speed that can be reached is about 67 m/s, which corresponds to 241 km/h.

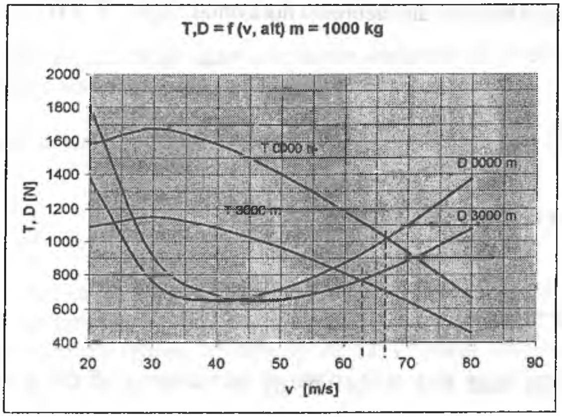

condition at min drag condition at max speed

In the next step it will be examined how the maximum speed depends on the aeroplane's mass. The thrust over speed will be unchanged, but the drag, which is the required thrust will vary.




As expected, the maximum speed will increase with decreasing mass.

The next question is how the maximum speed changes with increasing altitude. For that, different altitudes will be looked at with one mass of 1000 kg.




The maximum speed decreases with altitude as the available thrust decreases faster than the drag does.

### *II.4.6 Range in horizontal flight*

Range considerations are about the distance that can be flown with a certain amount of fuel or the amount of fuel that is required for a certain distance.

distance with given fuel or fuel required for a given distance

Used fuel as well as flown distance are both related to time. The fuel, that is used in a certain time span **dm$_F$** is the thrust specific fuel consumption **b$_T$** multiplied with the thrust **T** and the time span **dt**:

$$dm_F = b_T \cdot T \cdot dt$$

The distance that is flown in a certain time span is the product of speed and time:

$$dx_g = v \cdot dt \qquad \Rightarrow \qquad dt = \frac{dx_g}{v}$$ $x_g$: horizontal distance over ground

Now the formula for the used fuel can be written as:

$$dm_F = b_T \cdot T \cdot \frac{dx_g}{v}$$

With $\frac{D}{L} = \frac{D}{m \cdot g} = \frac{c_D}{c_L} \qquad \Rightarrow \qquad D = \frac{c_D}{c_L} \cdot m \cdot g$

and the condition for uncelebrated horizontal flight **T = D** the used fuel is :

$$dm_F = b_T \cdot \frac{c_D}{c_L} \cdot m \cdot g \cdot \frac{dx_g}{v}$$

and with the gliding number $E = \frac{c_L}{c_D}$

$$dm_F = \frac{b_T \cdot m \cdot g}{E} \cdot \frac{dx_g}{v}$$

the mass of used fuel is the change of the mass of the entire aeroplane $: dm_F = \frac{dm}{m}$

$$dx_g = -\frac{v \cdot E}{b_T \cdot g} \cdot \frac{dm}{m}$$

In this term for the range, one factor describes the flight condition and the other names an amount of fuel.

In the next step, the a.m. general equation for range shall be adapted to the conditions of a propeller aeroplane.

As demonstrated before, the power of the engine can be calculated from the engine speed and the atmospheric conditions, and the efficiency from the progress rate, which depends on aeroplane and engine speed. (**$b_T$** will now be replaced by **$b_P$**)

$$dm_F = b_P \cdot P \cdot dt \qquad P = \frac{T \cdot v}{\eta_P}$$

$$dm_F = b_P \cdot \frac{T \cdot v}{\eta_P} \cdot \frac{dx_g}{v}$$

$$\frac{dm_F}{dx_g} = b_P \cdot \frac{T \cdot v}{\eta_P} \cdot \frac{1}{v} = b_P \cdot \frac{T}{\eta_P} \qquad T = \frac{m \cdot g}{E}$$

$$\frac{dm_F}{dx_g} = b_P \cdot \frac{m \cdot g}{E \cdot \eta_P}$$

$$dx_g = \frac{E \cdot \eta_P}{b_P \cdot g} \cdot \frac{dm}{m}$$

In this term for the range again one factor describes the flight condition and the other names an amount of fuel.

The factor describing the flight condition is called the "range factor" ***RF***.

$$RF = \frac{E \cdot \eta_P}{b_P \cdot g} \quad [m]$$

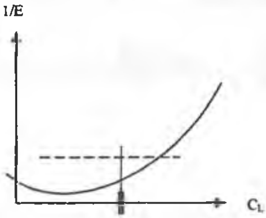
At this point, an important simplification will be implemented. The propeller efficiency and the power specific fuel consumption are considered to be constant. This is allowable, as the typical cruising speed for a normal aeroplane is within a small range of the possible speeds, and **$b_P$** and **$\eta_P$** show only small variations.

For the following calculations the propeller efficiency will be assumed as **$\eta_P$** = 0,75 and the power specific fuel consumption as given under I.4 with **$b_P$** = 0,075 g/kW/s = 0,000075 kg/kW/s.

With **$\eta_P$**, **$b_P$** and g as constant parameters, the flight condition only depends on the gliding number E. Consequently, the best range factor (and the best range) will be achieved with the best possible gliding number **E**, which is the best ratio **$c_L/c_D$**.

For the operation of the aeroplane the speed, at which this best gliding number **E** is achieved, is of interest. However, the speed is not an explicit element of the ***RF*** – formula. It can, however, be calculated via the $c_L$, which is an element of E. If the $c_L$ is found, that gives the best **E**, the corresponding speed can easily be computed.

The required gliding number and the respective $c_L$ can be found in a diagram, where **E** is depicted as a function of $c_L$. The analytical process, however is easier, when the invers value **1/E** is looked for as a function of $c_L$. The horizontal tangent touches the graph at **$(1/E)_{min}$**.




$$\frac{1}{E} = \frac{c_{D0} + \frac{c_L^2}{e \cdot \pi \cdot \Lambda}}{c_L} = \frac{c_{D0}}{c_L} + \frac{c_L^2}{c_L \cdot e \cdot \pi \cdot \Lambda}$$

$$\frac{1}{E} = \frac{c_{D0}}{c_L} + \frac{c_L}{e \cdot \pi \cdot \Lambda} = c_{D0} \cdot c_L^{-1} + \frac{c_L}{e \cdot \pi \cdot \Lambda}$$

$$\frac{d\left(\frac{1}{E}\right)}{dc_L} = c_{D0} \cdot (-1) c_L^{-2} + \frac{1}{e \cdot \pi \cdot \Lambda} = 0$$

$$-c_L^{-2} \cdot c_{D0} + \frac{1}{e \cdot \pi \cdot \Lambda} = 0$$

$$-\frac{c_{D0}}{c_L^2} = -\frac{1}{e \cdot \pi \cdot \Lambda}$$

$$c_{D0} = \frac{c_L^2}{e \cdot \pi \cdot \Lambda}$$

$$c_L^2 = c_{D0} \cdot e \cdot \pi \cdot \Lambda$$

$$c_L = \sqrt{c_{D0} \cdot e \cdot \pi \cdot \Lambda}$$

From that $c_L$ the speed that needs to be flown for achieving the best range factor RF can be calculated for any mass and altitude.

$$c_{L\,RF\max} = \sqrt{c_{D0} \cdot e \cdot \pi \cdot \Lambda} = \sqrt{0{,}022 \cdot 0{,}8 \cdot \pi \cdot 8} = 0{,}665$$

$$v_{RF\max} = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot 0{,}665 \cdot S}}$$

For a mass of 1000 kg at sea level (ρ = 1,225 kgm$^{-3}$) this speed then is:

$$v = \sqrt{\frac{2 \cdot 1000 \cdot 9{,}81}{1{,}225 \cdot 0{,}665 \cdot 15}} = 40{,}1\ ^{m}\!/_{s}$$

The influence of mass and altitude on the speed for best range can be recognized easily.

$$v = \sqrt{\frac{2 \cdot g}{\rho \cdot 0{,}665 \cdot S}} \cdot \sqrt{m} \qquad v = \sqrt{\frac{2 \cdot m \cdot g}{0{,}665 \cdot S}} \cdot \sqrt{\frac{1}{\rho}}$$

With the constant range factor $RF = \dfrac{E \cdot \eta_P}{b_P \cdot g}$ the equation for range can be integrated.

$$\int_0^R dx_g = -RF \cdot \int_{m_0}^{m_e} \frac{1}{m} dm$$

$$R = -RF \cdot (\ln m_0 - \ln m_e)$$

$$R = -RF \cdot \ln(1 - \frac{m_F}{m_0})$$

The remaining factor **ln (1 – $m_F$/$m_0$)** describes the amount of fuel that is available to be used, but also describes the influence of the overall mass on the range of the aeroplane. While $m_F$ is the mass used, $m_0$ is the overall mass of the aeroplane at the beginning of the segment to be flown. It has been shown earlier, that the drag of an aeroplane in horizontal flight varies with mass. Consequently, an aeroplane with less mass requires less fuel. This influence must not be underestimated, as the following example shows.

Calculation example for the best achievable range:

$c_{L\,RF\,max}$ = 0,665

$$c_{D\,RF\,max} = 0{,}022 + \frac{0{,}665^2}{0{,}8 \cdot \pi \cdot 8} = 0{,}044$$

$$E_{RF\,max} = \frac{0{,}665}{0{,}044} = 15{,}11$$

**Range factor:** $RF_{max} = -\dfrac{15{,}11 \cdot 0{,}75}{0{,}000075 \cdot 9{,}81 \cdot 10^3}\, m = -15402000\, m$

The factor $10^{-3}$ has to be used for the transformation from kW to **W** = (Nm/s)

The range at a mass of $\mathbf{m_0}$ = 1000 kg and an available fuel mass of $\mathbf{m_F}$ = 100 kg is:

$$R = -15402000 m \cdot \ln(1 - \frac{100}{1000}) = -15402000\, m \cdot (-0{,}105) = 1617000\, m = 1617\, km$$

It has been shown, that this maximum range can only be achieved at one specific speed. The power that is required to fly at this speed is relatively low. It can be found in the following way:

$$E = 15{,}11 = \frac{c_L}{c_D} \quad \Rightarrow \quad c_D = \frac{0{,}665}{15{,}11} = 0{,}044$$

At sea level with the standard density of 1,225 kg $m^{-3}$ :

$$D = 0{,}044 \cdot \frac{1{,}225}{2} \cdot 40{,}5^2 \cdot 15 = 663\ N = T$$

$$P = \frac{T \cdot v}{\eta_P} = \frac{663 \cdot 40{,}5}{0{,}75} = 35{,}8\, kW$$

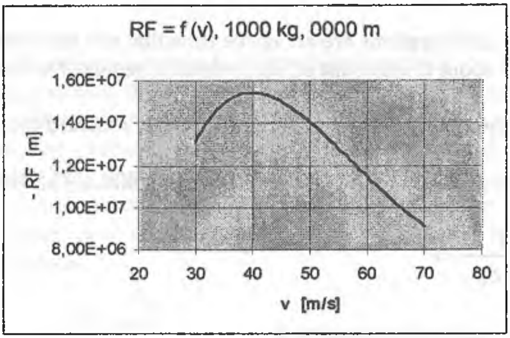
The achievable maximum speed was calculated as 67 m/s. Therefore, the next step is to examine the influence of speed on the achievable range (factor).

To be independent of altitude, the lift coefficients will be calculated. The range of questionable lift coefficients can be identified by examining the range of 30 m/s to 70 m/s at a mass of 1000 kg and at sea level.

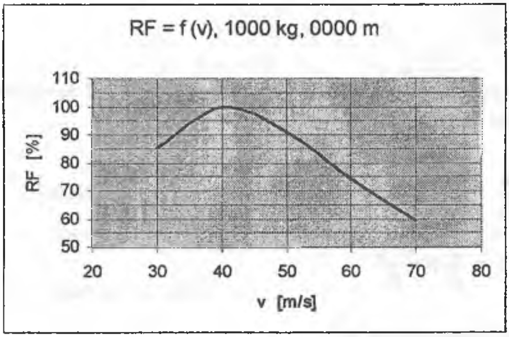
| v [m/s] | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|
| $c_L$ [-] | 1,186 | 0,667 | 0,427 | 0,297 | 0,218 |

With these $c_L$s the $c_D$s and gliding numbers **E** and range factors *RF* can be found:

| v | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|
| $c_L$ [-] | 1,186 | 0,667 | 0,427 | 0,297 | 0,218 |
| $c_D$ [-] | 0,0920 | 0,0442 | 0,0311 | 0,0264 | 0,0244 |
| E [-] | 12,89 | 15,11 | 13,74 | 11,24 | 8,94 |
| RF [m] | 13139584 | 15404286 | 14009423 | 11462150 | 9117546 |




It is, however, of higher interest to see the relative change:




It has, however, to be pointed out again, that simplifying assumptions of constant specific fuel consumption and propeller efficiency have been made. On a real aeroplane, due to the increase of specific fuel consumption and the decrease of propeller efficiency the range loss at higher speeds would be higher.

From this diagram the following can be read:

An increase of cruising speed from 40 m/s to 50 m/s would mean an increase of 25 % with a range loss of only 10 %. Considering time related operating costs, this should be meaningful and make sense. The economic cruise speed always depends on the mix of fuel and time related costs and is higher than the one for maximum range.

### *II.4.7 endurance in horizontal flight*

Endurance considerations are about the time that can be flown with a certain amount of fuel, or about the amount of fuel, which is required to fly a certain time.

Time with given fuel or fuel required for a certain time

From the range considerations, the following is already known:

$$dm_F = b_P \cdot \frac{T \cdot v}{\eta_P} \cdot dt$$

which now shall be dissolved to ***dt***:

$$dt = \frac{dm_F \cdot \eta_P}{T \cdot v \cdot b_P}$$

Again the used fuel mass equals the decrease in total aeroplane mass ($dm_F = - dm$) and $T$ will be substituted as follows:

$$E = \frac{c_L}{c_D} = \frac{L}{D} \qquad T = D \qquad (condition\ of\ horizontal\ flight)$$

$$E = \frac{L}{T} \qquad T = \frac{L}{E} = \frac{m \cdot g}{E}$$

Then ***dt*** can be expressed as:

$$dt = \frac{- dm \cdot \eta_P \cdot E}{b_P \cdot v \cdot m \cdot g} = - \frac{\eta_P \cdot E}{b_P \cdot v \cdot g} \cdot \frac{dm}{m}$$

Analogue to the range factor ***RF*** a time factor ***TF*** can be recognized in that equation:

$$TF = -\frac{\eta_P \cdot E}{b_P \cdot v \cdot g}$$

and the equation for time reads:

$$dt = TF \cdot \frac{dm}{m} = TF \cdot \ln(1 - \frac{m_F}{m_0})$$

That equation again can be transformed to find the required fuel for a given time span:

$$m_F = m \cdot \left(1 - e^{\frac{t}{TF}}\right)$$

While these equations are good for any flight condition, in accordance to the range factor that particular time factor shall be found, which allows the longest flying time with a given amount of fuel. Like for the range factor the specific fuel consumption and the propeller efficiency will be considered as constant for simplification. Then the only variables in the equation for the time factor are gliding number E and speed v. As any flight condition can be identified by the relevant **$c_L$** and there is a direct dependence of **$c_L$** and **$c_D$**, E and v must be substituted by **$c_L$** and **$c_D$**.

$$E = \frac{c_L}{c_D} \qquad v = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot c_L \cdot S}}$$

$$TF = -\frac{\eta_P}{b_P \cdot v \cdot g} \cdot \frac{c_L}{c_D} = -\frac{\eta_P}{b_P \cdot g} \cdot \sqrt{\frac{\rho \cdot c_L \cdot S}{2 \cdot m \cdot g}} \cdot \frac{c_L}{c_D} = -\frac{\eta_P}{b_P \cdot g} \cdot \sqrt{\frac{\rho \cdot S}{2 \cdot m \cdot g}} \cdot \frac{\sqrt{c_L} \cdot c_L}{c_D}$$

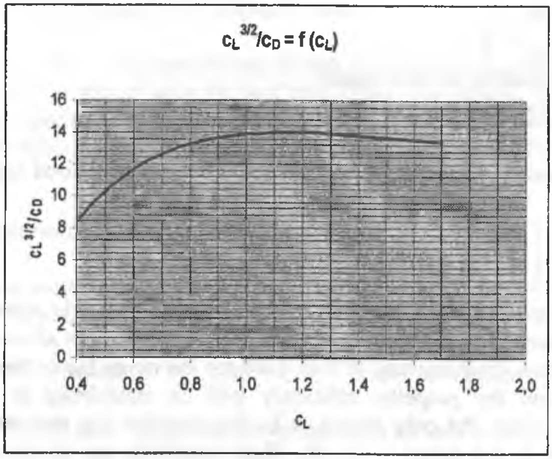
$$TF = -\frac{\eta_P}{b_P \cdot g} \cdot \sqrt{\frac{\rho \cdot S}{2 \cdot m \cdot g}} \cdot \frac{c_L^{3/2}}{c_D}$$

To find the speed, that gives the maximum time factor, a diagram $\left(\frac{c_L^{3/2}}{c_D}\right) = f(c_L)$ can be designed, with

$$c_L = \frac{2 \cdot m \cdot g}{\rho \cdot v^2 \cdot S} \qquad and \qquad c_D = c_{D0} + \frac{c_L^2}{e \cdot \pi \cdot \Lambda}$$

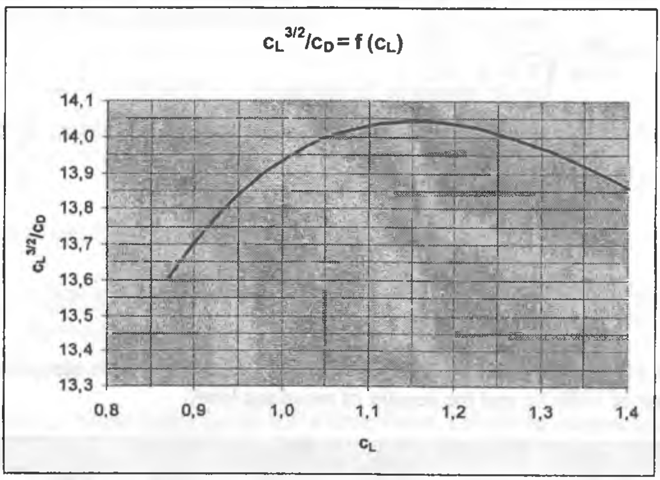
The calculation example will be performed using the sample aeroplane again with a mass of 1000 kg and the density of mean sea level.

| **$c_L$** | 1,708 | 1,186 | 0,872 | 0,667 | 0,527 | 0,427 | 0,353 |
|---|---|---|---|---|---|---|---|
| **$c_L^{3/2}/c_D$** | 13,35 | 14,04 | 13,61 | 12,34 | 10,68 | 8,98 | 7,44 |




As the curve is very flat around its maximum, a second calculation shall be done using smaller steps around the expected maximum at $c_L = 1,1$.

| $c_L$ [-] | 1,708 | 1,580 | 1,465 | 1,362 | 1,270 | 1,186 | 1,111 | 1,043 | 0,980 | 0,924 | 0,872 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $c_L^{3/2}/c_D$ [-] | 13,35 | 13,58 | 13,77 | 13,91 | 14,00 | 14,04 | 14,04 | 13,99 | 13,90 | 13,77 | 13,61 |



The $c_L$ for the best time factor can be identified with approximately $c_L$ = 1,15. It is remarkable, that this $c_L$ is very close to the maximum $c_L$ and therefore very close to the minimum possible speed.

Using $v = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot 1{,}15 \cdot 15}}$ the speed for the maximum endurance can be calculated under all operating conditions of mass and altitude.

An example will be calculated for the sample aeroplane, cruising at 1000 metres altitude at a mass of 1000 kg.

$\rho_{1000\,m} = 1{,}112\ kg\ m^{-3}$ $\quad b_P = 0{,}000075\ kg\ /\ kW\ /\ s$ $\quad \eta_P = 0{,}75$

$c_{L\,TF\,max} = 1{,}15$ $\quad c_{D\,TF\,max} = 0{,}0877$

$$TF_{max} = -\frac{\eta_P}{b_P \cdot g} \cdot \sqrt{\frac{\rho \cdot S}{2 \cdot m \cdot g}} \cdot \frac{c_L^{3/2}}{c_D} = -\frac{0{,}75}{0{,}000075 \cdot 9{,}81} \cdot \sqrt{\frac{1{,}112 \cdot 15}{2 \cdot 1000 \cdot 9{,}81}} \cdot \frac{1{,}15^{3/2}}{0{,}0877}$$

$$TF_{max} = -418\,000\,s$$

The fuel, that is used in one hour (= 3600 s) with a speed corresponding to $c_L$ = 1,15 then is:

$$m_F = m_0 \cdot \left(1 - e^{-\frac{t}{TF}}\right) = 1000\,kg \cdot \left(1 - e^{-\frac{3600}{418000}}\right) = 8{,}6\,kg$$

The maximum time, that can be flown with 10 kg fuel then is :

$$dt = -TF \cdot \ln\left(1 - \frac{m_F}{m_0}\right) = -418\,000 \cdot \ln\left(1 - \frac{10}{1000}\right) = 4201\,s \approx 01:10\,h$$

Under range considerations it has been found out, that the $c_L$ to achieve the max range factor is $c_L$ = 0,665. Now, time considerations shall be made using that $c_L$.

$c_L = 0{,}655 \quad \rightarrow c_D = 0{,}0433 \rightarrow \quad TF = -364\ 000\ s$

The fuel used within one hour at best range speed ($c_L$) is:

$$m_F = 1000\,kg \cdot \left(1 - e^{-\frac{3600}{364000}}\right) = 9{,}9\,kg$$

The time, that can be flown in that condition is:

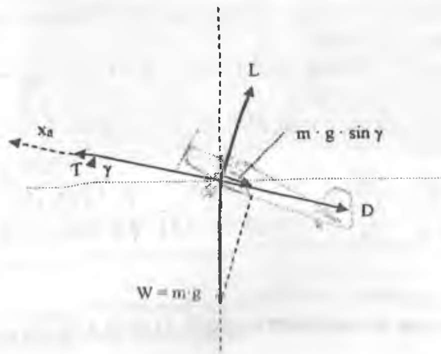
$$dt = -364000\,s \cdot \ln\left(1 - \frac{10}{1000}\right) = 3658\,s \approx 01:01\,h$$

Again it must be pointed out, that propeller efficiency and specific fuel consumption have been assumed as constant!

## II.5 Climbing flight

### II.5.1 Forces in climbing flight

The forces, that are already known from horizontal flight, are also present in climbing flight, but their distribution is different.




The lift still acts perpendicular to the $x_a$-axes.

The weight still acts direct to the centre of earth.

Drag and thrust still act parallel to the $x_a$-axes.

Some portion of the weight (m·g·sin γ) is acting along the negative $x_a$-axes ("downhill-force")
The sum of forces in the $x_a$ - direction is:

$$\sum F_{xa} = 0 = T - (D + m \cdot g \cdot \sin\gamma)$$

### II.5.2 Flight path angle

This balance can be transformed to:

$$m \cdot g \cdot \sin\gamma = T - D \qquad or \qquad \sin\gamma = \frac{T - D}{m \cdot g}$$

For small angles γ in arc or radian measure it can be stated: **sin γ** = γ,

so

$$\gamma = \frac{T - D}{m \cdot g}$$

This equation shows, that the path angle $\gamma$ increases, when the difference ***T-D*** increases. The higher the thrust and the lower the drag, the higher the path angle will be.

A simple example:

$m = 1.000$ kg $\qquad \varphi = 1{,}225$ kgm$^{-3}$ $\qquad v = 40$ m/s

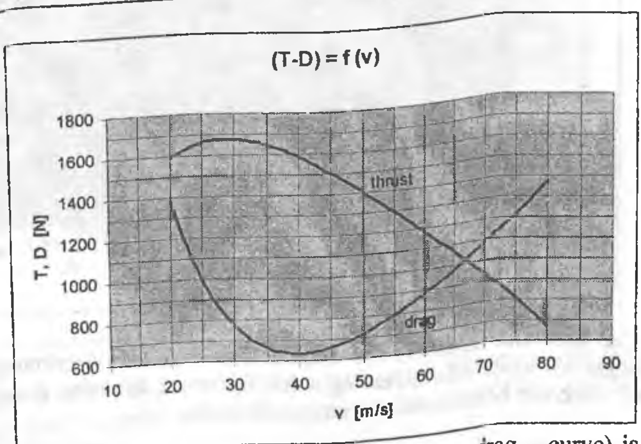
$c_L = 0{,}667 \qquad c_D = 0{,}0441 \qquad D = 649$ N

from II.4.5 $\qquad T = 1.580$ N

$$\gamma = \frac{1580\,N - 649\,N}{(1000 \cdot 9{,}81)N} = 0{,}0949 \;\; (5{,}44°)$$

The speed, that, for a given mass, results in the greatest difference of ***T – D***, is that one, which provides the best angle of climb. It can be analytically found by replacing ***T*** with the expression from formula for available thrust as a function of speed, which would be a very long term, or with a suitable diagram.

| *v [m/s]* | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| *T [N]* | 1590 | 1680 | 1580 | 1400 | 1190 | 930 | 670 |
| *D [N]* | 1384 | 761 | 649 | 714 | 872 | 1097 | 1375 |
| *T - D [N]* | 206 | 919 | 931 | 686 | 318 | -167 | -705 |

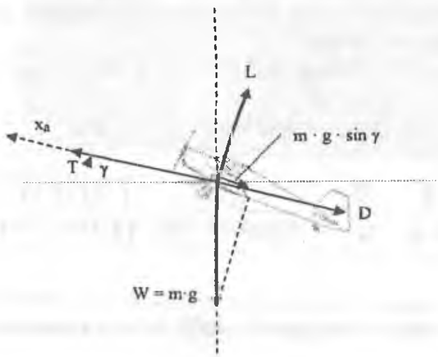



The largest difference (distance from thrust – to drag – curve) is found at about 36 m/s.

## II.5 Climbing flight

### *II.5.1 Forces in climbing flight*

The forces, that are already known from horizontal flight, are also present in climbing flight, but their distribution is different.




The lift still acts perpendicular to the $x_a$-axes.

The weight still acts direct to the centre of earth.

Drag and thrust still act parallel to the $x_a$-axes.

Some portion of the weight (m·g·sin γ) is acting along the negative $x_a$-axes ("downhill-force")

The sum of forces in the $x_a$ - direction is:

$$\sum F_{xa} = 0 = T - (D + m \cdot g \cdot \sin\gamma)$$

### *II.5.2 Flight path angle*

This balance can be transformed to:

$$m \cdot g \cdot \sin\gamma = T - D \qquad or \qquad \sin\gamma = \frac{T - D}{m \cdot g}$$

For small angles γ in arc or radian measure it can be stated: **sin γ = γ,** so

$$\gamma = \frac{T - D}{m \cdot g}$$

This equation shows, that the path angle $\gamma$ increases, when the difference ***T-D*** increases. The higher the thrust and the lower the drag, the higher the path angle will be.

A simple example:

$m$ = 1.000 kg $\qquad \varphi$ = 1,225 kgm$^{-3}$ $\quad v$ = 40 m/s

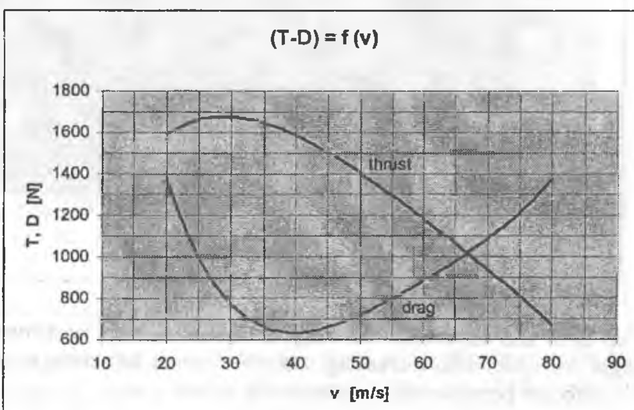
$c_L$ = 0,667 $\qquad c_D$ = 0,0441 $\qquad D$ = 649 N

from II.4.5 $\qquad T$ = 1.580 N

$$\gamma = \frac{1580\,N - 649\,N}{(1000 \cdot 9{,}81)N} = 0{,}0949 \;\; (5{,}44°)$$

The speed, that, for a given mass, results in the greatest difference of ***T – D***, is that one, which provides the best angle of climb. It can be analytically found by replacing ***T*** with the expression from formula for available thrust as a function of speed, which would be a very long term, or with a suitable diagram.

| *v [m/s]* | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| *T [N]* | 1590 | 1680 | 1580 | 1400 | 1190 | 930 | 670 |
| *D [N]* | 1384 | 761 | 649 | 714 | 872 | 1097 | 1375 |
| *T - D [N]* | 206 | 919 | 931 | 686 | 318 | -167 | -705 |




The largest difference (distance from thrust – to drag – curve) is found at about 36 m/s.

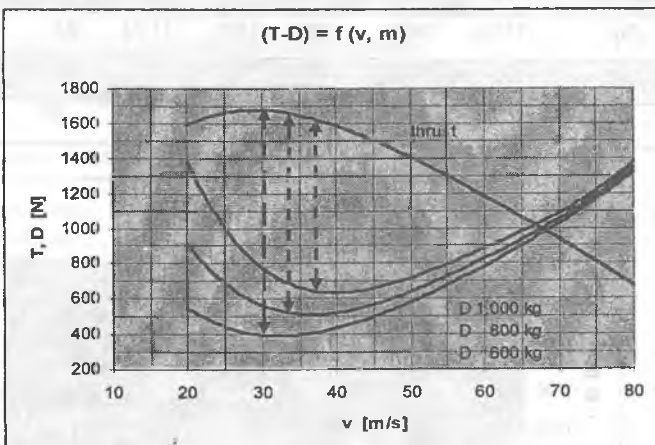
Again the maximum achievable speed is found to be 67 m/s. For a further speed increase, a descent must be initiated. The descent angle at 80 m/s would be:

$$\gamma = \frac{670\,N - 1375\,N}{(1000 \cdot 9{,}81)N} = -0{,}0719 \quad (-4{,}12°)$$

The influence of the aeroplane's mass on the climb angle and the speed for the best climb angle shall be examined next.

The environmental conditions remain unchanged and the masses will be again 600 kg, 800 kg and 1.000 kg. The available thrust is the one at mean sea level.

| v [m/s] | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| T [N] | 1590 | 1680 | 1580 | 1400 | 1190 | 930 | 670 |
| $D_{1000\,kg}$ [N] | 1384 | 761 | 649 | 714 | 872 | 1097 | 1375 |
| $D_{800\,kg}$ [N] | 915 | 553 | 532 | 639 | 820 | 1058 | 1346 |
| $D_{600\,kg}$ [N] | 550 | 390 | 441 | 580 | 780 | 1029 | 1323 |




The table and especially the diagram shows that the maximum achievable climb angle decreases with increasing mass. The speed, however, at which the best angle of climb can be achieved, increases with mass.

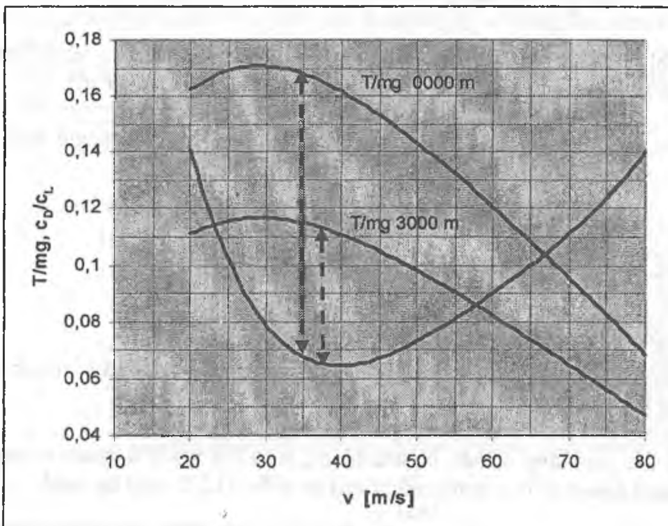
Finally, the influence of the density (altitude) shall be examined. The conditions at 0.000 m and 3.000 m will be looked at.

To make the diagram easier to construct and more clear, lift L (m·g) and drag D, that otherwise have to be calculated for every condition, will be substituted.

$$\gamma = \frac{T-D}{m \cdot g} = \frac{T}{m \cdot g} - \frac{D}{L} = \frac{T}{m \cdot g} - \frac{c_D}{c_L}$$

Now only one curve is required for the aerodynamic condition that is valid for all altitudes. The decreasing thrust, however, has to take into account for every altitude separately.

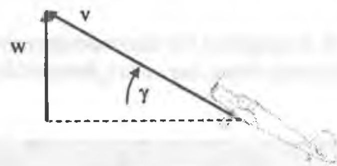
| *v [m/s]* | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| $c_D/c_L$ *[-]* | 0,1411 | 0,0776 | 0,0662 | 0,0728 | 0,0889 | 0,1118 | 0,1402 |
| $T_{0000\,m}/mg$ *[-]* | 0,1621 | 0,1713 | 0,16[illegible] | 0,1427 | 0,1213 | 0,0948 | 0,0683 |
| $T_{3000\,m}/mg$ *[-]* | 0,1111 | 0,1162 | 0,1111 | 0,0979 | 0,0815 | 0,0652 | 0,0469 |




It can be seen, that the achievable climb angle decreases with increasing altitude, and that the speed, that is necessary for the best climb angle, increases slightly with altitude.

### *II.5.3 Vertical speed*

Besides the flight path angle, the achievable vertical speed of an aeroplane is a very important performance criteria. It can be found from the foregoing conclusions.




The following applies:

$$\sin\gamma = \frac{w}{v} \qquad \Rightarrow \qquad w = v\cdot\sin\gamma \qquad w = v\cdot\gamma \qquad (\sin\gamma = \gamma)$$

$$w = v\cdot\frac{T-D}{m\cdot g} = \frac{T\cdot v - D\cdot v}{m\cdot g}$$

While the climb angle depends on the difference "thrust – drag", the climb rate is dependent on "power available – power required" (***P* = *T v***)!

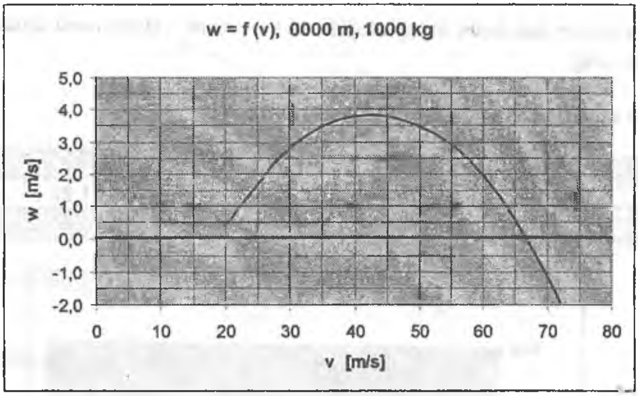
The same numbers as in the example for the flight path angle shall be used (from $v$ = 40 m/s):

$$w = \frac{1580\,N\cdot 40\frac{m}{s} - 649\,N\cdot 40\frac{m}{s}}{1000\,kg\cdot 9{,}81\frac{m}{s^2}} = 3{,}8\frac{m}{s}$$

The sample aeroplane can achieve a vertical speed of + 3,8 m/s at mean sea level and 1000 kg with a speed of 40 m/s.

In the next step it shall be examined, how the vertical speed w depends on the forward speed v. The same conditions as under II.5.2 shall be used.

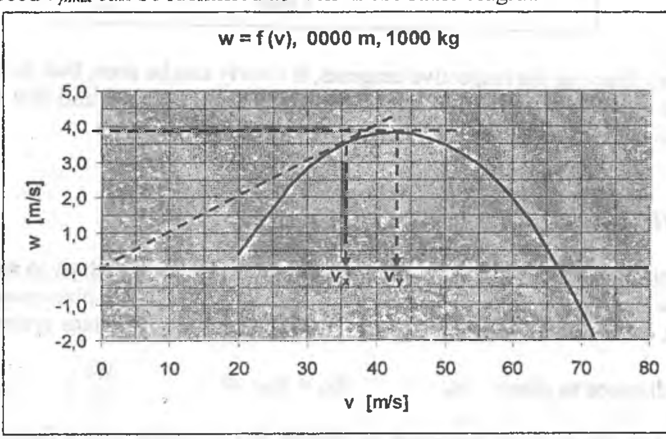
| *v [m/s]* | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| *$P_{available}$ [kW]* | 31,8 | 50,4 | 63,2 | 70 | 71,4 | 65,1 | 53,6 |
| *$P_{required}$ [kW]* | 27,7 | 22,8 | 26,0 | 35,7 | 52,3 | 76,8 | 110,0 |
| *w [m/s]* | 0,42 | 2,81 | 3,80 | 3,50 | 1,94 | -1,19 | -5,75 |




It can be easily seen, that the climb speed also very much depends on the flight speed. The achievable top speed is identical to the one that was found with "flight-path-angle-method". To fly faster, as seen earlier, a descent must be flown.

In a second step, the maximum achievable vertical speed $w_{max}$ and the flight speed $v_{wmax}$ shall be identified. This can be done by drawing the horizontal tangent to the curve.

But by drawing its tangent from the origin, the best climb angle $\gamma_{max}$ and its flight speed $v_{\gamma max}$ can be identified as well in the same diagram.



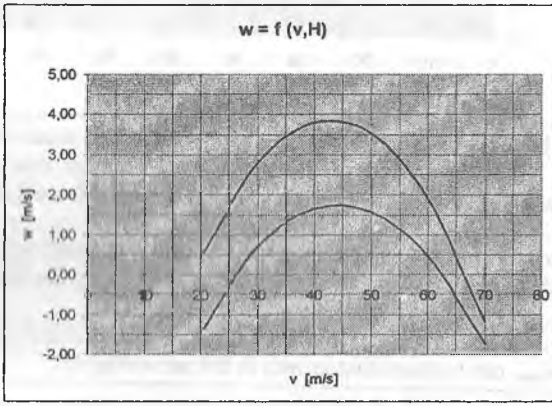

In aviation, the speed to achieve the best angle of climb is called $v_X$, the one to achieve the best vertical speed is called $v_Y$.

The next step shows, that and how far the achievable vertical speed and the achievable climb angle decrease with increasing altitude.

The power has been calculated from the thrust values used under II.5.2, as well as the drag.

The required formulas are not repeated again!

| $v$ *[m/s]* | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|
| $w_{6000\,m}$ *[m/s]* | 0,40 | 2,79 | 3,79 | 3,51 | 1,88 | -1.19 |
| $w_{3000\,m}$ *[m/s]* | -1,49 | 0.71 | 1,64 | 1.55 | 0,45 | -1,72 |



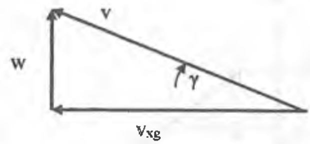

By drawing the respective tangents, it clearly can be seen, that the best angle and the best vertical speed decrease with increasing altitude, and that $v_X$ and $v_Y$ increase slightly with increasing altitude.

### *II.5.4 Distance to climb*

Now the distance, that is required to climb from one altitude to another one, shall be be calculated. As that distance is a distance in the x-y-plane parallel to the surface of the earth, the calculation refers to the geostationary axes system.

distance to climb : $dx_g$ $\qquad dx_g = v_{xg} \cdot dt$

Any climb of an aeroplane, of course, is performed with a flight speed $v$, a vertical speed $w$ and a flight path angle $\gamma$.




the following applies : $$\tan\gamma = \frac{w}{v_{xg}}$$

with the allowable simplification : $$\tan\gamma = \gamma$$

$$\gamma = \frac{w}{v_{xg}} \quad \Rightarrow \quad v_{xg} = \frac{w}{\gamma}$$

the vertical speed w is : $$w = \frac{dH}{dt}$$

$$v_{xg} = \frac{w}{\gamma} = \frac{dH}{\gamma\, dt} \qquad v_{xg} = \frac{1}{\gamma} \cdot \frac{dH}{dt}$$

$$dx_g = v_{xg} \cdot dt = \frac{1}{\gamma} \cdot \frac{dH}{dt} \cdot dt$$

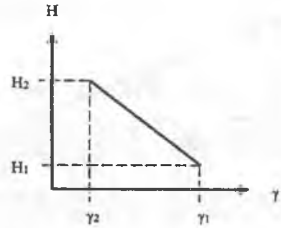
$$dx_g = \frac{dH}{\gamma}$$

That equation can be integrated, and the distance, that is required to climb, will be found:

$$\int_{x_{g1}}^{x_{g2}} dx_g = \int_{H_1}^{H_2} \frac{dH}{\gamma}$$

It is, however, already known, that the climb angle $\gamma$ is not constant throughout a climb and decreases with increasing altitude: $\gamma = f(H)$

In an initial assumption, $\gamma = f(H)$ can be assumed as a straight line:




At lower altitudes, the achievable climb angle is greater than at higher altitudes. $\gamma_1$ and $\gamma_2$ at their respective altitudes are known from II.5.2.

The equation for a straight line through two points reads: $\frac{H-H_1}{\gamma-\gamma_1}=\frac{H_2-H_1}{\gamma_2-\gamma_1}$

To find an expression for $\gamma$, that equation has to be transformed:

$$H-H_1=(\gamma-\gamma_1)\cdot\frac{H_2-H_1}{\gamma_2-\gamma_1}=\gamma\cdot\left(\frac{H-H_1}{\gamma_2-\gamma_1}\right)-\gamma_1\cdot\left(\frac{H_2-H_1}{\gamma_2-\gamma_1}\right)$$

$$\gamma\cdot\left(\frac{H_2-H_1}{\gamma_2-\gamma_1}\right)=(H-H_1)+\gamma_1\cdot\left(\frac{H_2-H_1}{\gamma_2-\gamma_1}\right)$$

$$\gamma=(H-H_1)\cdot\left(\frac{\gamma_2-\gamma_1}{H_2-H_1}\right)+\gamma_1\cdot\left(\frac{H_2-H_1}{\gamma_2-\gamma_1}\right)\cdot\left(\frac{\gamma_2-\gamma_1}{H_2-H_1}\right)$$

$$\gamma=\gamma_1+\frac{\gamma_2-\gamma_1}{H_2H_1}\cdot(H-H_1)$$

$$\gamma=\gamma_1-\frac{\gamma_1-\gamma_2}{H_2-H_1}\cdot(H-H_1)$$

This expression is used in: $dx_g=\frac{dH}{\gamma}$

$$dx_g=\frac{dH}{\gamma_1-\left(\frac{\gamma_1-\gamma_2}{H_2-H_1}\right)\cdot(H_2-H_1)}$$

The denominator is substituted by $z$: $z=\gamma$ $\quad dx_g=\frac{dH}{z}$

$$z = \gamma_1 - \left(\frac{\gamma_1 - \gamma_2}{H_2 - H_1}\right) \cdot (H - H_1)$$

$$dz = -\frac{\gamma_1 - \gamma_2}{H_2 - H_1} dH \qquad \Rightarrow \qquad dH = -\frac{H_2 - H_1}{\gamma_1 - \gamma_2} dz$$

$$dx_g = \frac{dH}{dz}$$

$$dx_g = -\frac{H_2 - H_1}{\gamma_1 - \gamma_2} \cdot \frac{dz}{z}$$

$$\int_{x_{g1}}^{x_{g2}} dx_g = \int_{z_1}^{z_2} -\frac{H_2 - H_1}{\gamma_1 - \gamma_2} \cdot \frac{dz}{z}$$

$$R = -\frac{H_2 - H_1}{\gamma_1 - \gamma_2} \cdot \int_{z_1}^{z_2} \frac{dz}{z} = -\frac{H_2 - H_1}{\gamma_1 - \gamma_2} \cdot \ln \frac{z_2}{z_1}$$

and with the reverse substitution $z = \gamma$

$$R = -\frac{H_2 - H_1}{\gamma_1 - \gamma_2} \cdot \ln \frac{\gamma_2}{\gamma_1}$$

For a sample calculation the values γ for the flight path angle can be taken from II.5.2:

$T_0$ = 1580 N $D_0$ = 649 N $T_{3000}$ = 1090 N $D_{3000}$ = 679 N at $v = 40$ m/s

$$\gamma_{1\,(0m)} = \frac{1580 - 649}{9810} = 0{,}0949 \qquad \gamma_{2\,(3000m)} = \frac{1090 - 679}{9810} = 0{,}0419$$

$$R = -\frac{(3000 - 0)\,m}{0{,}0949 - 0{,}0419} \cdot \ln \frac{0{,}0419}{0{,}0949} = 46.276\,m$$

### *II.5.5 Time to climb*

The time, that is required from one altitude to another altitude, can be calculated in a very similar manner.

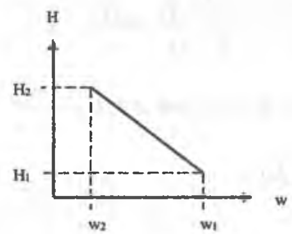
The vertical speed w is : $w = \frac{dH}{dt}$ $\qquad dt = \frac{dH}{w}$

That equation can be integrated and the time, that is required, will be found:

$$\int_{t_1}^{t_2} dt = \int_{H_1}^{H_2} \frac{dH}{w}$$

It is also known, that the vertical speed is not constant in a climb and decreases with increasing altitude, $w = f(H)$.

In an initial assumption, $w = f(H)$ also can be assumed as a straight line.




Like the climb angle, the achievable vertical speed is greater at lower altitudes due to the higher available engine power. $w_1$ and $w_2$ at their respective altitudes are known from II.5.3.

In this case the equation for the straight line reads:

$$\frac{H - H_1}{w - w_1} = \frac{H_2 - H_1}{w_2 - w_1}$$

As under II.5.4 for the path angle, it can be transformed to express $w$:

$$w = w_1 - \left( \frac{w_1 - w_2}{H_2 - H_1} \right) \cdot (H - H_1)$$

This expression is used in $\qquad dt = \frac{dH}{w}$

$$dt = \frac{dH}{w_1 - \left(\frac{w_1 - w_2}{H - H_1}\right) \cdot (H - H_1)}$$

The denominator is substituted by $z$ $(z = w)$:

$$dt = \frac{dH}{z}$$

$$z = w_1 - \left(\frac{w_1 - w_2}{H - H_1}\right) \cdot (H - H_1)$$

$$dz = -\frac{w_1 - w_2}{H_2 - H_1} dH \qquad \Rightarrow \qquad dH = -\frac{H_2 - H_1}{w_1 - w_2} dz$$

$$dt = -\frac{H_2 - H_1}{w_1 - w_2} \frac{dz}{z}$$

$$t = -\frac{H_2 - H_1}{w_1 - w_2} \cdot \int_{z_1}^{z_2} \frac{1}{z} dz$$

$$t = -\frac{H_2 - H_1}{w_1 - w_2} \cdot \ln \frac{w_2}{w_1}$$

An example can be calculated using the sample aeroplane:

$w_{0m} = 3{,}79\ m/s \qquad w_{3000m} = 1{,}64\ m/s$

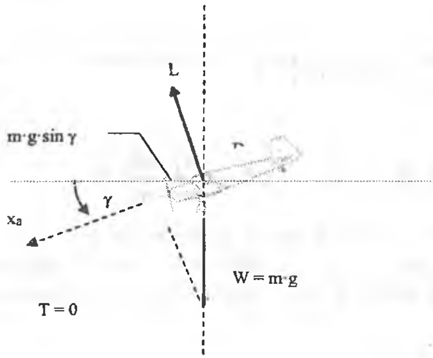
$$t = -\frac{3000 - 0}{3{,}79 - 1{,}64} \cdot \ln \frac{1{,}64}{3{,}79} = 1168\ s = 19 \text{ min } 28\ s$$

The reasonability of this result can be found easily: 3000 m:2,5 m/s = 1200 s = 20 min

## II.6 Descending flight

### *II.6.1 Forces in descending flight*

The forces, that are already known from horizontal flight, are also present in descending flight.




Thrust is considered to be zero!

The lift still acts perpendicular to the $x_a$-axes.

The weight still acts direct to the centre of earth.

Drag (and thrust, if any) still act parallel to the $x_a$-axes.

Some portion of the weight ($m \cdot g \cdot \sin\ \gamma$) is acting along the $x_a$-axes ("downhill-force")

The sum of forces in the $x_a$ - direction is:

$$\sum F_{xa} = 0 = m \cdot g \cdot \sin(-\gamma) - D$$

### *II.6.2 Flight path angle*

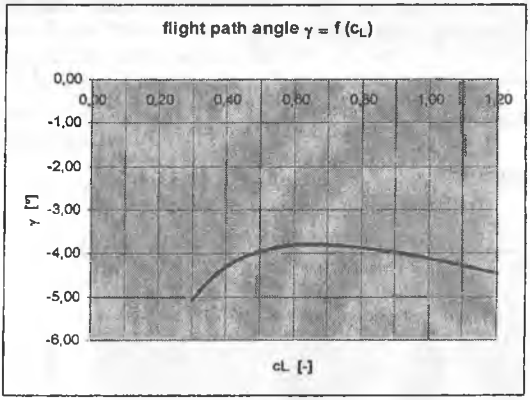
This balance can be transferred to $m \cdot g \cdot \sin(-\gamma) = D$

and with the known simplification $\sin \gamma = \gamma$ to

$$\gamma = -\frac{D}{m \cdot g} = -\frac{c_D}{c_L}$$

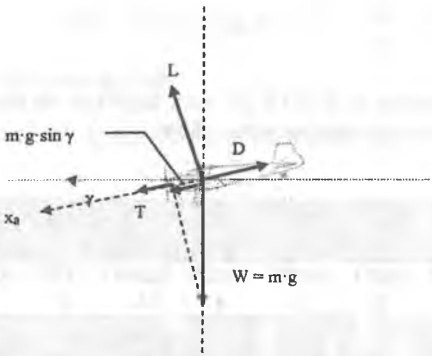
As $c_D$ is a function of $c_L$ the flight path angle can be shown as a function of $c_L$. An example for the sample aeroplane shows:

| $c_L$ [-] | 1,2 | 1,1 | 1 | 0,9 | 0,8 | 0.7 | 0,6 | 0,5 | 0,4 | 0,3 |
|---|---|---|---|---|---|---|---|---|---|---|
| $c_D$ [-] | 0,0937 | 0,0822 | 0,0718 | 0,0623 | 0,0538 | 0,0464 | 0,0399 | 0,0344 | 0,0300 | 0,0265 |
| $\gamma$ [rad] | -0,078 | -0,075 | -0.072 | -0,069 | -0,067 | -0,066 | -0,067 | -0,069 | -0,075 | -0,088 |
| $\gamma$ [°] | -4,47 | -4,28 | -4,11 | -3,97 | -3,86 | -3,80 | -3,81 | -3,95 | -4,29 | -5,06 |




The diagram shows, that the smallest flight path angle can be achieved with a $c_L$ of about 0,65. In II.4.8 (range considerations) the $c_L$ for the maximum range factor, which is the one for the best gliding number (best gliding angle), was found to be 0,665.

The balance of forces in $x_a$ – direction changes, if some remaining thrust has to be considered due to a partial power setting of the engine.




$$\sum F_{xa} = 0 = m \cdot g \cdot \sin(-\gamma) + T - D$$

The more thrust is added, the lower the flight path angle will be. The flight path angle will be zero (horizontal flight), when thrust equals drag and no downhill force is required to maintain a constant speed.

### *II.6.3 Vertical speed*

The following applies:

$$\sin(-\gamma) = \frac{-w}{v} \qquad \Rightarrow \qquad w = v \cdot \sin(-\gamma) \qquad w = v \cdot (-\gamma)$$

$$w = v \cdot \frac{T - D}{m \cdot g} = \frac{T \cdot v - D \cdot v}{m \cdot g}$$

For T = 0 the equation reads: $w = -\dfrac{D \cdot v}{m \cdot g}$

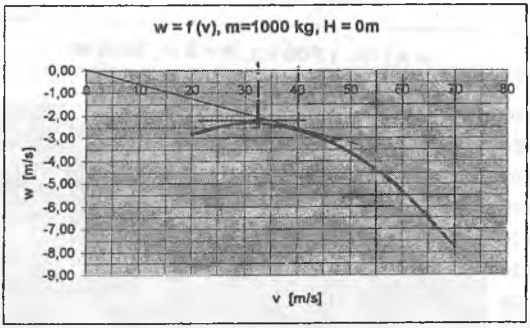
A calculation for $w = f(v)$ shows, how the vertical speed $w$ depends on the flight condition (flight speed).

| *v [m/s]* | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|
| $c_L$ *[-]* | 2,669 | 1,186 | 0,667 | 0,427 | 0,297 | 0,218 |
| $c_D$ *[-]* | 0,3766 | 0,0920 | 0,0442 | 0,0311 | 0,0264 | 0,0244 |
| *D [N]* | 1384 | 761 | 649 | 714 | 872 | 1097 |
| *w [m/s]* | -2,82 | [illegible] | -2,65 | -3,64 | 5,34 | -7,83 |

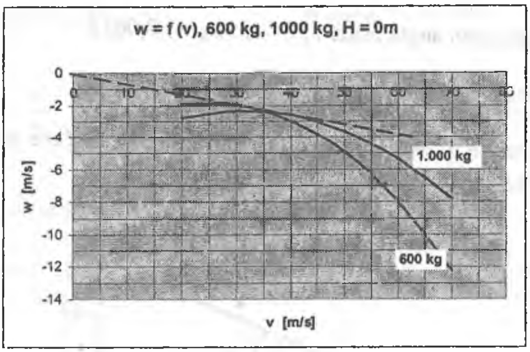
The diagram shows two important conditions.

The best angle and the speed to achieve it can be found with the tangent from the origin.

The lowest vertical speed and the speed to achieve it can be found with the horizontal tangent.



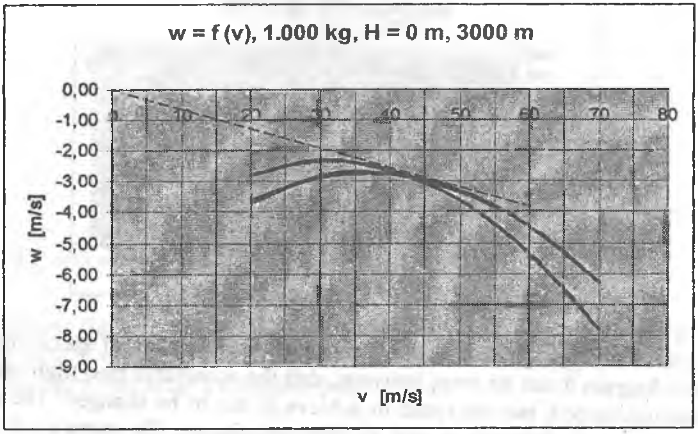

With a differing mass, the vertical speed as a function of speed changes. In another diagram it can be seen, however, that the achievable best flight path angle remains unchanged, but the speed to achieve it, has to be changed. The reason is that the best ratio between $c_D$ and $c_L$ does not change. The same $c_L$ at different masses, however, depends on the flown speed. This is also valid for flights at various altitudes (densities).




The achievable minimum vertical speeds are different under all conditions!

In preparation for the following chapters "distance to descend" and "time to descent" the above shown diagram shall be developed for the conditions of different altitudes. As in the chapter for climbing flight, 0 m and 3000 m are chosen. The formulas to find the curves are already known. Again the sample aeroplane with a mass of 1000 kg will be chosen.

| $v$ [m/s] | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|
| $w_{0000\,m}$ [m/s] | -2,82 | -2,33 | -2,65 | -3,64 | -5,34 | -7,83 |
| $w_{3000\,m}$ [m/s] | -3,70 | -2,80 | -2,77 | -3,34 | -4,50 | -6,27 |

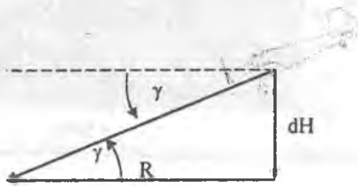



Flying the respective speed for the best flight path angle, the following vertical speeds are found : $w_{0000\,m}$ = 2,64 m/s $w_{3000\,m}$ = 3,07 m/s

The flight path angle reads : $\gamma = -c_D / c_L$ = - 0,0652 = - 3,73°

### *II.6.4 Distance to descent*

The distance to descent can be easily determined, if the speed is maintained for the best flight path angle.




$$\tan\gamma = \frac{dH}{R} \qquad \tan\gamma = \gamma \qquad \Rightarrow \qquad R = \frac{H}{\gamma}$$

$c_L = 0{,}665$ for the best flight path angle $\rightarrow$ $c_D = 0{,}0433$

$$\gamma = -\frac{c_D}{c_L} = -\frac{0{,}0433}{0{,}665} = -0{,}0652$$

$$R = \frac{3000\,m}{0{,}0652} = 46.012\,m$$ (the negative prefix can be neglected)

Another way to find the result is to multiply ***dH*** with the gliding number:

$$E = \frac{c_L}{c_D} = \frac{0{,}665}{0{,}0433} = 15{,}36 \qquad R = dH \cdot E = 3000m \cdot 15{,}36 = 46.073m$$

(Minor difference through rounding error)

It will be shown later, that the pilot will keep a constant flight path angle for operational reasons!

### *II.6.5 Time to descent*

To find the "time to descent", the same formula as for "time to climb" can be used.

In the example in II.6.3 vertical speeds for the descend have been found:

$w_{3000\,m} = 3{,}07$ m/s = $w_1$ $\qquad$ $w_{0000\,m} = 2{,}64$ m/s = $w_2$

$$t = -\frac{H_2 - H_1}{w_1 - w_2} \cdot \ln\frac{w_2}{w_1} = -\frac{3000\,m - 0\,m}{3{,}07\frac{m}{s} - 2{,}64\frac{m}{s}} \cdot \ln\frac{2{,}64}{3{,}07} = 1053\,s = 17\,\text{min}\;33\,s$$

The reasonability can be easily checked:

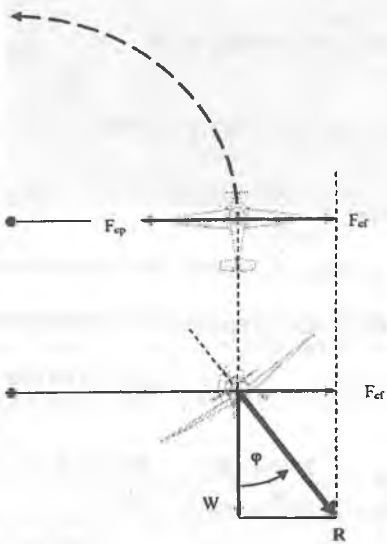
the average vertical speed is approximately 2,8 m/s. 3000 m: 2,8 m/s = 1071 s = 17 min 51 s

## II.7 Turning flight

### *II.7.1 Forces in turning flight*

In order to maintain a mass point (an aeroplane) on a circle track, a force must act on that mass, which is directed to the centre of the circle, the centripetal force.

The mass point itself, due to its inertia (that wants to keep it on a straight track), exerts an opposite (radial) force away from the centre, the centrifugal force.

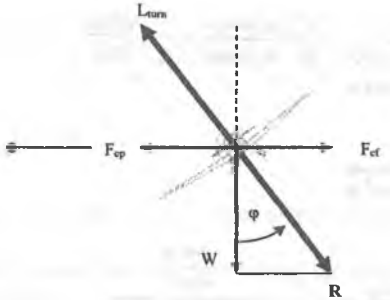



With the weight **W** still pointing to the centre of earth ($+z_g$-direction) and the centrifugal force $\mathbf{F_{cf}}$ the force acting on the aeroplane in **z** – direction is the resulting force **R**.

$$\cos\varphi = \frac{W}{R} \quad \Rightarrow \quad R = \frac{W}{\cos\varphi} = W \cdot \frac{1}{\cos\varphi}$$

The factor $\dfrac{1}{\cos\varphi}$ is called the load factor **n**.

$$n = \frac{1}{\cos\varphi} \quad \Rightarrow \quad R = W \cdot n = m \cdot g \cdot n$$




the lift **L**, that shows in negative $z$ – direction has to be equal in amount to **R**.

$$L_{turn} = R = W \cdot n = m \cdot g \cdot n = m \cdot g \cdot \frac{1}{\cos\varphi}$$

the $z_g$ – component of **L**turn is : $z_g = L_t \cdot \cos\varphi = m \cdot g \cdot \frac{1}{\cos\varphi} \cdot \cos\varphi = W$

the $y_g$ – component of **L**turn is: $y_g = L_t \cdot \sin\varphi = m \cdot g \cdot \frac{1}{\cos\varphi} \cdot \sin\varphi = m \cdot g \cdot \frac{\sin\varphi}{\cos\varphi}$

### *II.7.2 Turning radius*

The general equation for centrifugal force reads: $F = m \cdot \frac{v^2}{r}$

It can be used to determine the turning radius: $r = m \cdot \frac{v^2}{F}$

with $F = m \cdot g \cdot \frac{\sin\varphi}{\cos\varphi}$ it reads

$$r = m \cdot \frac{v^2}{m \cdot g \cdot \frac{\sin\varphi}{\cos\varphi}} = \frac{v^2}{g} \cdot \frac{m \cdot \cos\varphi}{m \cdot \sin\varphi} = \frac{v^2}{g} \cdot \frac{\cos\varphi}{\sin\varphi}$$

$$\sin\varphi = \frac{1}{\sqrt{1-\cos^2\varphi}}$$

$$r = \frac{v^2}{g} \cdot \frac{\cos\varphi}{\sqrt{1-\cos^2\varphi}} = \frac{v^2}{g} \cdot \frac{\frac{\cos\varphi}{\cos\varphi}}{\sqrt{\frac{1}{\cos^2\varphi} - \frac{\cos^2\varphi}{\cos^2\varphi}}}$$

$$n = \frac{1}{\cos\varphi}$$

$$r = \frac{v^2}{g} \cdot \frac{1}{\sqrt{n^2-1}}$$

An example shall be calculated for the sample aeroplane at a speed of 50 m/s and various bank angles $\varphi$.

| $\varphi$ [°] | 0 | 15 | 30 | 45 | 60 |
|---|---|---|---|---|---|
| $n$ [-] | 1,00 | 1,04 | 1,15 | 1,41 | 2,00 |
| $R$ [m] | ∞ | 951 | 442 | 255 | 147 |

### *II.7.3 Load factor and minimum speeds*

With $L_t = R = m \cdot g \cdot n$ it has been shown, that more lift is required in a turning flight. With a constant speed $v$ the lift coefficient $c_L$ must be increased to produce more lift.

The maximum $c_L$, that can be achieved by a wing is unchanged, whether in straight or turning flight. The minimum speed, that can be flown, is depending on that $c_L$.

$$v_{min} = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot c_{L_{max}} \cdot S}}$$

In a turning flight the lift **L** has to be equal to **R**, which is $R = L \cdot n = m \cdot g \cdot n$.

$$v_{\min turn} = \sqrt{\frac{2 \cdot R}{\rho \cdot c_{L_{\max}} \cdot S}} = \sqrt{\frac{2 \cdot m \cdot g \cdot n}{\rho \cdot c_{L_{\max}} \cdot S}} = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot c_{L_{\max}} \cdot S}} \sqrt{n}$$

$$v_{\min turn} = v_{\min straight} \cdot \sqrt{n}$$

The minimum speed in a turn is $\sqrt{n}$ times higher than in a straight flight.

The following table shows the load factor n and the increase of the minimum speed at various bank angles:

| $\varphi$ [°] | 0 | 15 | 30 | 45 | 60 |
|---|---|---|---|---|---|
| n [-] | 1,00 | 1,04 | 1,15 | 1,41 | 2,00 |
| $\Delta v_{min}$ [%] | 0 | 2 | 7 | 19 | 41 |

### *II.7.4 Drag and thrust*

The increased $c_L$ that must be maintained in a turning flight with a constant speed also increases the coefficient drag $c_D$.

$$c_D = c_{D,0} + \frac{c_L^{\ 2}}{e \cdot \pi \cdot \Lambda}$$

The coefficient lift $c_L$ increases by the factor n in turning flight.

$$L_{turn} = m \cdot g \cdot n = c_L \cdot n \cdot \frac{\rho}{2} \cdot v^2 \cdot S$$

The coefficient drag $c_D$ increases accordingly: $c_D = c_{D,0} + \frac{(c_L \cdot n)^2}{e \cdot \beta\pi \cdot \Lambda}$

and so does the drag : $D = (c_{D,0} + \frac{(c_L \cdot n)^2}{e \cdot \pi \cdot \Lambda}) \cdot \frac{\rho}{2} \cdot v^2 \cdot S$

The effect of turning flight is the same as of an increased mass, which was shown under II.4.2. The mass increase in the given examples there has the same effect like the load factor n here.

The thrust increase, that is necessary to maintain airspeed, is identical to the drag increase.

## II.8 Operational procedures

This chapter will examine the effects that have been discussed so far, on the operation of an aeroplane from a pilot's view.

### *II.8.1 Indicated and true airspeed*

So far, it has been discussed, at which speeds an aeroplane has to be operated to fly certain points in the performance range. All speeds have been speeds through the air.

It has not been discussed, which airspeeds are indicated to a pilot in the cockpit.

Under I.1.4 it has been shown, that the airspeed is measured via the dynamic pressure that is the total pressure minus the static pressure. The dynamic pressure has been expressed as

$$q = \frac{\rho}{2} \cdot v^2$$

That formula can be transformed to find the speed: $v = \sqrt{\frac{2q}{\rho}}$

It can be seen immediately, that with the same dynamic pressure q the calculated airspeed changes with the density of the air.

An example shall make this clear.

At mean sea level with a density of 1,225 kg/m$^3$ at a speed of 50 m/s the dynamic pressure is:

$$q = \frac{1{,}225}{2} \cdot 50^2 \; Pa = 1531{,}25 \; Pa$$

When the same dynamic pressure is measured at 3.000 metres with a density of 0,910 kg/m$^3$ the airspeed is:

$$v = \sqrt{\frac{2 \cdot 1531{,}25}{0{,}910}} \frac{m}{s} = 58{,}01 \frac{m}{s}$$

This increase in speed through the air, the "true airspeed", can be shown via the equation for speed: $v = \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot c_L \cdot S}}$

It also has been mentioned, that flight conditions are identical, when the coefficient lift $c_L$ is identical.

$$\frac{v_{3000m}}{v_{0000m}} = \frac{\sqrt{\dfrac{2 \cdot m \cdot g}{\rho_{3000m} \cdot c_L \cdot S}}}{\sqrt{\dfrac{2 \cdot m \cdot g}{\rho_{0000m} \cdot c_L \cdot S}}}$$

With m, g, $c_L$ and S being the same at both altitudes, the only different parameter is density ρ.

The equation then reads:

$$v_{3000m} = v_{0000m} \cdot \sqrt{\frac{\rho_{0000m}}{\rho_{3000m}}} = 50 \, {}^{m}\!/\!_{s} \cdot \sqrt{\frac{1{,}225}{0{,}910}} = 58{,}01 \, {}^{m}\!/\!_{s}$$

or in general:

$$v = v_0 \cdot \sqrt{\frac{\rho_0}{\rho}}$$

But what does the pilot see on the airspeed indicator?

At mean sea level the dynamic pressure at 50 m/s is:

$$q = \frac{1{,}225}{2} \cdot 50^2 \, Pa = 1531 \, Pa$$

At 3000 metres the dynamic pressure at 58,01 m/s is .

$$q = \frac{0{,}910}{2} \cdot 58{,}01 \, Pa = 1531 \, Pa$$

With the same dynamic pressure, the pilot has the same indication on his airspeed indicator, the same "indicated airspeed"!

$$v_{true} = v_{indicated} \cdot \sqrt{\frac{\rho_0}{\rho}}$$

Now it has been shown, that a pilot can maintain any flight condition that depends on a certain coefficient lift cL, when he maintains a constant indicated airspeed, which is very beneficial for the operation of an aeroplane.

This will be demonstrated with two examples!

The minimum speed of the sample aeroplane with a mass of 1000 kg at mean sea level with a $c_L = 1{,}2$ is :

$$v_{min} = \sqrt{\frac{2 \cdot 1000 \cdot 9{,}81}{1{,}225 \cdot 1{,}2 \cdot 15}} = 29{,}83 \, {}^{m}\!/_{s}$$

With the same parameters, however at 3000 m ($\rho = 0{,}910$ kg/m$^3$) it is:

$$v_{min} = \sqrt{\frac{2 \cdot 1000 \cdot 9{,}91}{0{,}910 \cdot 1{,}2 \cdot 15}} = 34{,}61 \, {}^{m}\!/_{s}$$

The dynamic pressures in both conditions are:

$$q_{0000m} = \frac{1{,}225}{2} \cdot 29{,}83^2 = 545\,Pa \qquad q_{3000m} = \frac{0{,}910}{2} \cdot 34{,}61^2 = 545\,Pa$$

With the same dynamic pressure in both conditions the pilot has the same indication on his airspeed indicator at all altitudes, although the true airspeeds are different.

This can also be shown with the formula $v_{true} = v_{indicated} \cdot \sqrt{\frac{\rho_0}{\rho}}$.

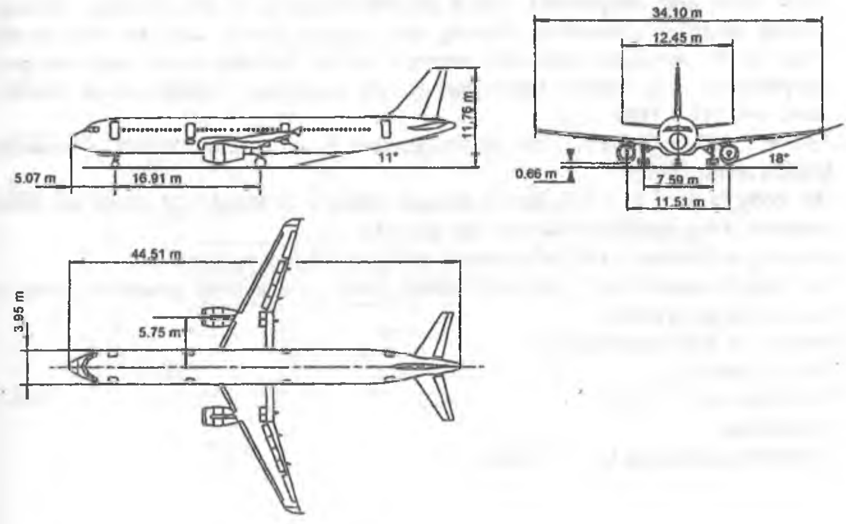
$$v_{indicated} = \frac{v_{true}}{\sqrt{\frac{\rho_0}{\rho}}} = \frac{34{,}61}{\sqrt{\frac{1{,}225}{0{,}910}}} = 29{,}83 \, {}^{m}\!/_{s}$$

# APPENDEX

**Annex 1.**

## A320 Family Aircraft Systems


***Additional Information for self-study***



**Airbus A321: Cockpit**

**Air Conditioning (ATA 21)**

***Those units and components, which furnish a means of pressurizing, heating, cooling, moisture controlling, filtering and treating the air, used to ventilate the areas of the fuselage within the pressure seals. Includes cabin supercharger, equipment cooling, heater, heater fuel system, expansion turbine, valves, scoops, ducts, etc. (ATA 100)***

Above 11000 m (36089 ft) the air temperature is at constant -56.5 °C (=> cabin heating and cooling).

Our body is used to a low partial oxygen pressure of about 0.21 times sea level pressure. If we want to survive at high altitudes

the oxygen fraction has to be increased (using an oxygen system) or

the total pressure has to be maintained close to sea level pressure (using a pressurization system).

Purpose of Air Conditioning

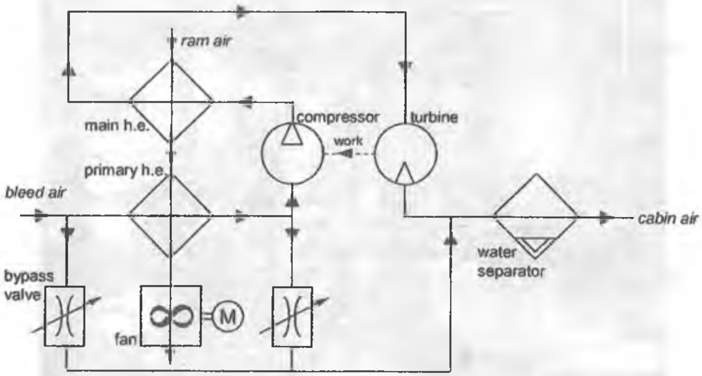
Heating, cooling

Pressurization

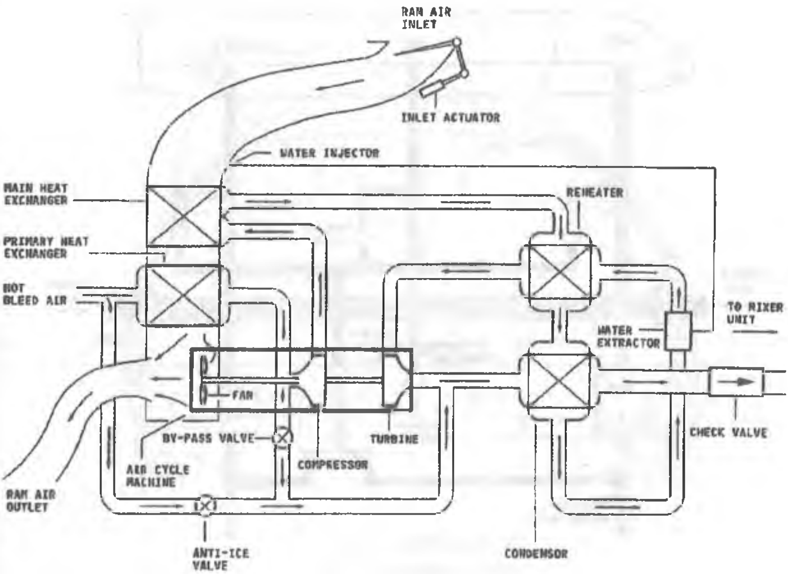
Ventilation

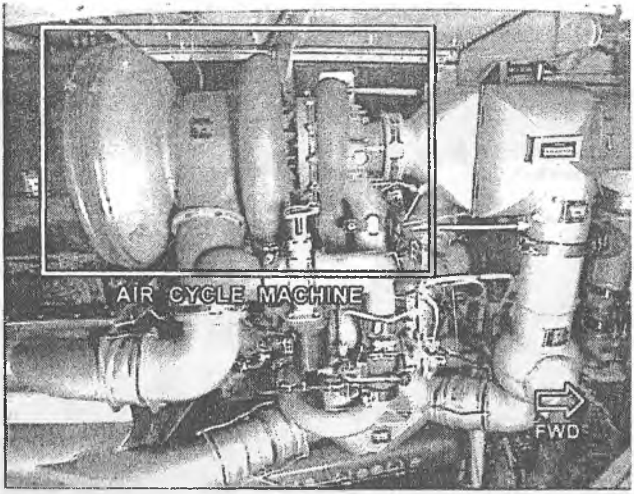
Humidity control (as far as possible)

## Open bootstrap air cycle system

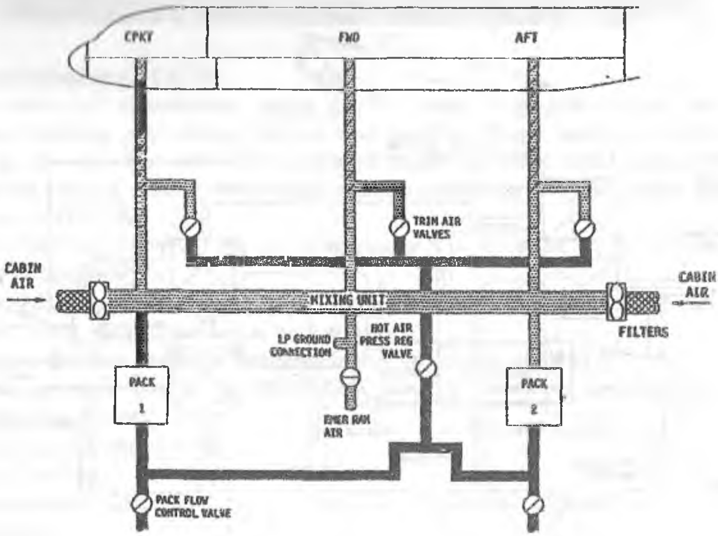


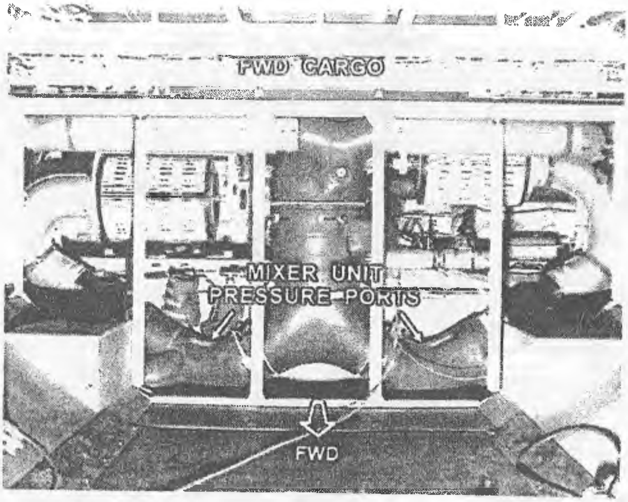

## Air cycle system



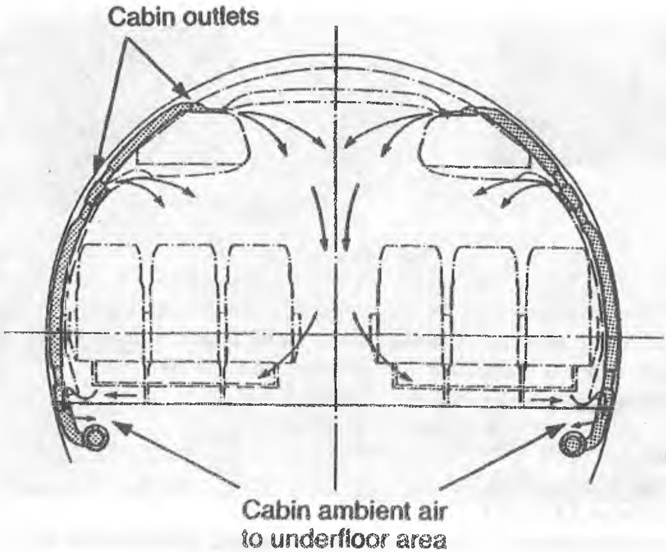


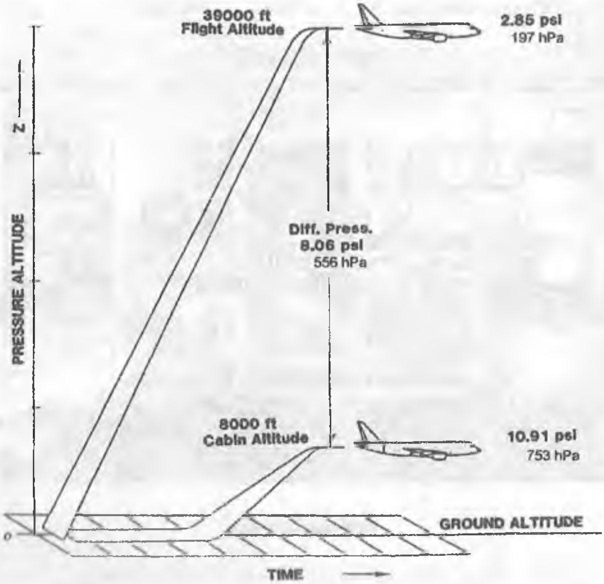

**Ventilation**

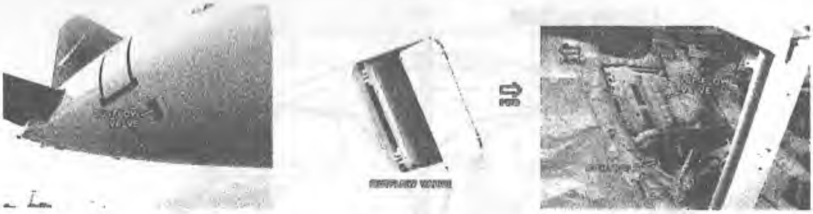





**Pressurization**





## Auto Flight (ATA 22)

Those units and components which furnish a means of **automatically controlling the flight of the aircraft**. Includes those units and components which control direction, heading, attitude, altitude and speed. **(ATA 100)**

**Flight Management and Guidance System (FMGS)**

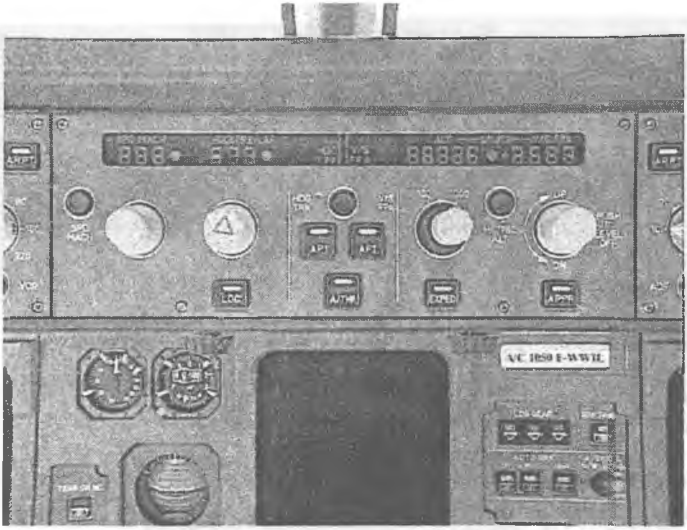
The FMGS performs the functions given below:

Autopilot (AP)

Flight director (FD)

auto thrust (A/THR)

Flight management which includes navigation, performance and processing of displays.



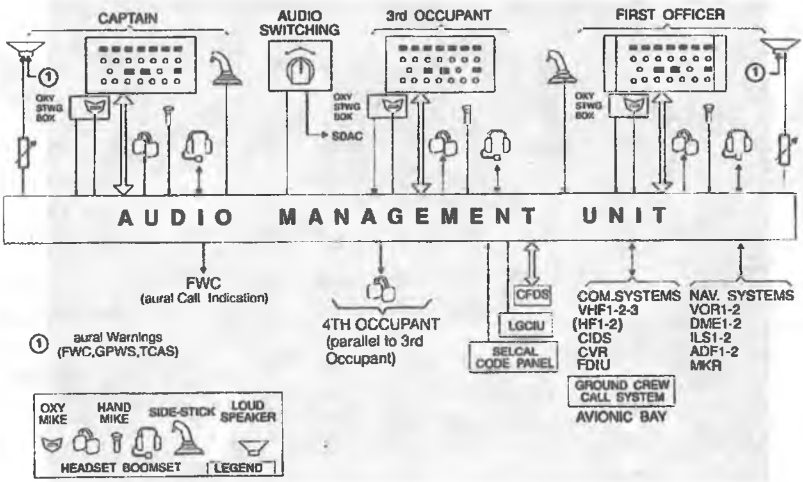

**A321Flight Control Unit (FCU) Interface to Auto Pilot**

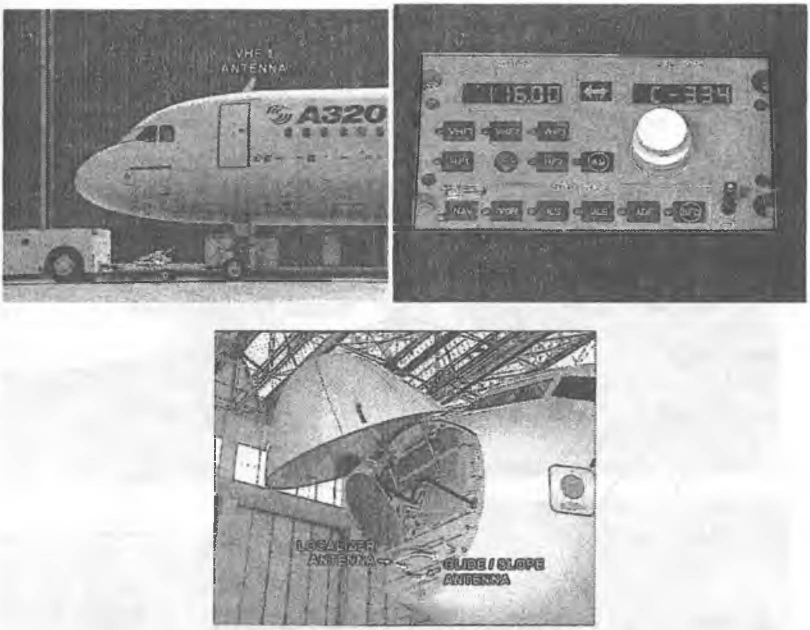

**A321 Multipurpose Control & Display Unit (MCDU)**
**Interface to Flight Management System**

## Communication (ATA 23)

Those units and components which furnish a means of communicating from one part of the aircraft to another and between the aircraft or ground stations, includes voice, data, PA [Passenger Address] system, intercom and tape reproducer-record player. (ATA 100)

Very High Frequency (VHF) System
Aircraft Communications Addressing and Reporting System (ACARS)
High Frequency (HF) System
Radio Management Panels (RMP)
Audio Management System
Mechanic Call System
Service Interphone
Passenger Address System
Cabin and Flight Crew Interphone System
Cabin Intercommunication Data System (CIDS)
Announcement - Music Tape Reproducer System
Passenger Entertainment System (Video)
Cockpit Voice Recorder (CVR) System
Static Discharging





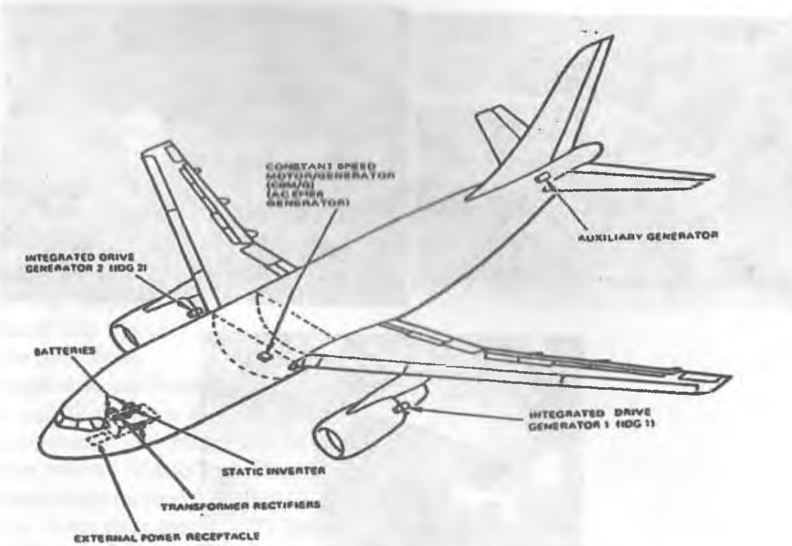

**ATA 23 – Communications**

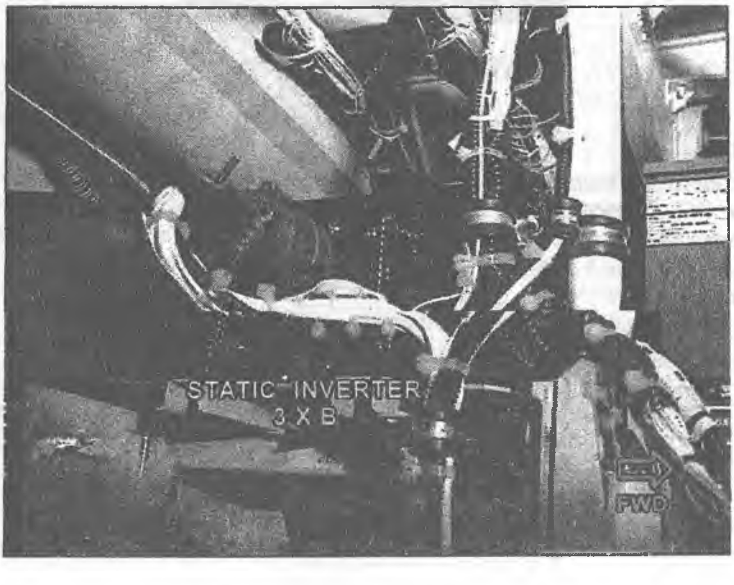
## Electrical Power (ATA 24)

*Those electrical units and components which generate, control and supply AC and/or DC electrical power for other systems, including generators and relays, inverters, batteries, etc., through the secondary busses. Also includes common electrical items such as wiring, switches, connectors, etc. (ATA 100)*

Power generation:

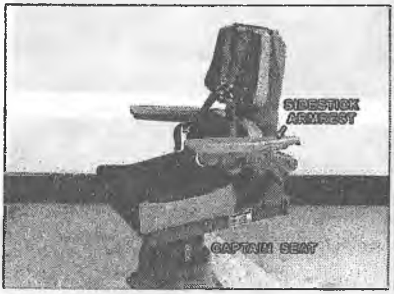
Generator drive systems: constant speed drives (CSD)
Alternating current (ac) generation
Direct current (dc) generation
External power distribution:
Alternating current (ac) electrical load distribution
Direct current (dc) electrical load distribution.

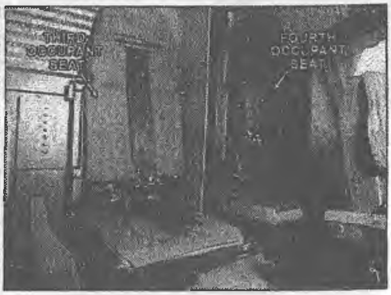

CONSTANT SPEED MOTOR/GENERATOR (CSM/G) (AC EMER GENERATOR)
AUXILIARY GENERATOR
INTEGRATED DRIVE GENERATOR 2 (IDG 2)
BATTERIES
INTEGRATED DRIVE GENERATOR 1 (IDG 1)
STATIC INVERTER
TRANSFORMER RECTIFIERS
EXTERNAL POWER RECEPTACLE

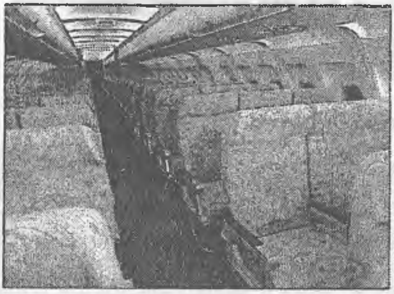

STATIC INVERTER
3 X B
FWD

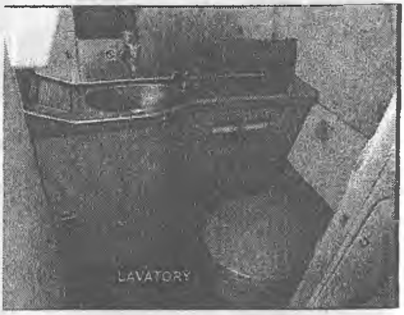
## Equipment / Furnishings (ATA 25)

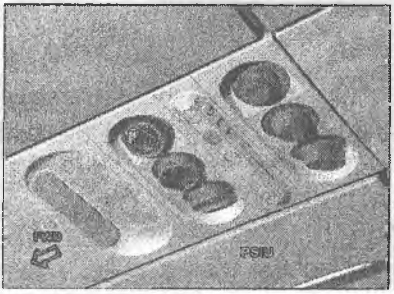
Those removable items of equipment and furnishings contained in the flight and passenger compartments. Includes emergency, galley and lavatory equipment. Does not include structures or equipment assigned specifically to other ... [systems]. *(ATA 2200)*

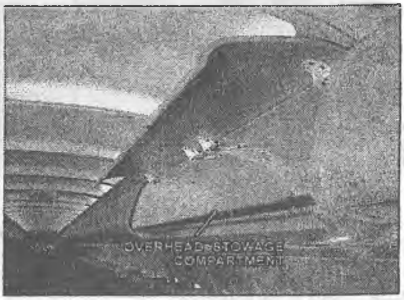


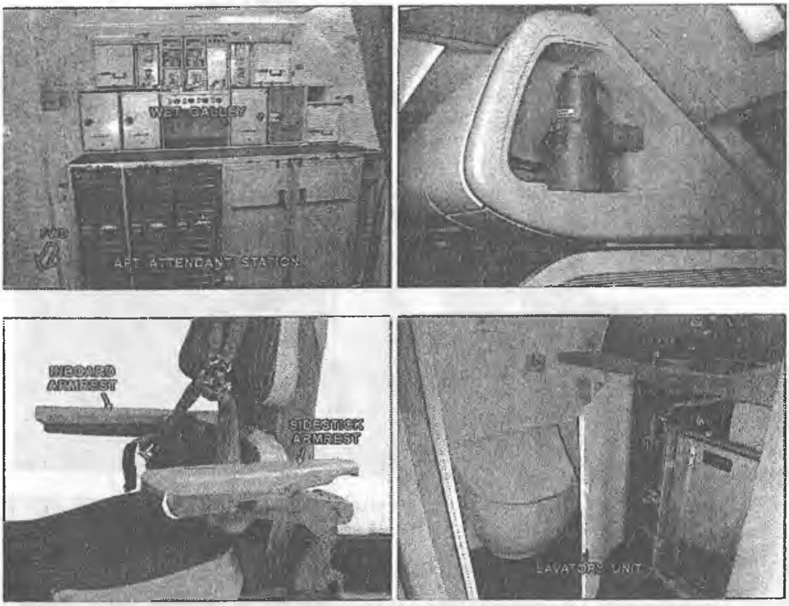



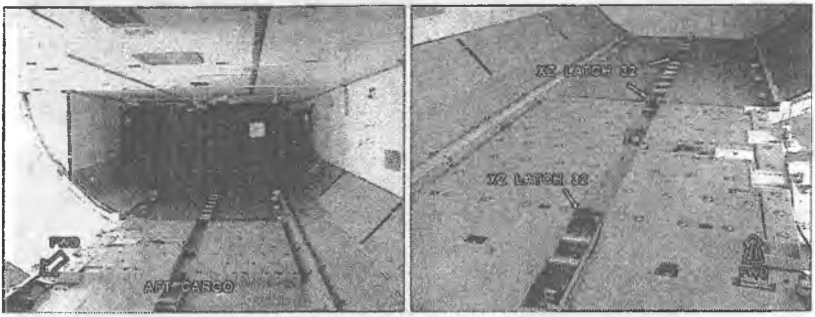




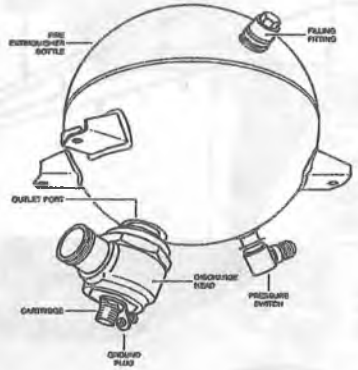

**ATA 25 –Equipment /Furnishings**




## Fire Protection (ATA 26)

*Those fixed and portable units and components which detect and indicate fire or smoke and store and distribute fire extinguishing agent to all protected areas of the aircraft; including bottles, valves, tubing, etc. (ATA 100)*

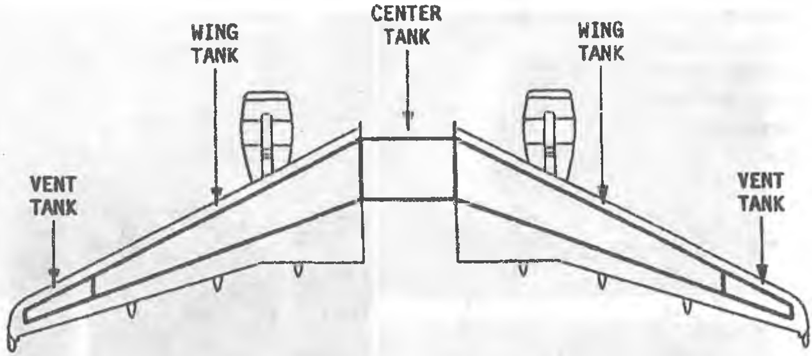
Detection:
Direct observation
Overheat detector
Smoke detector
Extinguishing:
Halon






## Fuel (ATA 28)

Those units and components which **store and deliver fuel** to the engine. Includes engine driven fuel pumps for reciprocating engines, includes tanks (bladder), valves, boost pumps, etc., and those components which furnish a means of dumping fuel overboard. Includes integral and tip fuel tank leak detection and sealing. Does not include the structure of integral or tip fuel tanks and the fuel cell backing boards which are ... [part of the structure], and does not include fuel flow rate sensing, transmitting and/or indicating, which are covered ... [by the power plant systems]. **(ATA 100)**


CENTER
TANK
WING
TANK
WING
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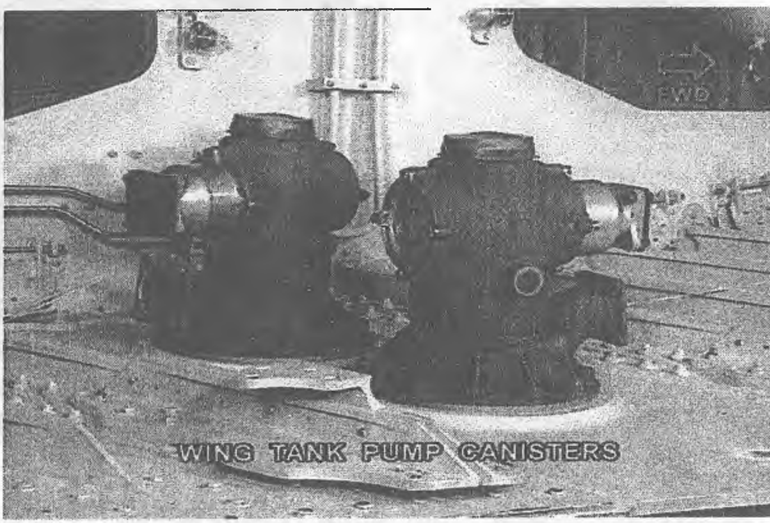
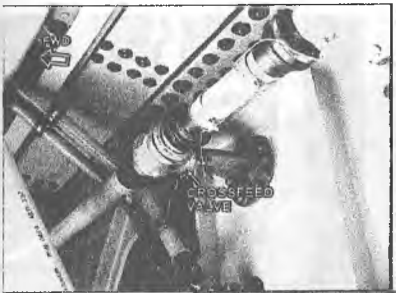
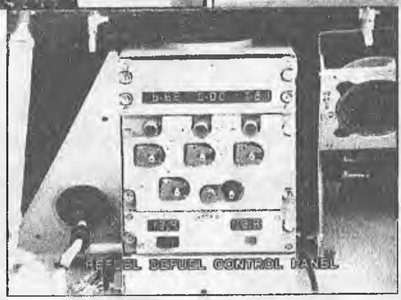
VENT
TANK
VENT
TANK


FWD
WING TANK PUMP CANISTERS



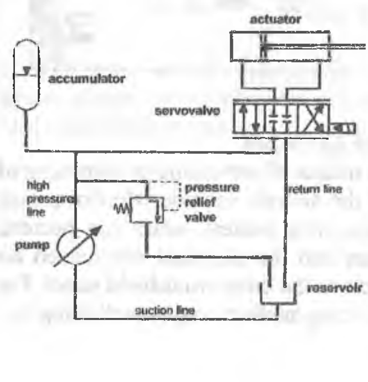


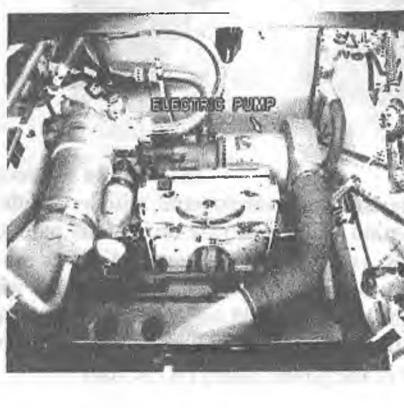


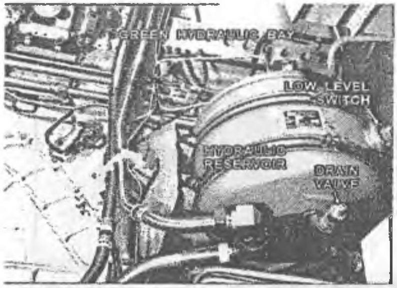

**ATA 28 – Fuel**

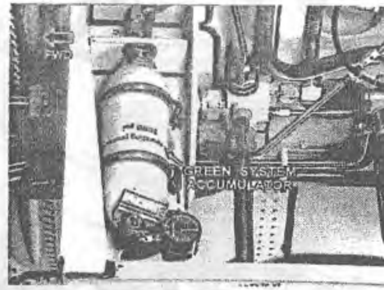
**Hydraulic Power (ATA 29)**

*Those units and components which **furnish hydraulic fluid under pressure** (includes pumps, regulators, lines, valves, etc.) to a common point (manifold) for redistribution to other defined systems. (ATA **100**)*

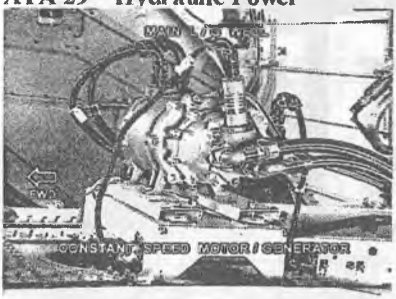


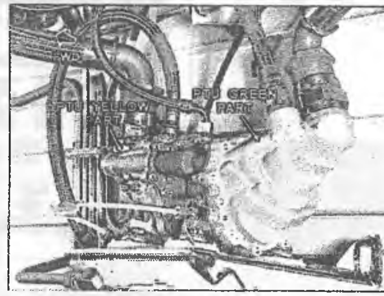



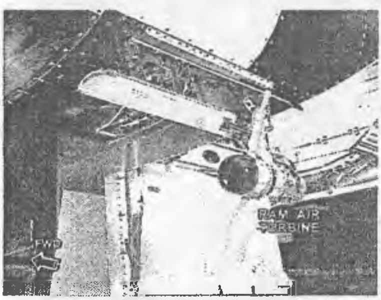


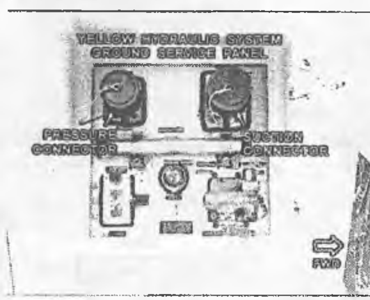




**ATA 29 – Hydraulic Power**



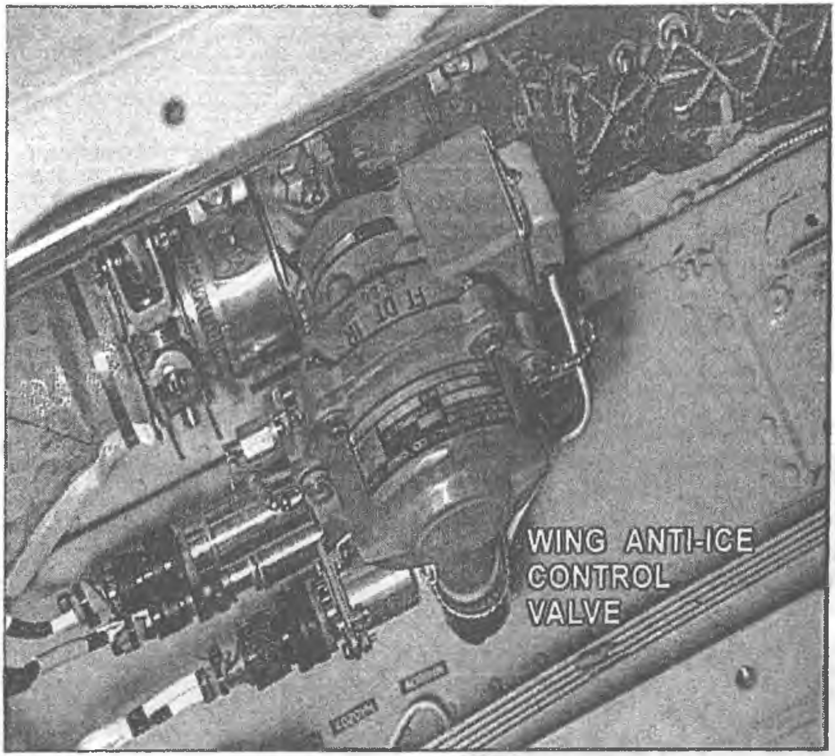




**ATA 29 – Hydraulic Power**

## Ice & Rain Protection (ATA 30)

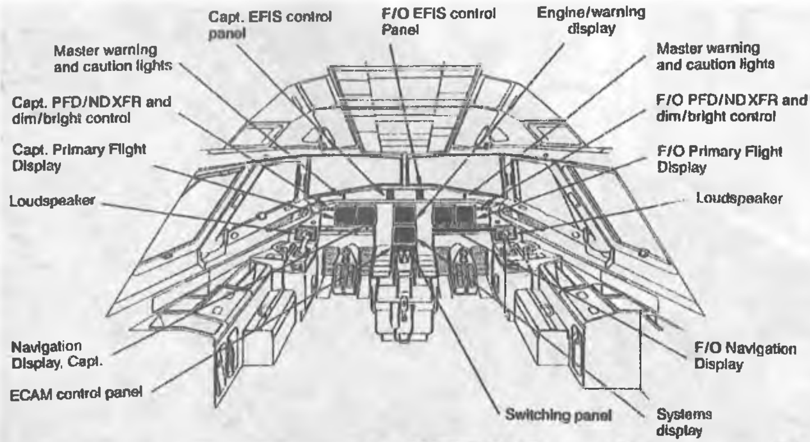
Those units and components, which provide a means of preventing or disposing of formation of ice and rain on various parts of the aircraft. Includes alcohol pump, valves, tanks, propeller/rotor anti-icing system, wing heaters, water line heaters, pitot heaters, scoop heaters, windshield wipers and the electrical and heated air portion of windshield ice control. Does not include the basic windshield panel. For turbine type power plants using air as the anti-icing medium, engine anti-icing is... [part of the power plant] (ATA 100)

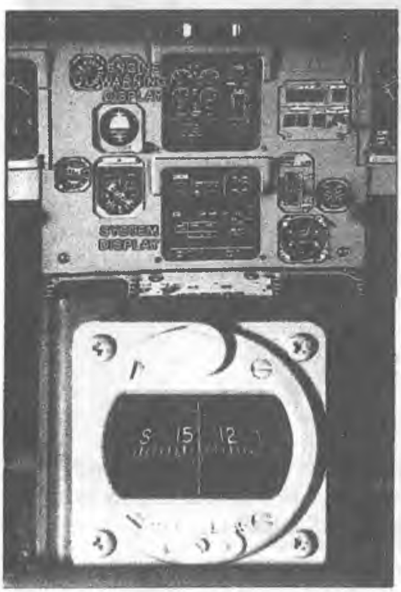



**ATA 30 – Ice & Rain Protection**

Indicating & Recording (ATA 31)

... Coverage of all instruments, instrument panels and controls... Includes systems/units which integrate indicating instruments into a central display system and instruments not related to any specific system. (ATA 100)



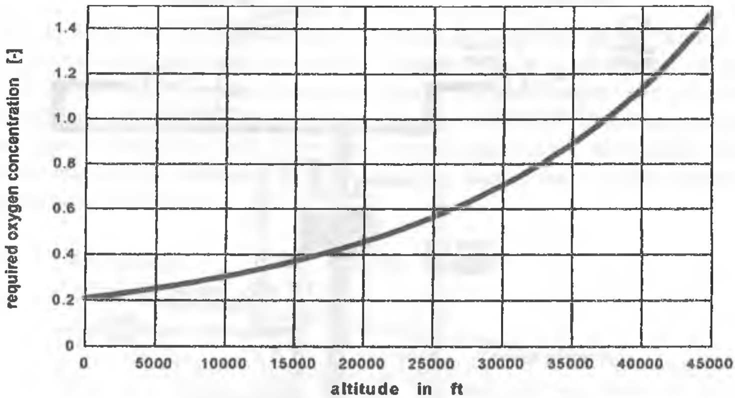



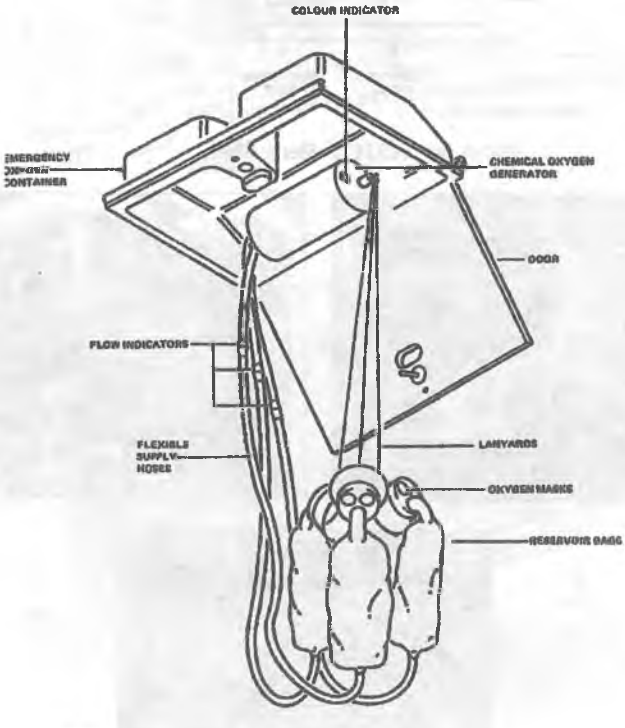



**ATA 31 – Indicating/Recording Systems      ATA 34 – Navigation**

**Oxygen (ATA 35)**

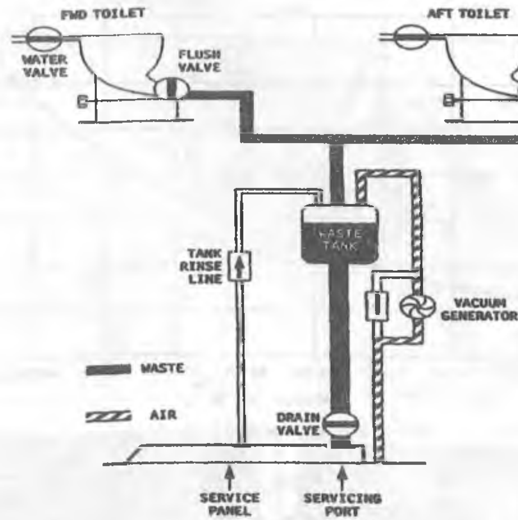
Those units and components which store, regulate, and **deliver oxygen to the passengers and crew**, including bottles, relief valves, shut-off valves, outlets, regulators, masks, walk-around bottles, etc. **(ATA 100)**



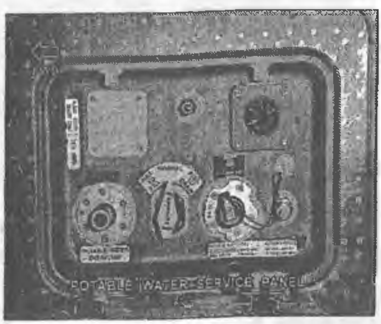




NOTE: 3 MASK CONTAINER SHOWN

**A321 passenger oxygen system**




**A321 Toilet system**

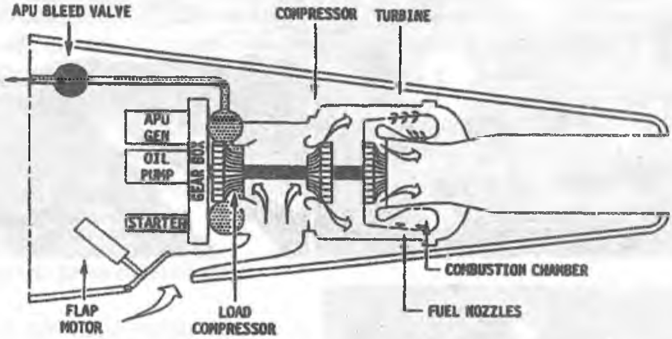




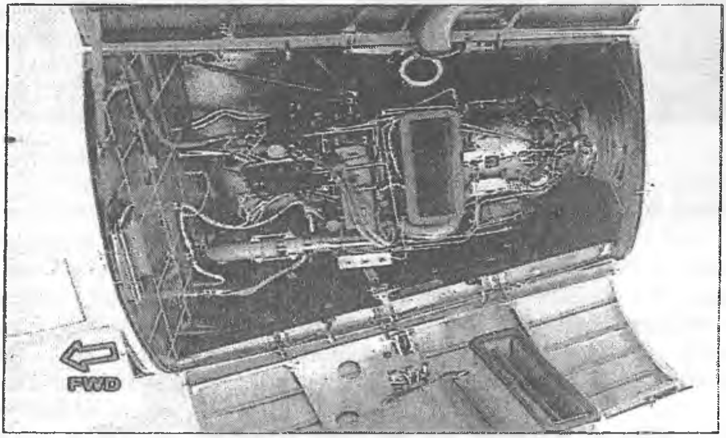

**ATA 38 – Water / Waste**

## Auxiliary Power (ATA 49)

Those airborne power plants (engines) which are installed on the aircraft for the purpose of generating and supplying a single type or combination of auxiliary electric, hydraulic, pneumatic or other power. Includes power and drive section, fuel, ignition and control systems; also wiring, indicators, plumbing, valves, and ducts up to the power unit. Does not include generators, alternators, hydraulic pumps, etc. or their connecting systems which supply and deliver power to their respective aircraft systems. (ATA 100)




**A321 Auxiliary Power Unit**




**ATA 49 – Airborne Auxiliary Power**

Summary and Beyond

Aircraft systems: 1/3 of the total aircraft

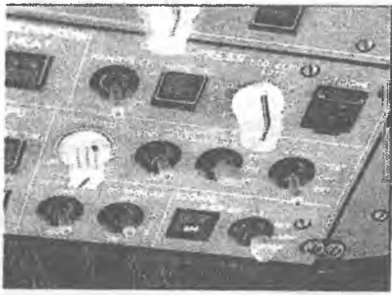
(related to purchase price, weight and costs)

Aircraft Systems are ordered by "ATA Chapters"

from ATA 21 (Air Conditioning)

to ATA 49 (Auxiliary Power)

The A321 uses state of the art aircraft system technology only be topped by the A380.



**Annex 2**

**Basic of helicopter performance**

*1 Features of Helicopter*

**1.1 VTOL (Vertical Take-Off and Landing)**

Lift that is higher than weight of aircraft is needed in static state for VTOL. As it needs extra power and equipments for VTOL, its cruise performance is worse than one of fixed-wing aircraft. Like this, hovering and cruising are each conflicted. Therefore, there are various types of VTOL for its usage. But it's widely classified like these; vectored thruster which changes the direction of jet thrust to vertical, tilt rotor which changes the direction of rotor for cruise or take-off and landing, and helicopter which gets the both lift and thrust using rotor whose shaft is fixed in body.

Vectored thruster and tilt rotor types have the fixed wing for cruise, and change the direction of thrust temporarily for take-off and landing. However, helicopter has no fixed wing. Therefore, helicopter is defined that the aircraft, which gets the most lift force from rotor in whole flight, range not only take-off and landing but also cruise.

**1.2 Basic form of helicopter**

There are some considerations to solve for flight of helicopter; efficient occurrence of force, which can overcome the drag and weight using only rotor, canceling of reaction torque, which is caused by torque of rotor, and controlling of the direction of rotor for forward flight with safety. The solutions for these problems have changed with development of technology and manufacture.

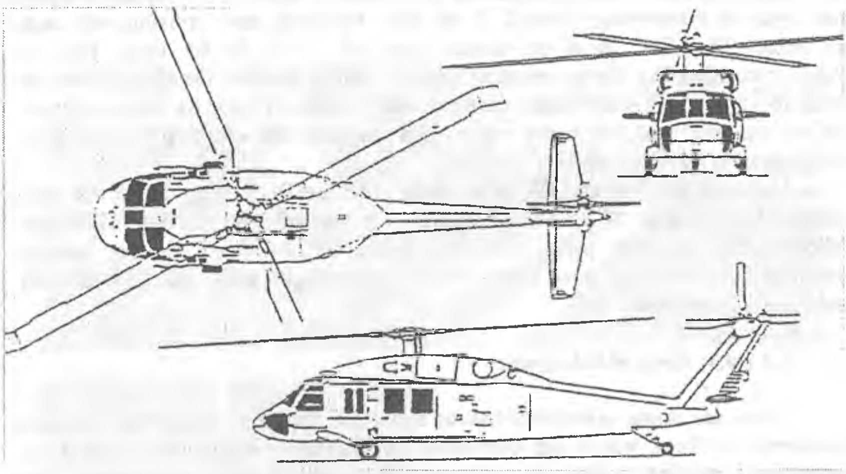
Basic form of helicopter is decided by the way of canceling of reaction torque. Complexity, safety, utility and aerodynamic efficiency are also considered for selection of form.

(1) Single rotor helicopter

This form is the most widely used, and it is composed of one main rotor and tail rotor. As this helicopter has only one rotor, control and transfer of power is relatively simple. Tail rotor, however, is not contributed to thrust and lift at all, and it only cost the 8~10% of total power in hovering and 3~4% of total power in forward flight. However, it can compensate that loss as relatively low weight, because its structure is simple. It can also install the vertical stabilizer for efficiency of forward flight and horizontal stabilizer for pitch stability of body. Ducted tail rotor whose rotor is surrounded by tail is also sometimes used for high efficiency and low noise.

When single-rotor helicopter that has the horizontal rotor-plane flies forward, relative speed of rotor-blade that rotates in the reverse way of forward is low. In addition, when rotor-blade rotates in the direction of forward, its tip reaches drag coefficient Mach number. Therefore, required power is suddenly increased and vibration is also increased. Therefore, maximum speed of helicopter is usually limited as 380km/h by this problem.

Moreover, tail-rotor needs to be separated far away from center of mass to get big anti-torque. Therefore, this is connected to long length of helicopter that is difficult to hangar in small place. Long tail is also disadvantage for flight at low altitude, because it is easy to be broken by ground object. Actually, lots of helicopter accident is caused by trouble of tail rotor.

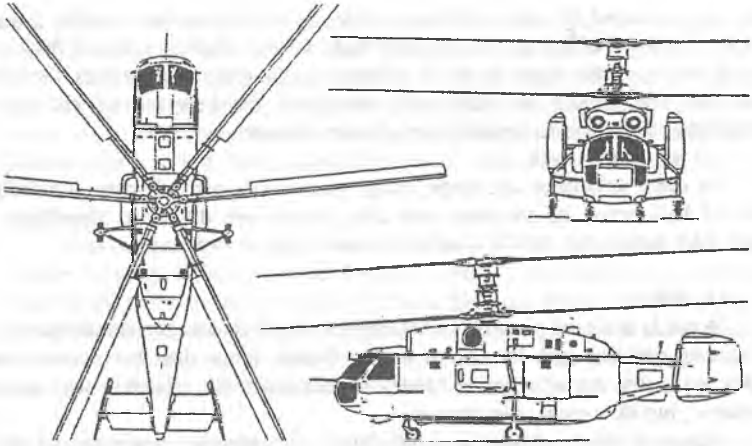


**[Figure 1]** Form of single rotor helicopter

(2) Coaxial rotor helicopter

It cancels the torque using two rotors that rotates in the opposite direction with same shaft. As it does not need tail rotor, the length of rotor decides its full length. This type is suitable for naval vessel that does not have enough places.

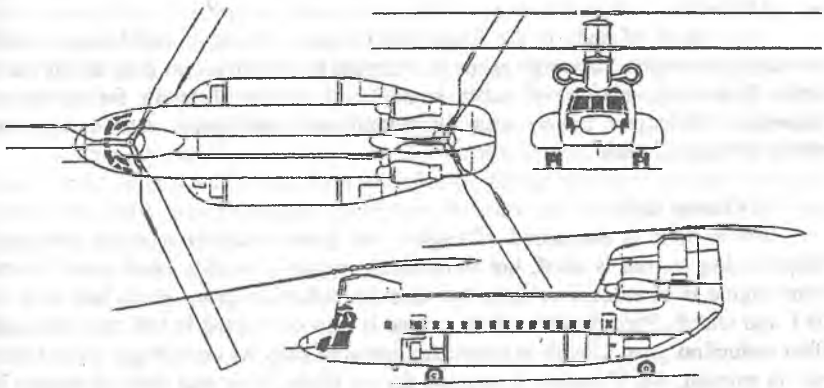
In aerodynamic respect, lower rotor is in the wake of upper rotor. Therefore, its efficiency is lower than if two rotors are operated separately. Nevertheless, there is no dissymmetry of left and right when helicopter flies forward, because each rotor rotates in the opposite direction. Therefore, its maximum velocity is increased. As center section of rotor and control system are very complex, its efficiency is decrease because of weight. Therefore, it is very difficult to design efficient helicopter. For these reasons, mainly Russia can manufacture this type of helicopter for practical use.



[Figure 2] Form of coaxial rotor helicopter

(3) Side by side rotor helicopter

It cancels the torque with set of left and right rotors which rotate in the opposite direction. Its shape is similar with coaxial rotor helicopter, but its drive shaft is simpler than coaxial rotor helicopter. It has high cruise efficiency because side rotors are worked like high aspect ratio. However, its power-transfer and control systems are complex, so its weight is increased. And its size is also increased, so that it's not spotlighted as helicopter.



[Figure 3] Form of tandem rotor helicopter

(4) Tandem rotor helicopter

It cancels the torque using reverse rotation of two rotors which are set front and rear. The most advantage of this type is its wide allowable range of center of mass. So it's used for carry of heavy objects. But its power transfer system is complex, and lift efficiency of forward flight is low. During forward flight, front rotor is below of rear rotor not to be affected by the wake. Sometimes, the sizes of front and rear rotors are differently designed for increase of stability and comfortable. But they designed to absorb same torque.

**1.3 Structure part**

Its main structures are rotor, body, power-unit and tail-part. According to types of helicopter, its structure and components are different. Therefore, only single rotor helicopter, which is most common type, is expresses in here.

(1) Rotor

Rotor is the core part of the helicopter, which makes the aerodynamic force for ascend and forward. It's composed of blades more than two. Sometimes, 8 blades are used. As number of blades is increased, its vibration and noise are decreased, but its weight is increased.

Shape of blade section is airfoil. Most of blades are symmetry airfoil, but sometimes airfoil with camber is also used for high efficiency. Shape of blade has aspect ratio of 15~20. And taper and sweepback angle are sometimes used. Sometimes, center of rotor and tip of blade have different shapes, but it is not easy to manufacture. All rotor has twist from center to tip with twist angle of 8~12°. In aerodynamic respect, it is the most efficiency that distribution of twist angle is inverse proportion to distance. Nevertheless, it is very difficult to manufacture, so twist angle is decreased as regular stages.

Material of early day's rotor is veneer board, wood or aluminum alloy. Nowadays, honeycomb structure and composite materials used for low weight.

(2) Body

Basic form of body is the frame that connects the cabin and tail part with fuel tank and engine. Its whole shape is designed to get minimum drag for forward flight. Especially, bottom of cabin is made of double structure for safety of passenger. Helicopter which uses for surveillance and patrol uses transparent plastic for skin of cabin.

(3) Power unit

Power unit is composed of engine and power transfer gear. In the past, reciprocating engine is used, but turbo-shaft engine is usually used now. Power from engine is connected to rotor hub through reduction gear, which has ratio of 10:1 and clutch. Part of power from engine is also connected to tail rotor through other reduction gear. Clutch is connected automatically as centrifugal force when rpm is enough. So, if engine is stopped during flight, rotor and shaft of engine is automatically disconnected for safe landing with autorotation.

(4) Tail part

Tail part is usually composed of tail-rotor and horizontal stabilizer. Multiple of thrust of tail-rotor and distance between tail-rotor and center of mass must be same with torque from main rotor. If these values are different, yaw-moment is occurred. And direction of helicopter is controlled by this moment using rudder pedals in front of pilot. Thrust of tail-rotor is decided by its blade angle.

When helicopter inclines its rotor for forward flight, the pitch moment, which inclines the nose down, is occurred. So, horizontal stabilizer is used for equilibrium of pitch. There is also vertical stabilizer in tail part. However, its aerodynamic role is so low that it is regarded as the supporter of tail-rotor.

### 1.4 Rotor hub

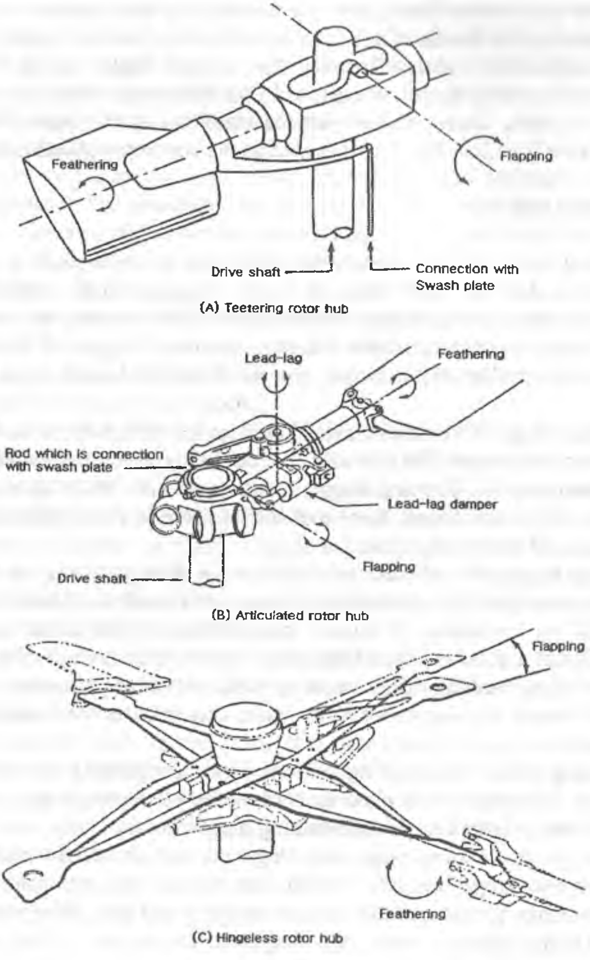
Center of rotor, which connects the rotor blade to drive shaft, is called rotor hub. In addition, there are three types of hinges; flapping hinge, which moves the rotor up and down, lead-lag hinge, which makes blade to move in same rotation plane, and feathering hinge, which changes the blade angle. With these three hinges, moments without drive torque are not transferred each other, blade and body.

Flapping hinge: When helicopter flies forward, left side of rotor and right have different relative speed. In this situation, as their lifts are dissymmetry, rolling moment is occurred. So, flapping hinge, which moves the blade up and down for zero moment of lift, centrifugal force and inertia force of rotor blade, is used. On the other hands, all moments are zero at hinge.

Lead-lag hinge: When rotor starts to rotate or drive power is cut off, there is big moment in the direction of rotation between rotor shaft and blade. So, lead-lag hinge is used for canceling of impact load. When rotated blade has flapping movement, Coriolis acceleration which makes the blade move to front or rear is occurred. For this, lead-lag hinge is also used. Moreover, damper is used for prevention of excess lead-lag movement, which concentrates the center of mass of rotor on one point.

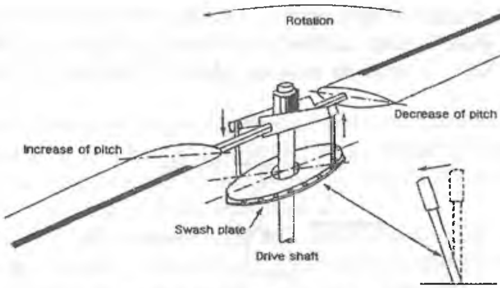
Feathering hinge: Controlling of helicopter is operated by inclination of rotation plane. Therefore, it is need to control the blade angle according to the position of blade. For this function, feathering hinge is used.

As the type of flapping hinge, teetering rotor and articulated rotor is widely used. There is also hinge less rotor which uses elastic structure instead of hinge. Hinge less rotor has advantages that simple structure and long life span. So, it can be developed in the future.




[Figure 4] Kinds of rotor hub and hinge

Swash plate: In teetering rotor, shaft of hinge and center of drive shaft are agreed. However, there is hinge offset which can't make perfect canceling of moment between these two shafts in articulated rotor. In addition, this factor is good for control of helicopter. Therefore, proper hinge offset is designed using swash plate.

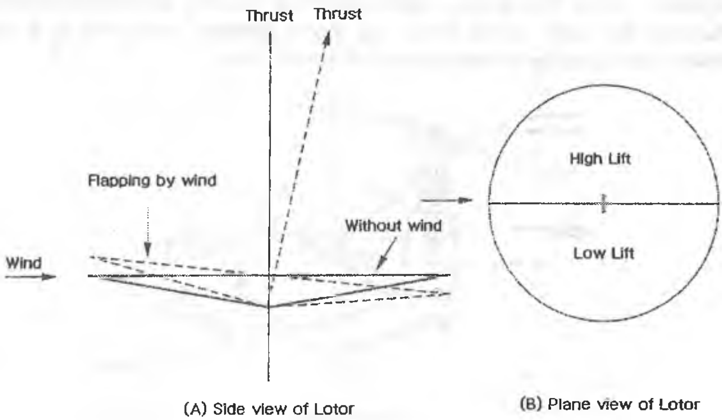



[Figure 5] Principle of swash plate

### 1.5 Principle of control

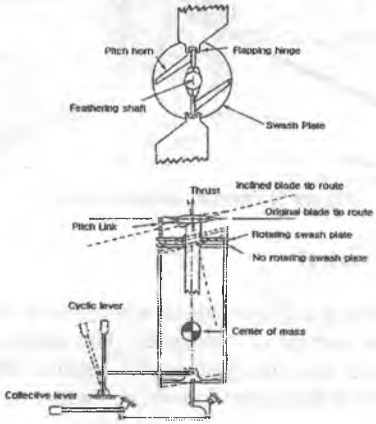
Change of direction and magnitude of aerodynamic force, which is occurred by rotor, is needed for control of helicopter. The easiest way is changing the rotation shaft to forward like auto-gyro which rotation shaft has no transfer of power. However, shaft of helicopter has drive torque, so it is very difficult to change that direction.

The direction of thrust which rotor makes is the vertical of rotation plane. Using flapping it is possible to change the direction of aerodynamic force with change of rotation plane. If blade has higher position at the rear and lower position at the front than rotation plane, the direction of thrust is inclined to front. So, control of blade angle is needed for direction and magnitude of aerodynamic force with control of flapping.




[Figure 6] Flapping by wind

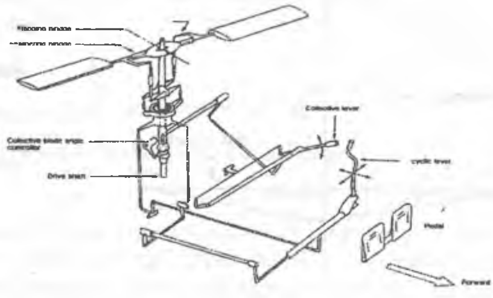
Real control system of helicopter is like Figure 7. If cyclic lever is inclined to forward, swash plate is also inclined to forward. In figure, angle between rotor and pitch horn is 90°, so that its rotation plane is inclined to forward not like Figure 6.




**[Figure 7]** Control system of cyclic pitch and collective pitch

If collective lever is raised up, all blade angles are uniformly increased without any concern of its position. So, this lever is used for control of total lift. In addition, tail rotor also has swash plate which is controlled by pedals for control of blade angle, but there is no cyclic lever for tail rotor.

It is the control system of teetering rotor. It is composed of collective lever, cyclic lever and pedal. Left of cockpit, there is collective lever for control of total aerodynamic force. Nowadays, engine governor is used, so collective lever does not have to be used. Cyclic lever can move forward, back, left and right for controlling the direction of aerodynamic force of rotor.



**[Figure 8]** Control system of helicopter

*2 Principle of Flight*

**2.1 Hover**

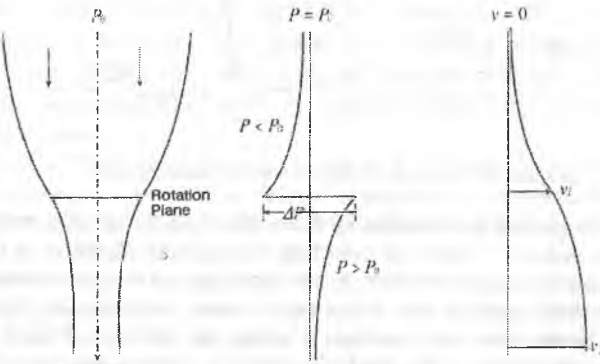
The peculiar advantage of helicopter is hover, no movement of forward, back, left and right with no change of altitude. It is compared to the straight and level flight of fixed wing aircraft as aerodynamic respect. In addition, blade of rotor is compared to the wing of fixed-wind aircraft.

The fundamental difference of wing and blade is that relative speed of air is different from center of rotor to tip of blade. Its velocity is almost zero at the center of rotor. On the other hand, the tip has the velocity of transonic. Therefore, lift is very different from center to tip, and range of Reynolds number is also from 1 to 6 x 106.

There are momentum theory and blade element theory to get the whole thrust of rotor. Momentum theory calculates the thrust using change of momentum of whole airflow, so it is convenient for rough analysis of whole rotor plane. Blade element theory calculates the thrust and torque with integration of lift and drag according to each blade element. So, it is used for design of actual rotor or calculation of its performance.

(1) Momentum theory

When helicopter does hover, rotor thrust is same with its weight. As thrust of rotor is occurred to upward, the air is force to downward. So, the flow is happened from the upper space of rotor to lower space of rotor plane. When air flows through rotor plane, it has induction velocity of vi, and it's accelerated to wake velocity of vj after rotor plane. Here is assumption that induction velocity of whole rotor plane is same as vi.




[Figure 9] Velocity-static pressure by momentum theory

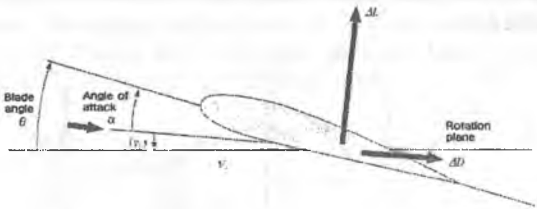
When air flow approaches to rotor from upper side of plane, its velocity is increased and static pressure is decreased. Decreased static pressure gets the energy through rotor, so its pressure is higher than atmospheric pressure. And this high energy should be change to kinetic energy for equilibrium with atmospheric pressure. So, its velocity becomes wake velocity vj. If Bernoulli's theory is applied to upper and lower side of rotor planes, it is calculated that wake velocity is twice than induction velocity; vj=2vj.

Thrust of rotor is same with the change of momentum in unit time. So, the amount of air mass, which flows through the rotor plane in unit time, is proportioned to air density, area of rotor plane and induction velocity. Change of momentum is calculated by the multiple of air mass in unit time and speed of flow, and this is same with thrust. In the respect of pressure difference, thrust is calculated by the multiple of disk area and pressure difference between upper and lower part.

Total thrust of rotor and weight of helicopter must be same in hover. Moreover, wind area of fixed wing aircraft is compared to the disk area of rotor. So wing loading is compared to disk loading which weight of helicopter is divided by the disk area. The maximum value of disk loading is 10lb/ft2 now. High disk loading means high efficiency and easy controllability, but it is dangerous in the ground, because it blows off the stone and sand.

(2) Blade element theory

In blade element theory, lift and drag of one section is calculated from airflow, and its vertical factor is integrated from center of rotor to tip of rotor and multiplied by the number of blades. Then thrust is calculated. In addition, the torque is calculated by multiple of integrated horizontal factors and distance to center of rotor, and multiplied by the number of blades.



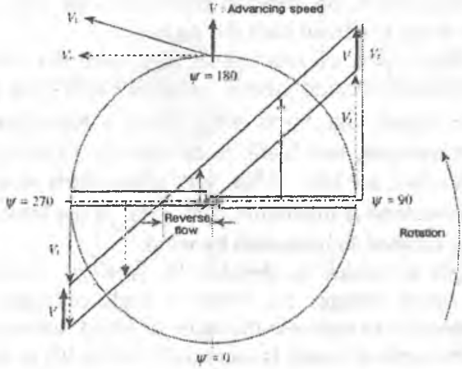

**[Figure 10]** Direction of relative wind on blade element

Blade angle ($\theta$) is controlled by the swash plate. In addition, relative airflow is the vector sum of $V_i$ which is induction velocity and $V_r$ which is the speed of rotor. So, angle of attack is smaller than blade angle. As the aerodynamic character decides induction velocity and blade angle should consider the twist, angle of attack is different from each section. Usually, the blade is divided into 20~50 sections for calculation. At the blade tip, blade tip wake is also occurred, because of pressure difference between upper and lower sides of blade. This wake is caused to blade tip loss. Therefore, thrust of rotor is calculated with consideration of blade tip loss.

The most proper means that it has high thrust compared to torque and shows high performance in the wide range of flight. Moreover, this is decided by the airfoil of rotor section, twist and solidity ratio, which shows the ratio of blade area to disk area. Usually solidity ratio is 6~10%.

Actually, there is loss, which is caused by blades each other, but it is very difficult to be calculated. The wake which front blade makes affects the angle of rear blade. This phenomenon decreases the aerodynamic efficiency and causes vibration and noise. Sometimes, tail-rotor also affects to rotor blade.

### 2.2 Forward flight




**[Figure 11]** Velocity vector during forward flight

When helicopter does forward flight, relative airflow, which affects to blade is changed periodically according to its position. Like [Figure 11], direction and magnitude of relative wind ($V_t$) is decided by the sum of rotation speed ($V_r$) and forward speed ($V$). Position of rotor blade is decided by $\psi$. When $\psi$ is 90°, it has maximum relative speed, and when $\psi$ is 270° its relative speed is minimum. In other angles, vector of relative velocity is inclined to blade, and it makes the effect like sweepback wing.

Around $\psi$=270°, there is some place that $V_r$ is lower than $V$ near center of rotor. In this case, relative wind is affected to airfoil in the opposite way. This area is called reverse flow. As forward speed is increased, its area is also increased. This area does not make lift but big drag.

At $\psi$=90°, relative speed which affects to airfoil is maximum, so it reaches the drag divergence Mach number. In addition, it is caused that blade tip has shock wave which increase the drag suddenly. In this case, there is limitation that forward speed cannot be increased any more. And, maximum speed is also limited around $\psi$=270°, because of reverse flow area and excessive flapping which makes angle of attack increase to stall angel of attack. If blade angle is not change according to its position, its maximum speed should be limited in short range.

Solution of these problems is the change of blade angle according to position. When blade is turned to forward, its blade angle is decreased. And wh it is turned to backward, its angle is increased. Nowadays, flapping hinge is us for this function.

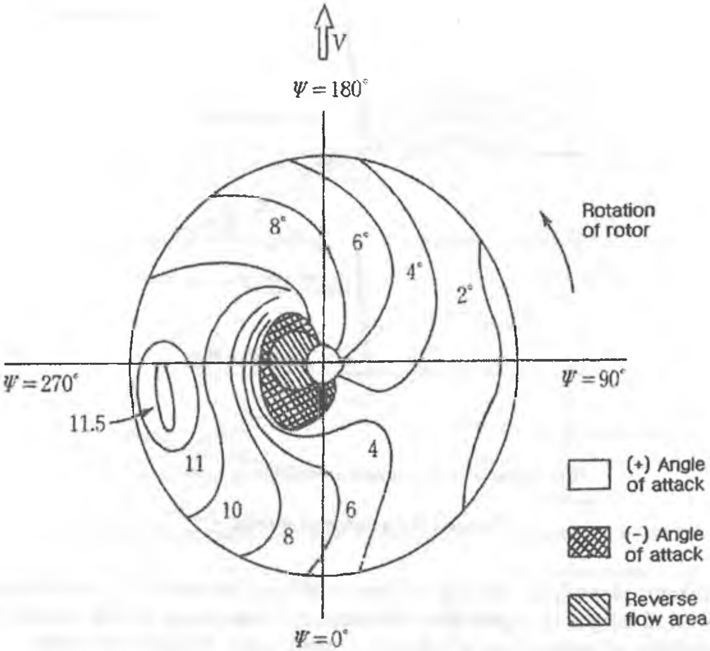
Role of flapping hinge is making the condition of zero moment. Momen which are affected to hinge, are lift, centrifugal force and weight of blade. B moment by weight of blade can be ignored because its effect is very sma Moment by centrifugal force is occurred for agreement with rotation plane wh blade is in the escaped position of rotation plane. In hover state, blade is in t equilibrium condition of flapping angle that hinge moment by lift cancels the hin moment by centrifugal force. So, rotation plane of blade tip is little higher th center of rotor with shape of turned corn during hover.

In forward flight, sum of moment is also needed to be zero. Howev flapping angle is different, because there is imbalance of lift. At $\psi$=0°, lift which affected on blade is bigger than hover state, blade is forced to raise. At $\psi$=9 flapping speed is maximum, and blade is continually lifted up. So, the high position of blade is when $\psi$=180°. After that, blade starts to lower. At $\psi$=27 flapping speed to downward is maximum, and $\psi$=0° is the lowest position. This showed as if rotor is inclined to backward by wind.

Effective angle of attack is decided by forward velocity and speed rotation. Flapping speed changes the value of angle of attack. And if blade raised, it is also connected to decrease the angle of attack which means decrease lift. If blade is lower, angle of attack is increased, so that lift is also increased. Li these principles, moment change, which is caused by imbalance of lift, automatically cancelled by flapping without any control of pilot.

Control of rotation plane is same with above principles. In hover state, swash plate is inclined to have minimum blade angle at $\psi$=90° and maximu angle at $\psi$=270°, lift starts to increase at $\psi$=180°. At $\psi$=270°, flapping speed upward is maximum, and maximum flapping angle is showed at $\psi$=0°. As resul rotor plane is inclined to front for thrust to forward. Totality, flapping is duplicat by the inclination of swash plate, so that control of rotor is always possible.

If forward speed is increased, flapping speed is also increased, so that stall occurred by increase of effective angle of attack. Stall angle of rotor is bigger th angle of fixed wing aircraft. Its process that separation is firstly occurred at t trailing edge and it is spread to leading edge. In this case, stall is occurred late than fixed wing, because its angle of attack is changed according to its positio Therefore, when angle of attack is bigger than stall angle at one point, it is r bigger than stall angle in other position after few times. This phenomenon is call dynamic stall.

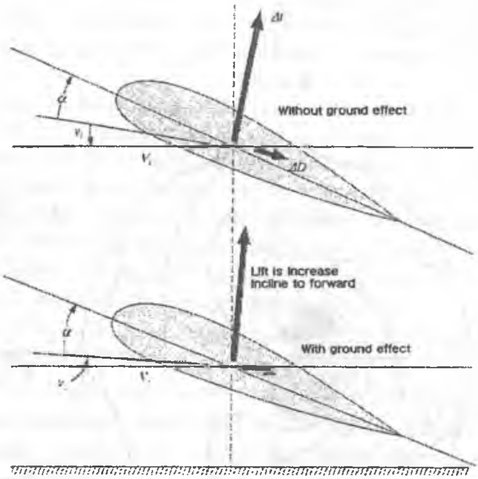



**[Figure 12]** Distribution of angle of attack during forward flight

The position of $\psi$=270° can happen the dynamic stall most easily with the biggest angle of attack. Because, its relative velocity is minimum and flapping speed to downward is the biggest. This [Figure 12] shows the distribution of angle of attack during forward flight.

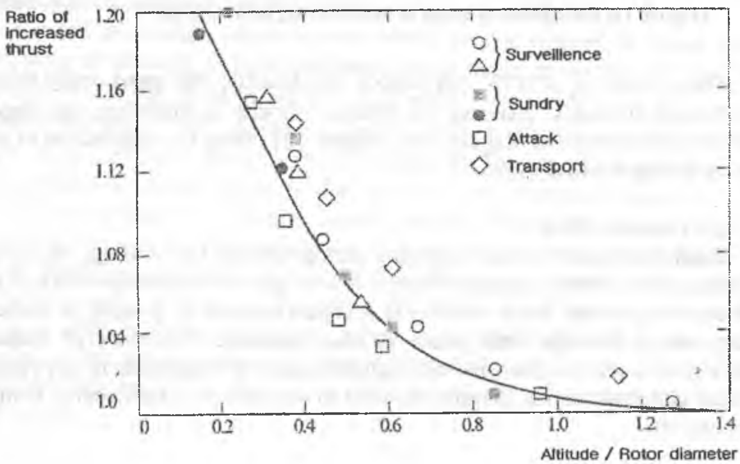
### 2.3 Ground effect

When helicopter is close to ground during take-off and landing, lift is bigger than usual. This is called ground effect. Close to ground means that wake of rotor is affected by ground. Wake velocity is decrease because of ground, so induction velocity which through rotor plane is also decreased. Decrease of induction velocity is connected to increase the angle of attack, so magnitude of lift vector is increased and its direction is more inclined to vertical. So its efficiency is higher than in the air.



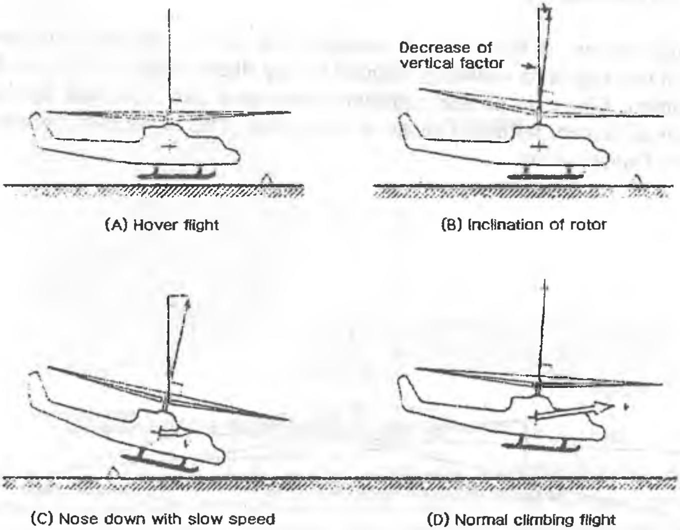

[Figure 13] Airflow near ground

If rotor plane is in the height of blade radius, increase of thrust is around 7%. Moreover, if height is higher than diameter of rotor, there is few ground effect. And condition of ground can affect to this effect; sand, meadow or water.




[Figure 14] Rate of increased thrust by ground effect

## 2.4 Transition flight




[Figure 15] Stages of transition flight

Transition flight means the transitional period of change from hover to forward flight. This situation can't be regarded as equilibrium condition, because speed is changed by acceleration according to time.

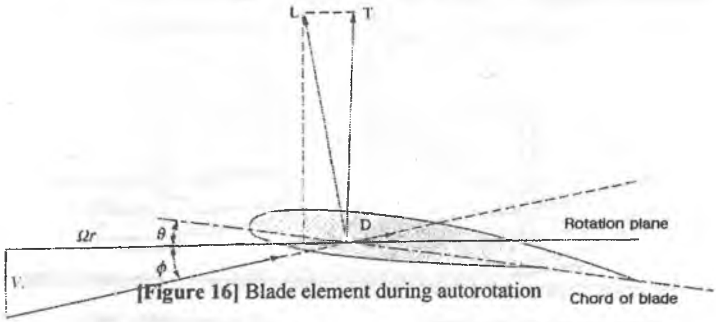
Like (A) of [Figure 15], there is assumption that helicopter hovers at the altitude of 1m. Because of ground effect, power of engine is lower than one of high altitude. With this condition, if pilot inclines the cyclic lever to front, rotation plane is inclined to forward without control of collective lever for steady total thrust. Then, forward factor of thrust is occurred and vertical factor of thrust is decreased.

Therefore, there are accelerations to forward and downward. After few times, forward speed is increased, and vertical force is not changed because of ground effect as altitude is little decreased.

In the respect of body posture, the point of action of thrust vector is moved behind of center of mass. So, its nose is turned down. With increase of velocity, drag is also increase so that more inclination angle of rotation plane is needed for acceleration. And effect of horizontal stabilizer is increased with increase of velocity, so posture of helicopter is recovered with canceling of moment. Until forward velocity reaches some value, required power for acceleration is decreased. So, collective lever is also used together with increase of excess power. If speed is more increased, excess power is also increase and rate of climb is also increase for transition to normal forward flight.

## 2.5 Autorotation

Autorotation of helicopter is compared to power-off glide of fixed wing aircraft. When engine is suddenly stopped during flight, rotor is started to decrease its revolution. Decrease of some altitude, helicopter can have safe landing with stable rate of descent without change of revolution. This is possible because of the principle of autorotation.



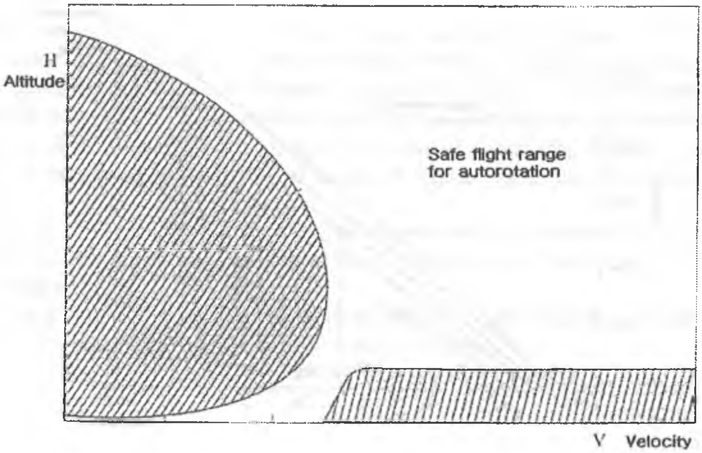

[Figure 16] Blade element during autorotation

Autorotation means that rotor shaft maintain stable revolution without torque. Rotor of helicopter, which is in the condition of autorotation makes thrust, which is same with weight of helicopter. Because, the energy for overcome of torque affected by blade and maintain of revolution is supplied by potential energy which is decrease of altitude.

Like [Figure 8-16], lift and drag is affected to the direction of vertical and horizontal by relative wind. Relative wind is decided by the vector sum of forward speed, descending speed, rotation speed and flapping speed. Lift and drag that is affected to blade section is decided by the magnitude and angle of attack. Angle of attack is the sum of angle which is caused by vertical descending velocity ($\phi$) and blade angle ($\theta$). Rotor should rotate without power, which means no torque, and torque is the multiple of the distance from center to its section and force of section, which is affected on the parallel direction of plane. Therefore, torque, which is affected on the drive shaft, is zero, because horizontal factor of lift cancels the drag.

Some part of blade needs power to rotate and some other part makes power like windmill. Therefore, if total torque is zero with canceling effect, this situation is called autorotation.

Blade angle and rate of descent are decided by the ratio of form drag coefficient to lift coefficient, so there is some conditions which autorotation cannot be operated. This conditions are expressed like this; V-H diagram.




[Figure 17] V-H diagram of single rotor helicopter

It shows the range of single engine helicopter. After increasing enough forward speed, it is good to increase the altitude together with forward speed for safe flight. A part does not have condition of blade angle for autorotation, and B part means that there is no enough time for autorotation.

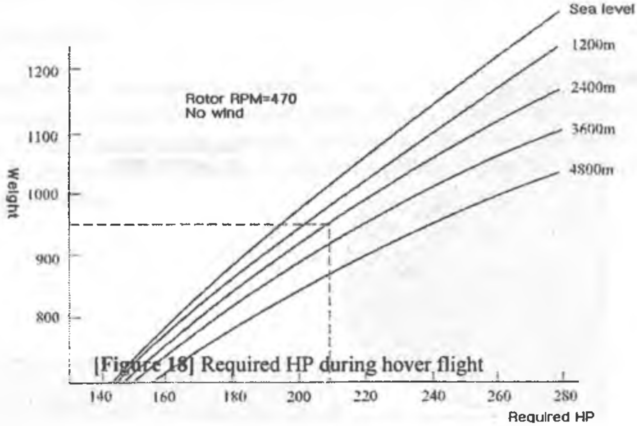
### *3 Flight Performance*

### 3.1 Required power for hover flight

When helicopter rotates its rotor, lift and drag is occurred in blade element. This drag makes the torque, which resists the rotation of rotor, and required power is calculated by the multiple of this torque and angular velocity. Torque is divided into profile drag and induced drag. So, required power is also divided into power for profile drag and induced drag.

Profile drag: Required power for profile drag is calculated like this. Integration from rotor-shaft to blade tip of multiple which is of profile drag at one section and the number of blades multiplies distance from center of rotor again. So, required power for overcome of profile drag is proportional to air density and triple multiplication of rotation speed of rotor.

Induced drag: Required power for induced drag is easily calculated by the results using momentum theory. In hover state, as vertical speed of relative wind (induced velocity) is occurred, required power is calculated by the multiple of weight and induced velocity. It is also possible to get the relation between induced speed and weight, so required power for induced drag can be expressed as weight, disk area and air density. As a result, require power for induced drag is proportioned to 3/2 multiplication of weight and inverse proportioned to square root of altitude.




[Figure 18] Required HP during hover flight

Total required power in hover state without ground effect is the sum of powers by profile drag and induced drag. Rotation speed is only concerned about required power for profile drag, and weight is about required power for induced drag. Actually, required power for induced drag is 65~75% of total required power. [Figure 18] shows the required power of small helicopter in hover state.

### 3.2 Required power for forward flight

When helicopter flies forward, induced drag and profile drag is different from one of hover flight, because relative wind and angle of attack are changed. In addition, parasite drag is also needed to be considered.

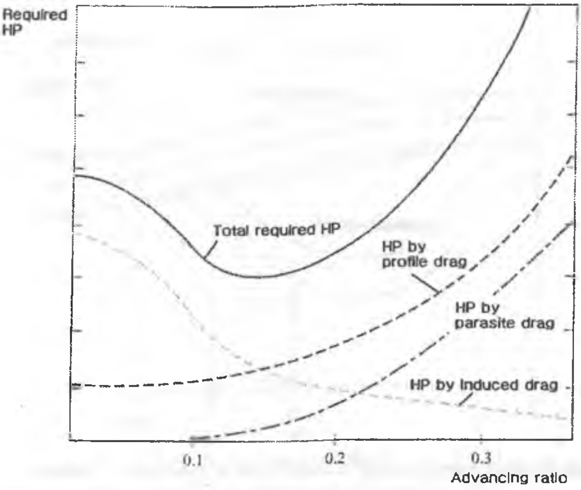
Induced drag: Required power for induced drag in forward flight is different from one of hover state. If forward speed was more 55km/h, it could be regarded same as fixed wing aircraft. Actually, wake far from forwarding helicopter is same with one of fixed wing aircraft, so disk area of helicopter is regarded as the fixed wing whose span is same with diameter of rotor. As a result, required power for induced drag is proportioned to 3/2 multiplication of weight in hover state, but it changes to square of weight in forward state. In addition, required power is in inverse proportioned to square root of air density in hover state, and inverse proportioned to air density itself during forwarding. It is very difficult to calculate the required power for induced drag in the range of forwarding velocity from 0~55km/h.

Profile drag: There are three reasons for difference from hover state. First, drag is increased by increase of relative wind through whole rotor. Second, distribution of angle of attack is changed by flapping movement, and average drag coefficient is increased overall. Third, drag by compression is also needed to be considered because rotor blade which turns to forward has high speed.

First reason is expressed as advancing ratio for increasing amount of required power by one of hover state. Advancing ratio is defined by the ratio of these; advancing speed: transitional velocity of blade tip. Advancing ratio is zero at hover state, it is always smaller than 1, because if ratio is 1, whole blades are in reverse flow area at $\psi$=270°. Advancing ratio at maximum velocity is around 0.35. Second reason, flapping effect is also function of advancing ratio. If advancing ratio was increased, distribution of angle of attack would have wide imbalance area.

Parasite drag: Parasite drag by air is proportioned to square of velocity, so required power for this drag is proportioned to triple multiplication of velocity like fixed wing aircraft.

Total required power for advancing flight is calculated by the sum of the power by induced drag, profile drag and parasite drag.




**[Figure 19]** Curve of required HP on forward flight

If velocity is low, required power for induced drag is high, but it will be decrease with increase of advancing speed. In addition, powers for profile drag and parasite drag are increased. Therefore, there is minimum point of total required power that it is around the advancing ratio of 0.14. Minimum required power is 60% of power for hovering.

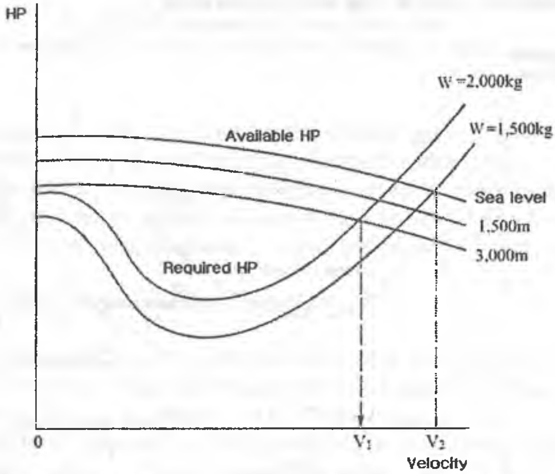
To get the altitude and velocity for cruise after take-off, these sequences are recommended; First, hovering in very low altitude. Second, it starts to advance in low altitude. Third, when advancing velocity is in minimum required power, helicopter increases its altitude. This way has low fuel consumption and high rate of climb. Even the helicopter, which is over the maximum take-off weight, can take-off using this way. Initially, over weight can be overcome by ground effect, and weight is decreased with consumption of fuel.

### 3.3 Maximum level velocity

Maximum level velocity of helicopter is decided by the loss of control by stall of rotor blade and limitation of available power. This maximum velocity is the cross point of this graph in [Figure 20].

Available power means the power of engine with consideration of loss and power used for accessories.

If altitude is increased, density is decreased so that available power is also decreased. But required power is not sensitive to altitude that much. On the other hands, required power is generally increased with increase of weight, so cross point of required and available powers are also changed by its weight. So, maximum level velocity is increased with low weight and low altitude.




[Figure 20] Maximum velocity of level flight by velocity-HP diagram

### 3.4 Maximum rate of climb

Like fixed wing aircraft, maximum rate of climb is calculated by excess power, which is the difference of required power and available power. In addition, the velocity gets this when required power has minimum value, because available power is not affected by velocity.

With increase of altitude, the velocity for maximum rate of climb is little slower than advancing speed for minimum required power, because ratio of required power is changed by parasite, profile and induced drag. If altitude is continuously increased, available power is decreased, so that rate of climb is also decreased to get maximum ceiling.

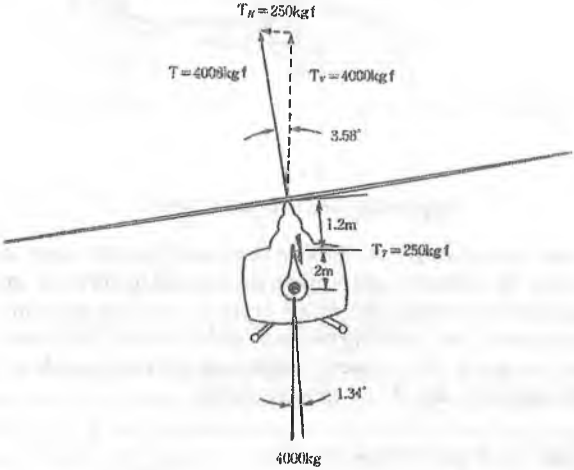
If excess power at the velocity, which can get the maximum rate of climb, is used for horizontal acceleration, maximum level acceleration is calculated.

## *4 Stability*

The factors which contribute to the stability of helicopter are aerodynamic force by rotor, tail-rotor, horizontal stabilizer and vertical stabilizer, and gyro effect by rotation of rotor. Speed range of helicopter is very wide from hover to maximum velocity, so analysis of stability is more complex than fixed wing aircraft. In this section, states of trim and static stability by wind from front and below are considered.

### 4.1 Trim for hover flight

Trim means the condition that there is no angular movement with canceling of all moments. So, trim is the necessary condition for equilibrium, and equilibrium state with fixed lever is also called trim.




[Figure 21] Trim in hover flight

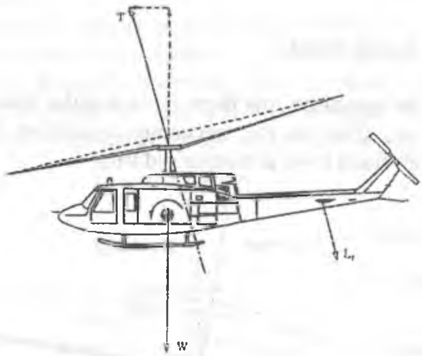
For trim hover flight, force and moments should be zero. A vertical factor of rotor thrust is same with weight of helicopter. In addition, horizontal factor is rotor must be same with thrust in tail rotor, so rotation plane of rotor is need to be inclined to side.

The action points of horizontal factor of rotor thrust and thrust of tail-rotor is different, so artificial moment is made with different axis between drive shaft of rotor and vertical factor of center of mass for equilibrium of moment. In outward form, body of helicopter is little inclined for this. Moment by reaction of rotor torque is cancelled by the multiple of tail-rotor thrust and distance between center of mass and tail-rotor.

It the difference of height between rotor-hub and tail-rotor is decreased, inclination angle of helicopter is also decreased. In addition, posture of helicopter is change by the total weight, position of center of mass and external force.

## 4.2 Trim for forward flight

Helicopter that flies in high speed needs equilibrium of force and moment. First, high thrust of rotor is needed for high speed. In addition, thrust of tail-rotor is also increased for canceling of reaction torque, or yaw moment by vertical stabilizer is used with inclination of helicopter to side for canceling of increased reaction torque.

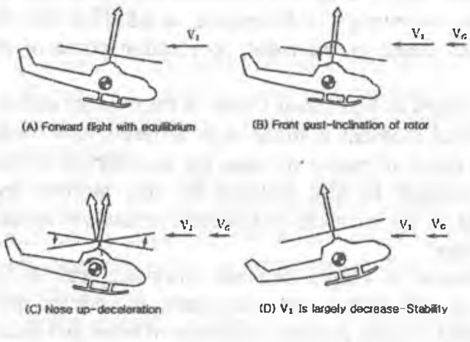



**[Figure 22]** Trim in forward flight

It is need to overcome the profile drag and parasite drag for maintain of forward velocity. In addition, rotor plane is inclined to forward for this. If rotor plane was inclined to forward, the action point of rotor thrust would be moved to behind of mass center. So, pitching moment which makes helicopter nose down is occurred. For cancel of this moment, horizontal stabilizer needs to make lift, so camber of this stabilizer airfoil is inclined to down.

8.4.3 Stability for advancing velocity

Here is the assumption that helicopter which flies forward meets the gust from front.

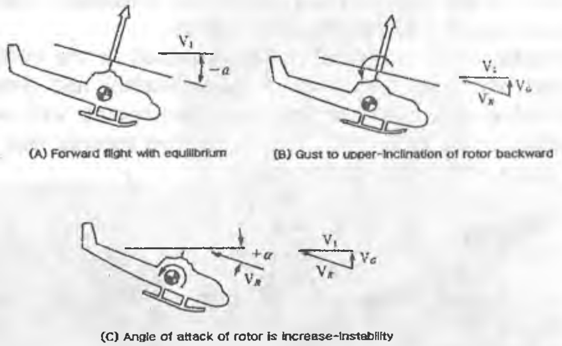



**[Figure 23]** Static stability for advancing velocity

If relative airspeed is increased from $V_1$ to $V_1+V_G$, flapping angle is increased, because imbalance of dynamic pressure is occurred. Rotation plane is inclined to backward, so that action point of thrust of rotor is moved to front of mass center. This is connected to that moment which makes nose up is occurred, and horizontal factor of thrust of rotor is decreased. Therefore, this makes $V_1$ decreased to return of the original relative airspeed. This situation means stable.

And there are phugoid and short-period motions like fixed wing aircraft in the respect dynamic stability.

### 8.4.4 Stability for angle of attack




**[Figure 24]** Static stability by angle of attack

Here is the assumption that helicopter meets the gust, which blows from down to up, or collective, pitch is suddenly controlled. Before gust, each side of rotor makes steady lift using flapping which blade turned to forward has low blade angle with big relative air speed, and blade turned to backward has opposite characters. After gust, relative wind has steady velocity, but angle of attack is increased. Therefore, increase of lift coefficient is same in all position of blades. However, lift itself is highly increased at the rotor blade which turns to forward, so that flapping angle is increased. Increase of flapping angle is connected to the inclination of rotation plane backward. Therefore, action point of thrust is moved front of mass center that makes moment to nose up. Therefore, angle of attack is more increased that it shows unstable. [Figure 8-24] shows the process of this sequence.

Stability for angle of attack should consider not only rotor but body and horizontal stabilizer. With increase of angle of attack, body is always unstable, but horizontal stabilizer always does stable role. So, total stability is decided by these three factors; rotor, body and horizontal stabilizer. If helicopter was in low speed, effect of horizontal stabilizer would be low. However, effect of stabilizer is increased in high speed flight, so that helicopter becomes stable with increase of velocity.

Actually, stability for angle of attack is stable if its displacement from trim is low. Nevertheless, if external effect was big, it would show unstable property.

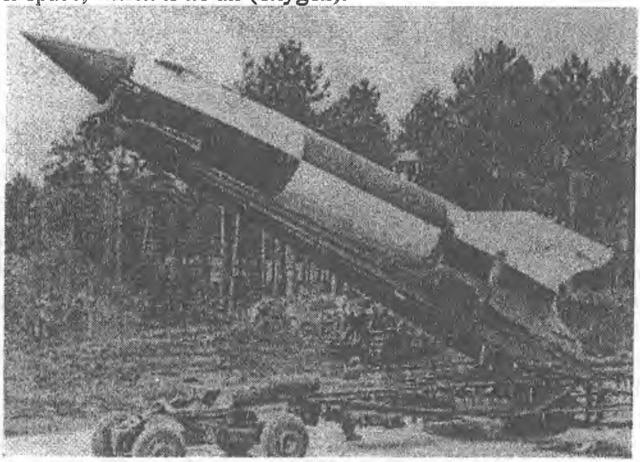
**Space Propulsion**

Rocket: Rocket is the only vehicle, which flies through space, no air, to carry payload. Rocket is classified into two groups by payloads; launch vehicle for satellite and space vehicle to do scientific and technological research and missile with nuclear and chemical weapons.

***1 History of Rocket Propulsion***

The first official document of rocket in the world is solid propellant rocket in the 1232 of China. Its fuel is called black powder and consisted of charcoal, $KNO_3$ and sulfur. This powder is used until the 19th century.

The substantial development of rocket is started in the 1940s because it needed high heat and fluid mechanics. Robert Goddard and Herman Oberth, forerunners of rocket researched complex rocket mechanism with not only solid but liquid propellant in the 1920s and 1930s. After their success, their rocket could reach the outer space, which is no air (oxygen).



**[Figure 1]** V-2 Rocket

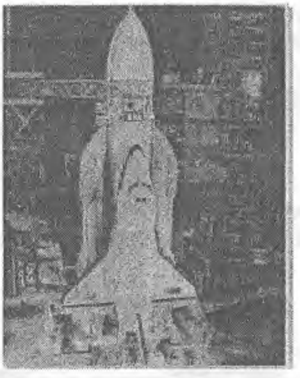
In German, they developed V-2 rocket in the end of 1930s and early of 1940s. This could carry military payloads such like bomb with supersonic for long distance. V-2 (A-10) is developed from A series which is the project of Wernher Von Braun's team and known that around 4,000 rockets are made in World War II.

V-2 is the origin of modern liquid propellant rocket. It used mixture of ethyl alcohol and water with extremely low temperature oxygen. The thrust of engine is 352,000 N and it means that V-2 can carry payload of 900kg to 320km.

After World War II, US, UK, France and Russia tried to develop rocket propellant system with all their strength. And most of program was processed by help of engineers from German who were prisoner of war.

In the October of 1957, Russia was succeeded to put the first satellite to the Earth orbit. Its name is Sputnik I and it has mass of 83.6kg. After two months, US also launched the Explorer I, which is mass of 14kg. After that the US and Russia have had competition in manned space flight, moon and planet exploration.

The US changed the policy of space development in the 1958; NACA (National Advisory Committee for Aeronautics) is changed into NASA (National Aeronautics and Space Administration) and all university and company are permitted to attend space project.



**[Figure 2]** Energiya and Buran

The president of US, John F. Kennedy presented the Apollo project, which is manned moon exploration during 1960s. Therefore, July of 1969, Apollo 11 reached the moon and the first human stepped on the moon. For this, Saturn-V rocket are used which has 6,000,000 lbf of thrust.

After that, US processed the project of Space Shuttle for reuse of spacecraft. The first flight was succeeded in 1981 and it has used until now more than 100 times. Russia also developed Energiya and Buran for space shuttle. This plan, however, is reserved because of collapse of Soviet Union and change for the worse of economy of Russia.

For the experiments in space, US used cargo partition of space shuttle for two weeks and Russia used space station MIR. MIR is the combination of multiple spacecraft's. This is made little by little by docking of spacecrafts. And MIR was disposed in the 2000 because of its life span. Nowadays, ISS (International Space Station) is constructed in the orbit. The 16 countries include US, Russia and ESA (Europe Space Agency) participate this program as international cooperation.

The commercial launch vehicle of ESA is Arian series and has high possession in space industry. US also have tried to recover the possession rate of launch vehicle with improvement of Titan, Atlas and Delta. And Russia has launched hundreds of rockets until 1960, so it has high reliability. Proton, Zenit, Soyuz, Rockot and Cosmos are used for commercial satellites.

### *2 Principle of Propulsion*

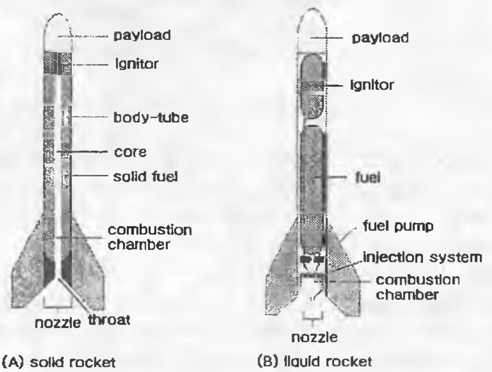
Basic principle of rocket is Action and Reaction, which is Newton's third law. It means that "for every action, there is an equal and opposite reaction". Rocket is consisting of combustion chamber, which is for making gas using fuel and nozzle for exhaust of gas. Rocket also loads the oxidizer for combustion because there is no oxygen in space. Therefore, combustion gas consists of oxidizer and fuel, which is carried by rocket itself. In addition, exhausting of gas, rocket is thrust for flight.

### *3 Kinds of Rocket Propulsion*

Rocket Propulsion system is largely classified into heat, electric and nuclear propulsion system. Heat propulsion system is divided into chemical, solar wind and laser system and electric propulsion system into electro-thermal, electrostatic and electromagnetic system. And chemical propulsion system is the most common system in those days.

#### 3.1 Chemical Propulsion System

This system is classified into various systems by the physical condition of propellant.




**[Figure 3]** Form of solid and liquid rockets

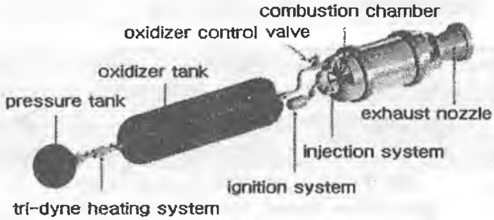
- Solid propellant system
- Liquid propellant system
- Gas propellant system
- Hybrid propellant system

Solid propellant system has used for around 1000 years for military use. Grain, which is mixture of fuel and oxidizer, is stored in inside of case or combustion chamber. Its system is simple so that it does not need complex mechanism and high cost. However, it is impossible to control of thrust and restart.

So this system is usually used to reinforce the launch vehicle.

Liquid propellant system is consisted of propellant tank and rocket engine. It uses the liquid propellant, which moves from tank to combustion chamber by pressure. Bipropellant propulsion system is consisted of liquid propellant and liquid oxidizer. In addition, Monopropellant propulsion system used catalyst instead of oxidizer and can make small impulse so that used to minutely control of position and orbit.

Gas propellant system uses high-pressure gas, which is stored for propellant or working fluid. Stored gas needs heavy and large tank. This engine was used for control of position in the early days.




**[Figure 4]** Hybrid rocket system

Hybrid propellant rocket is developed to overcome the defect of solid propellant; control of thrust. It is a mixture of two engines solid and liquid. Therefore, it burns with liquid oxidizer, which is sprayed upper side of solid fuel. As this system uses the outside air, it can save the oxidizer in the atmosphere. It also has flexibility and safety of operation. In addition, it does not exhaust any pollution objects.

### 3.2 Nuclear Propulsion System

There are three nuclear energy sources, which are classified by the way of heating working fluid (usually liquefied-hydrogen).

| | |
|---|---|
| **Fission solid/gas core system** | The heat is generated by the fission of Uranium in the solid atomic reactor and is transferred to working fluid. There was test-experience in the 1960. |
| **Radioactive isotope system** | If radioactive material radiates its radioactivity, it can be conversed to heat easily. This energy source has successfully used for generating of electricity in the spacecraft. |
| **Fusion gas core system** | Many concepts for this system have researched until now, however, nothing has tested and used. Because of its dangers and high cost. |

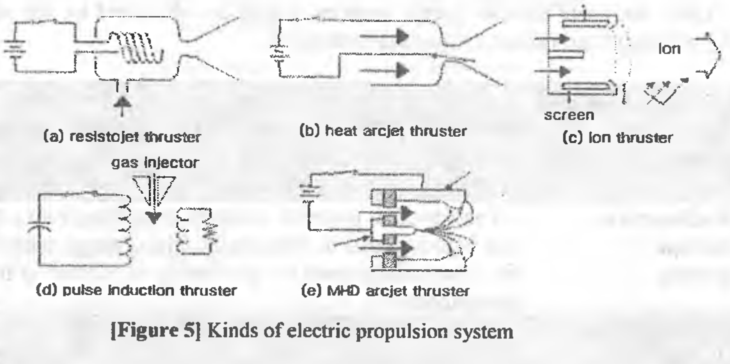
Above three systems are considered as the extension of chemical propulsion system. The only difference is the source of energy; chemical or nuclear. Nuclear propulsion system has not developed perfectly until now, so it does not put to practical use. However, the thrust of nuclear rocket is 2~5 times bigger than usual rockets so that it can reduce the time to 1/3 for round trip to Mars. Therefore, this system is lighted for the manned investigation of planets.

### 3.3 Electric Propulsion system

Electric propulsion system gets the thrust acceleration of exhaust gas by electrical heating, electrical force or magnetic force.

| | |
|---|---|
| **Electro thermal propulsion system** | In this system, the thrust is generated through nozzle by expanded gas whose enthalpy is increased. Power augmented hydrazine thruster, resistojet thruster and hydrazine arcjet thruster are kind of these ways. |
| **Electrostatic propulsion system** | The thrust is generated by accelerated ion in the thrust generation beam. Ion thruster and colloid thruster are these kinds. |
| **Electromagnetic propulsion system** | Magnetic field and huge electric current are used to generate the force, which functions to ionized propellant. There are magneto plasma dynamic thruster and magnetic arcjet thruster. |

The advantage of this system is high exhaust speed for saving of fuel, reduction of satellite mass, increase of payload capacity and increase of operation time with overcoming of chemical propulsion system. However, it also has weak points. For the operation of system, it needs high electricity so that it needs high efficiency solar cell and panel and battery or alternative energy.




[Figure 5] Kinds of electric propulsion system

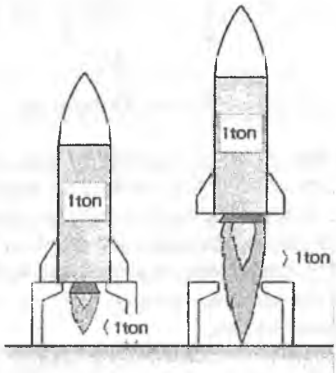
### 3.4 Comparison of propulsion system

| | Chemical Propulsion | Nuclear Propulsion | Electric Propulsion |
|---|---|---|---|
| **Specific Impulse** | Low<br>Solid (170-350 sec)<br>Liquid (250-525 sec) | Meddle<br>Fission (2,000 sec)<br>Fusion (5,000 sec) | High<br>Electro-thermal/static/magnetic<br>(300-10,000sec) |
| **Thrust** | High thrust in short time | Low thrust in long time | High thrust in long time |
| **Acceleration** | High | Low | High |
| **Operation Circumstance** | Launch in relatively high gravity | Orbit control in zero gravity | No concern about gravity |

There are also concepts of Solar Sail, which uses the radiant pressure of sun, and Photon Rocket.

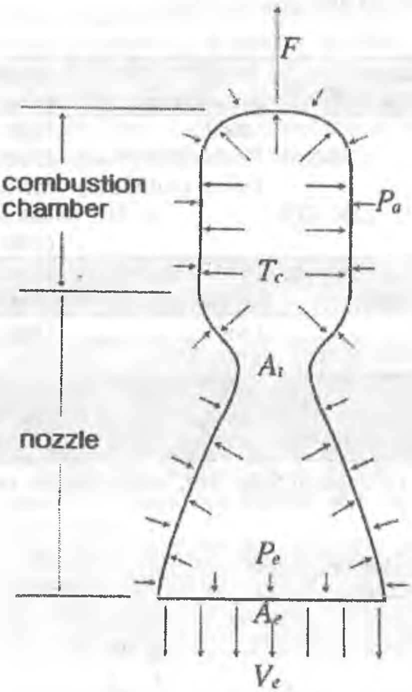
4 Theory of Rocket Propulsion

**4.1 Thrust of rocket**




**[Figure 6]** Thrust of rocket

The main factors of rocket performance are thrust and specific impulse. Thrust means the power, which pushes the rocket ahead; how much mass rocket carried. For example, if rocket which has one ton of mass needs thrust at least more than 1 ton. Thrust is proportioned to the amount of exhaust gas and its speed in each second.




**[Figure 7]** Form of exhaust gas

[Figure 7] shows the form of exhausting gas. Pressure distribution in combustion chamber is symmetric so that there is no pressure change in chamber. However, the pressure is decreased around nozzle and the force which is formed by gas pressure in the latter part of chamber isn't supplied from outside. Therefore, the force, which is caused by the difference of pressure between inside and outside, is called thrust and pushes the chamber upward.

The thrust is expressed like this.

$$F = \dot{m} \times V_e + (P_e - P_a) \times A_e$$

($\dot{m}$: mass rate of exhausted gas, $V_e$: speed of gas, $P_e$: gas pressure at the exit of nozzle, $P_a$: standard atmospheric pressure, $A_e$: area of nozzle exit)

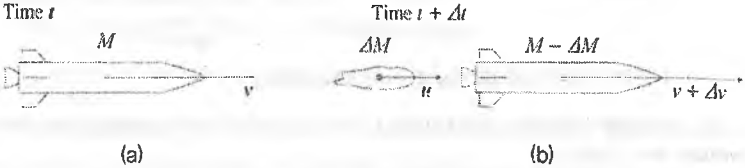
### 4.2 Conservation of Momentum

There is momentum (P) equation, which is concerned about the second law of Newton.

$$P = m \times v, \quad F = \frac{dP}{dt}$$

And if there was no exterior force, total momentum in this system is steady. This is the conservation of the momentum.

In the assumption, there is one rocket in non-gravity space. The engine is ignited in some time ($\Delta t$), and the rate of exhausting gas is steady, and exhausted gas has same relative velocity to rocket.




**[Figure 8]** Momentum Conservation

[Figure 8] shows the difference in time ($\Delta t$). There is no exterior force, so the equation of momentum is like this.

$$0 = \frac{\Delta P}{\Delta t} = \frac{(P_2 - P_1)}{\Delta t} = \frac{\{[(M - \Delta M) \times (v + \Delta v) + (\Delta M \times v)] - [M \times v]\}}{\Delta t}$$

If $\Delta t$ is converged to zero, $\frac{\Delta v}{\Delta t}$ is to $\frac{dv}{dt}$ and $\frac{\Delta M}{\Delta t}$ to $-\frac{dM}{dt}$ because total mass of rocket is decreased. And $u - (v + \Delta v)$ is $V_{rel}$. So the equation becomes like this.

$$M \times (\frac{dv}{dt}) = (u - (v + \Delta v)) \times (\frac{dM}{dt}) \text{ or, } M \times (\frac{dv}{dt}) = V_{rel} \times (\frac{dM}{dt})$$

$V_{rel} \times (\frac{dM}{dt})$ is the part of first equation in 10.4.1; $\dot{m} \times V_e$. This factor is called momentum thrust or velocity thrust. $(P_e - P_a) \times A_e$ is called pressure thrust and it has the best efficiency when $P_e = P_a$.

### 4.3 Impulse and Momentum

The equation of Newton's second law can be expressed like these.

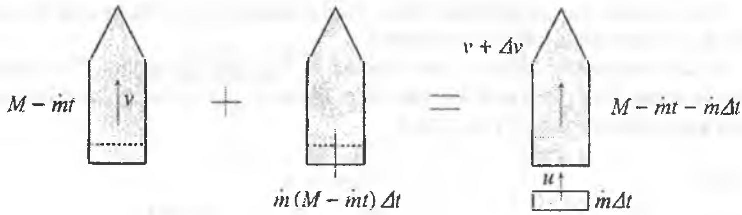
$$F \times dt = dP$$

$$\int F dt = P_2 - P_1, \text{ or } P_1 + \int F dt = P_2$$

And if there are several force which affect the object, these all force should be considered as vector sum.

$P_1 + \sum \int F dt = P_2$, or $P_1 + \sum (F \times \Delta t) = P_2$ if time interval is $\Delta t$.

In assumption, rocket has its initial mass M in time t=0. The rate of mass use is $\dot{m}$ and its relative velocity to rocket is $V_e$. After time t, the mass of left fuel is $M - \dot{m}t$ and velocity is $v$.




[Figure 9] Impulse and Momentum

The absolute velocity of exhausted fuel is u. Then the equation becomes like this without any friction.

$$(M - \dot{m}t) \times v - g \times (M - \dot{m}t) \times \Delta t = (M - \dot{m}t - \dot{m}\Delta t) \times (v + \Delta v) - \dot{m}\Delta t \times u$$

Dividing all factors in $\Delta t$ and using $V_e$. If $\Delta t$ converged to zero, the equation becomes like this.

$$-g \times (M - \dot{m}t) = (M - \dot{m}t) \times (\frac{dv}{dt}) - (\dot{m} \times V_e)$$

Rearrange of variables and integration from $t$=0, $v$=0 to $t$=$t$, $v$=$v$

$$\int dv = \int \left[ \frac{\dot{m} \times V_e}{M - \dot{m}t} - g \right] dt, \text{ so } \boxed{v = V_e \times LOG\left[ \frac{M}{M - \dot{m}t} \right] - g \times t}$$

The factor $-gt$ means the pulling of the gravity field. So if the rocket isn't vertical, this factor should be $-gt\ sin(T)$. $T$ means angle between thrust vector of rocket and surface of the earth.

And, if $v$ is integrated from $t$=0 to $t$=$t$ again, there is the distance of rocket during combustion.

$$d = V_e + \left( t + t \times LOG\left[ \frac{M}{M - \dot{m}t} \right] + M \times \frac{LOG\left( \frac{M - \dot{m}t}{M} \right)}{\dot{m}} \right) - g \times \frac{t^2}{2}$$

If rocket moves in orbit of the earth or non-gravity, $-gtsin(T)$ is zero. Therefore, the equation is like this.

$$v = V_e \times LOG\left[ \frac{M}{M - \dot{m}t} \right]$$

Moreover, we can get the time of combustion with rearrange of this equation.

$$t = M \times \frac{\left[1-\left(\frac{1}{10^{\left[\frac{\Delta t}{V_e}\right]}}\right)\right]}{\dot{m}}$$

### 4.4 Specific Impulse

The efficiency measure of rocket propulsion is specific impulse, which is mentioned by time. It means how much thrust can be generated by one pound (or kilogram) of propellant in one second.

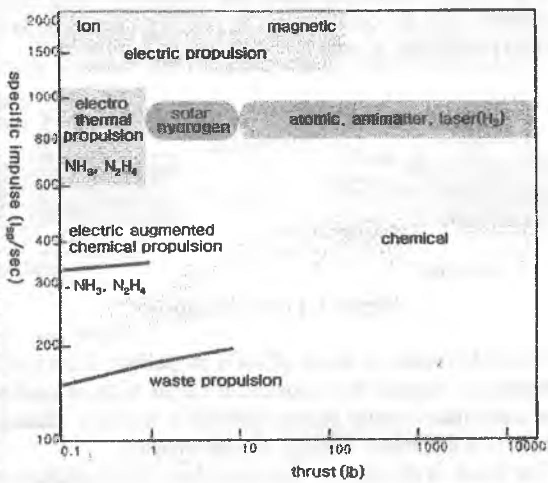
Physically specific impulse of rocket is $I_{sp}$.

$I_{sp} = \frac{F}{\dot{m} \times g}$ (F=thrust, $\dot{m}$=mass rate of fuel, g=gravity acceleration)

Specific impulse has different value in vacuum and in the atmosphere. If nozzle is ideal and best suited, the specific impulse in the ground is $\frac{V_e}{g}$.

This picture shows the specific impulse and thrust according to propulsion system.

According [Figure 10-10], chemical system is used for main propulsion system such like booster. In addition, electric system is used for assist-propulsion system such like fine control of satellite or transition of orbit.




[Figure 10] Thrust and specific impulse by propulsion system

*5 Rocket Propellant*

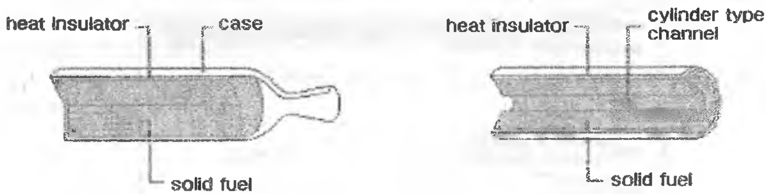
Propellant means the chemical mixture, which is oxidized to get the thrust, and this is consisted of fuel and oxidizer.

**5.1 Liquid propellant**

In liquid propulsion system, fuel and oxidizer is stored in different tanks and mix in the combustion chamber through pipe, valve and turbo-pump. This system is much more complex than solid propulsion system, but it has advantages. This system can control the engine; turn on or off, control of intensity by regulation of flowing propellant. Good liquid propellant has high specific impulse and makes high speed of exhaust gas. It means it has low molecular weight, high heating temperature and gas. Moreover, density of propellant is big factor because this is concerned with weight of rocket. Storage temperature and toxicity of propellant is also important because it needs special equipment's, which increase the weight of rocket.

**5.2 Solid propellant**

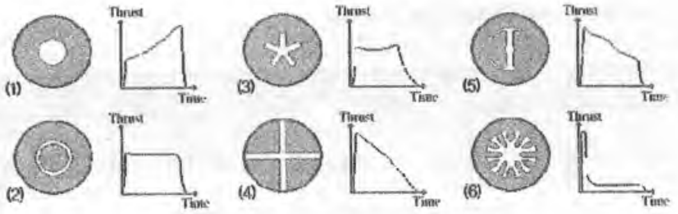
Solid propellant system has the simplest structure. This is consisted of casing, which is filled with solid-chemical mixture (fuel and oxidizer), and nozzle which gets the thrust by exhausting of gas. During ignition, solid propellant is oxidized from center to outline. Shape of center channel decides the combustion rate and pattern, so control the thrust. However, control of solid rocket is limited. Most of all, this cannot stop the combustion, so that if combustion is started, it will not finished until all propellant is used.




**[Figure 11]** Form of solid fuel

There are two main cases of shape of solid propellant. First is cylinder block, which the combustion is started from nozzle to upper side of casing, and this is called end burner and make steady thrust. Second is cylinder channel, which the surface of combustion is generated through center channel.

Shape of fuel block is decided by the operation. These pictures are examples of shape and their characters.




[Figure 12] Block shape of solid fuel

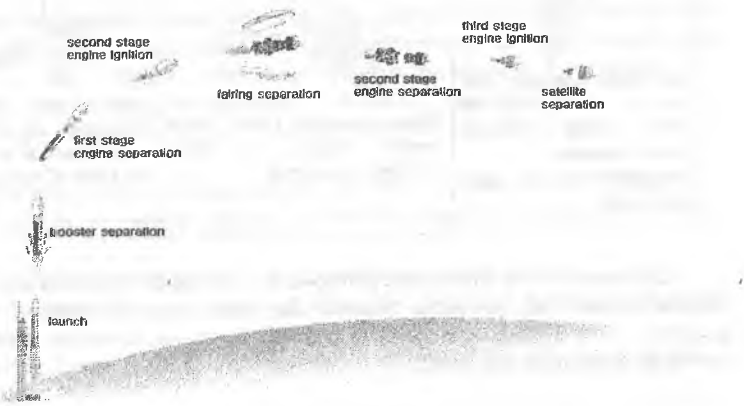
**5.3 Hybrid propellant**

Hybrid propellant is considered as the center group between solid and liquid propulsion engine. Usually, fuel is solid and oxidizer is liquid. Fuel tank has the role of combustion chamber like solid propulsion system and oxidizer is injected to this tank. Main advantage of this engine is that it has high performance like solid propellant rocket and has control of the engine like liquid propellant rocket.

It is very difficult to apply this concept to the rocket, which needs huge thrust. Therefore, this concept has not fit for practical use until now.

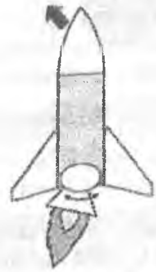
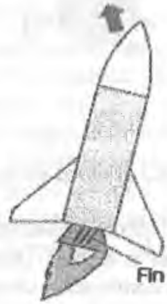
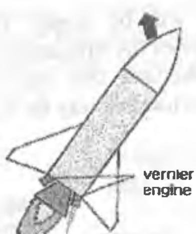
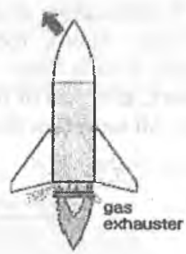
***6 Multi-Stage Rocket***

After launch, fuel tank will be empty. The empty tank is not needed anymore, what is more it decreases the efficiency of rocket because of its weight. Therefore, after use of fuel in the tank, this tank is separated to decrease the total weight. All rockets in those days use this way for better performance.



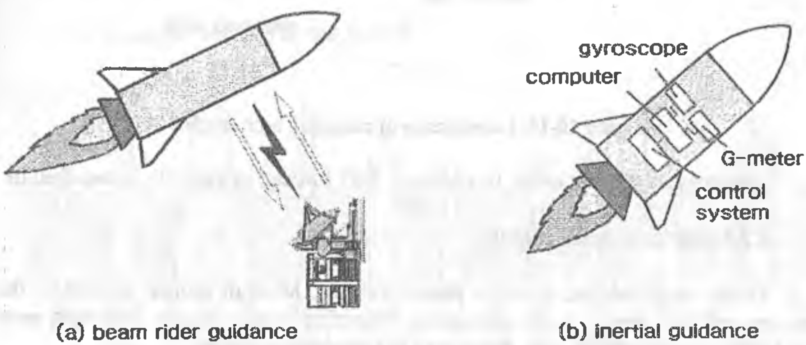

[Figure 13] Multi-stage rocket

# 7 Guidance and Control of Rocket

| Control | | |
|---|---|---|
|   Wing control: It changes the direction using wing of rocket in the atmosphere. |  Rotation of nozzle or engine: The whole direction of engine or nozzle is changed. Usually assist-booster of space shuttle used this way. |   Jet fin: Using fin in the nozzle, the direction of exhausting gas can be changed. |
|   Auxiliary injection: This way is usually used in solid rocket. Through assist-injection equipment gas or liquid is poured. |   Vernier engine: Using small vernier engine, the flight direction is changed. |   Gas exhaust: Gas is exhausted in cold-gas exhaust which is set on the body of rocket. |

Guidance means that rocket is moved to according to decided path and plan. Control means that computer changes the speed and direction of rocket for guidance. This system is absolutely needed for space operation because most operation in space is unmanned.

| Guidance | |
|---|---|
| Program guidance | The flight path is previously memorized in computer. |
| Beam rider guidance | It's remote control between rocket and ground center. |
| Inertial guidance | Using gyro and G-meter, rocket finds its path with calculation of its position by itself. |




**[Figure 14]** Guidance systems

*8 Launch Vehicle of Satellite*

This is the rocket system, which put the satellite into orbit. Performance and circumstance of launch vehicle can give great influence to design of satellite, and reliability and cost of launch vehicle is main factors for succeed of operation.

10.8.1 Launch stage and sequence of stationary orbit satellite

① Rise: Flight from ground to 200~300 km of parking orbit or around perigee.

② Parking orbit: If satellite reaches this orbit, satellite rises to perigee.

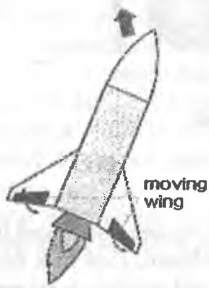
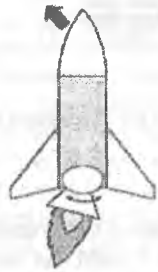
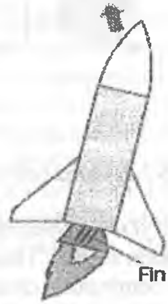
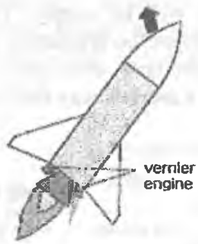
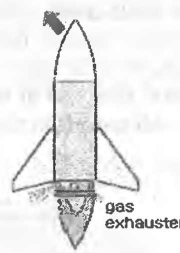
③ Ignition of perigee engine: Ignition the engine if satellite reaches this orbit.

④ Transfer orbit: Entry from parking orbit to transfer orbit.

⑤ Ignition of apogee engine: Ignition the engine if satellite reaches this orbit.

⑥ Drift orbit: Before satellite sets its position, it entries this orbit to check the all system.

⑦ Operational orbit: Using satellite propulsion system, satellite moves from drift orbit

| Control | | |
|---|---|---|
| <br>Wing control: It changes the direction using wing of rocket in the atmosphere. | <br>Rotation of nozzle or engine: The whole direction of engine or nozzle is changed. Usually assist-booster of space shuttle used this way. | <br>Jet fin: Using fin in the nozzle, the direction of exhausting gas can be changed. |
| <br>Auxiliary injection: This way is usually used in solid rocket. Through assist-injection equipment gas or liquid is poured. | <br>Vernier engine: Using small vernier engine, the flight direction is changed. | <br>Gas exhaust: Gas is exhausted in cold-gas exhaust which is set on the body of rocket. |

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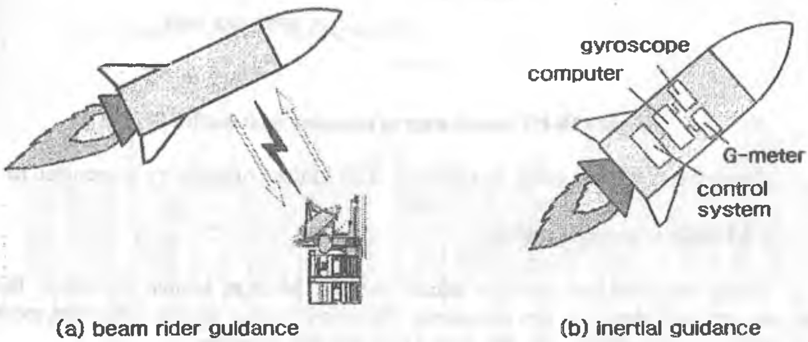


**[Figure 14]** Guidance systems

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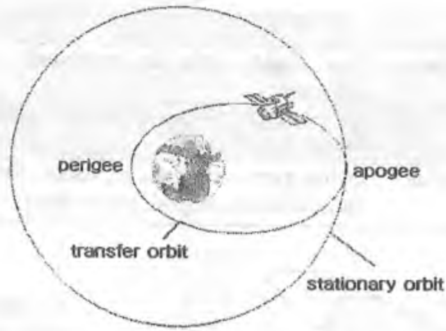



**[Figure 10-15]** Launch stage of stationary orbit satellite

to geosynchronous orbit. In addition, 3.07 km/sec of velocity is required to set.

### 8.2 Kinds of launch vehicle

There are reuse and attrition launch vehicle. Most of launch vehicle in the past was attrition that it is not economic. Therefore, space shuttle, which is most well-known reuse vehicle, was developed to solve this problem.

According to orbit, there are launch vehicles for stationary orbit and low orbit.

According to performance, there four classification like this table.

| Classification by payload capacity | Low orbit | Stationary Orbit | Representative Launch vehicle |
|---|---|---|---|
| Ex-large | > 10,000 kg | > 5,000 kg | Proton, Titan, Ariane... |
| Large | 5,000~10,000 kg | 2,000~5,000 kg | Atlas, Long March, H-2... |
| Medium | 2,000~5,000 kg | 1,000~2,000 kg | Atene-3, Molniya... |
| Small | < 2,000 kg | < 1,000 kg | PSLV, Taurus, Scout... |

### 8.3 Difference between missile and launch vehicle

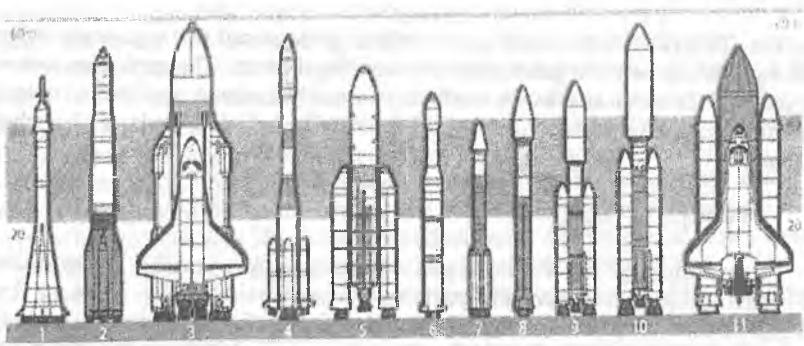
Technology in launch vehicle and missile is same without payloads and flight path. Propulsion system, structure, staging, guidance and control, launch equipment, ground support and system integration are common points of two crafts.

Historically, most of launch vehicles are developed from ballistic missiles; Titan, Atlas and Delta in U.S, Start in Russian and Long March in China.

The biggest difference between these two systems is payloads. Satellite is usually complex and delicate system, but it does not have to re-enter to atmosphere. However, warhead include nuclear, chemical and biological weapons should designed to endure the stress when missile tries to re-enter to the atmosphere.

Usually launch vehicle needs higher speed than missile to reach the orbit, which is desired. In addition, this high speed can be acquired by high-performance 3-stage rocket or kick motor for entry of orbit. However, missiles don't have to equip these things.

### 8.4 Launch vehicles in the world




**[Figure 10-16]** Launch vehicles in the world
(1: Soyuz, 2: Proton, 3: Energiya and Buran, 4: Ariane4 5: Ariane5 6. Long March, 7: Delta II, 8: Atlas II, 9: Titan III, 10: Titan IV, 11. Space Shuttle)

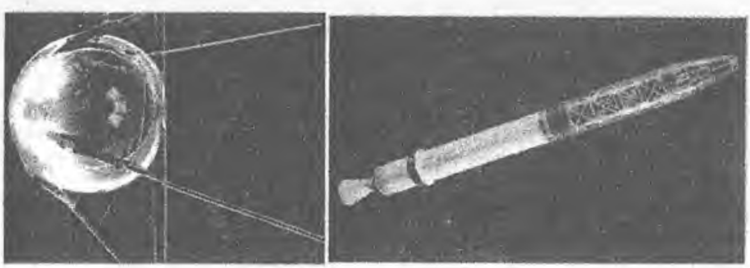
**Satellite**

Satellite: This is the small-mass object, which goes around the huge-mass object with equilibrium between gravitation and centrifugal forces. The earth goes around the sun, and the moon around the earth; these are called natural satellite. In contrast with these natural ones, there is also artificial satellite, which is made and launched by human.

1 Principle of Flight

Its principle is very simple. It just needs a minimum velocity, which makes the centrifugal force overcome the gravitation force. This velocity is called the first astronautical velocity (7.9 km/sec) which makes the satellite go around the earth in the case of no air-friction. In addition, the minimum velocity, which makes the spacecraft, goes to moon or other planet over the earth orbit is called escape velocity or the second astronautical velocity, which is value of 11.2 km/sec.

***2. Development History***



**[Figure 11-1]** Principle of satellite **[Figure 11-2]** Sputnik 1 (L) and Explorer 1 (R)

The 4th of October 1957, the Soviet Union launched the first satellite, Sputnik 1, into the low orbit of the earth (900km). It is the opening of the space development era and detonator of competition between United States and Soviet Union. Sputnik 1 is aluminum sphere satellite with diameter of 0.58 m and mass of

83.6 kg and went around the earth 16 times in 24hours. After two months, Soviet Union launched Sputnik 2 with the first living thing, the dog name of 'Laika'.

The first satellite by United States is Explorer 1 and was launched after 4 months of Sputnik 1. It has length of 1.2 m, diameter of 20.3 cm, and weight of 14.6 kg. It reached the altitude of 2,460 km and founded strong radiation belts, which are called Van Allen Radiation.

In the early of 1960, John F. Kennedy who was the president of United States announced Apollo project, which is manned moon exploration during 1960s. With lots of support 25 billion dollars in 9 years, NASA succeeded this project. The 21st of July 1969, Apollo spaceship reached the surface of the moon. Until Apollo 17, US had gotten great results from 6 times of exploration.

In 1970s, Soviet Union and United States tried to cooperate of space development in the field of science technology. July of 1975, Apollo 18 of United States and Soyuz 19 of Soviet Union had docking and did co-experience.

Soviet Union tried the space station for space journey, so they launched Salute in 1971 and MIR in 1986. United Stated developed the concept of space shuttle, which can be reused. So it has launched more than 100 times. In addition, ISS (International Space Station) has been built with attention of 16 countries.

From 1980s, space development is expanded to space industry, which is more commercial with launch vehicle and broadcasting, and telecommunication satellite industry. In the 1985, United Stated announced the policy of 'Open Skies' to be commercial with export of satellite and hardware technology.

After 1990s, the cold war between two countries is finished so that total budget of space development was decreased. Relatively, however, civil commercialization has widely developed because of the development of civil info-communication technology.

### *3. Kinds of Satellite*

#### 3.1 Classification by operation (payloads)

Earth observation satellite and metrological satellite are usually positioned in the low orbit for direct observation of surface and atmosphere. For the first time, the purpose of observation satellite was only for resources, but it uses for all phenomenon on the earth include increase of desertification, forest denuda, condition of ozone layer, El Nino of the earth, hurricane, flood and forest fire and so on. These things are closely connected to our lives.

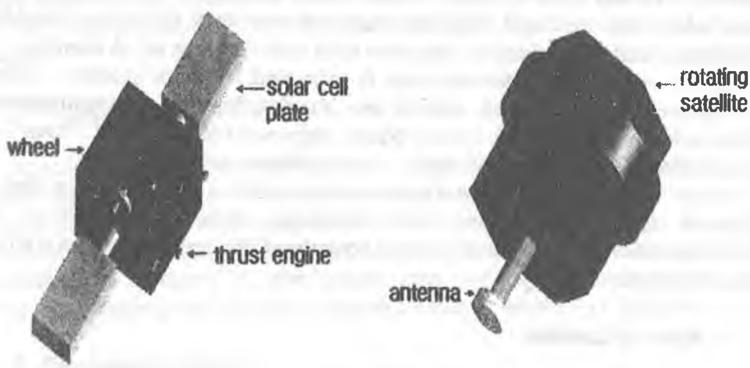
GPS satellite is proposed in United States in the 1973 for new concept of navigation guidance equipment in military use. It has 24 satellites, which has the period of 12 hours. Nowadays, GPS is used for navigation system of car, airplane and ship, ATC, surveillance of earthquake and rescue and so on.

The purpose of scientific satellite is scientific research. It researches atmosphere, sun, ground, energy and complex interaction between living things. It also investigates density of atmosphere, temperature, ionization degree, radiation from sun, magnetic field of earth and the cosmic ray. Pioneer, explorer, sky-lab, HST (Hubble space telescope) by United States and sputnik, cosmos by Soviet Union are well known scientific satellites.

During cold war, United States and Soviet Union launched many reconnaissance satellites for surveillance of various military actions. For example, there are infrared-light sensor for discovery of missile launching, radar for aircraft carrier and warship, visual surveillance of the ground and interception of telecommunication transmission. DSP satellites of United States which was launched for the first time in the early of 1970s are still good performance in the active service. Cosmos satellite of Soviet Union had launched around 2,000 times during cold war, and half of them are known as military satellite for surveillance.

### 3.2 Classification by shape or stabilization form

There are main two kinds of satellite; box-shape (3-axis stabilization satellite) and sphere-shape (rotation stabilization satellite).




**[Figure 11-5] 3-axis stabilization type and Rotation stabilization type**

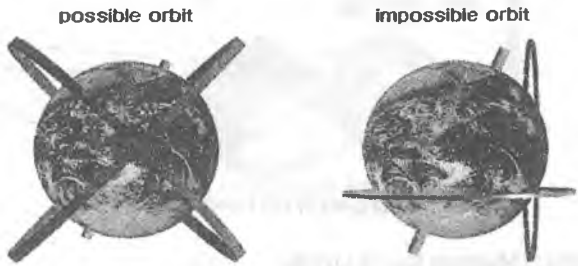
3-axis stabilization satellite, which is the most common way of stabilization, is usually consisted of body, which is shape of box, and solar cell plate, which is unfolded. This satellite is very stable without low-speed rotation to turn the antenna and sensors toward the earth. Moreover, solar-battery plate rotates inverse to cancel the inertia of rotation. This method much depends on the performance of sensor and driving machine, but it has high precision. In addition, this way uses reaction wheel to get gyroscopic stiffness to stabilize the satellite.

Rotation stabilization satellite can get the stabilization with rotation of satellite itself in constant speed. This method is simple and easy to set any direction. However, it has weak point that antenna, sensor and solar battery plate cannot head to inertia object.

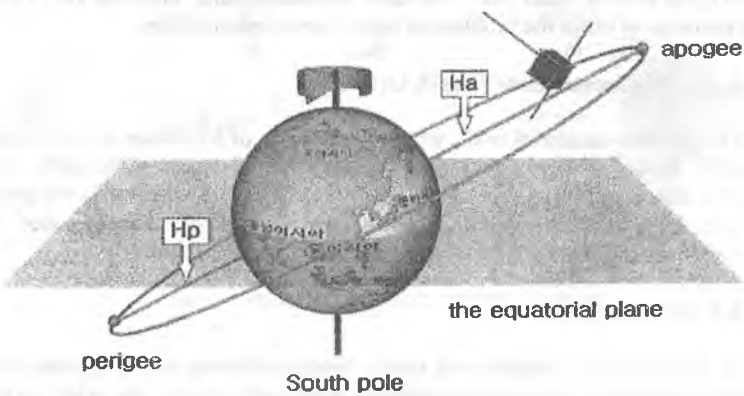
*4 Satellite Orbit*

Orbit is the path which satellite moves. Therefore, movement of satellite is fixed in one orbital plane, which always includes the center of the earth.

Orbit is consisted of $H_p$ (distance between surface of the earth and the perigee), $H_a$ (distance between surface and the apogee), $P$ and $I$ (angle between orbital and equatorial plane) (orbital period).



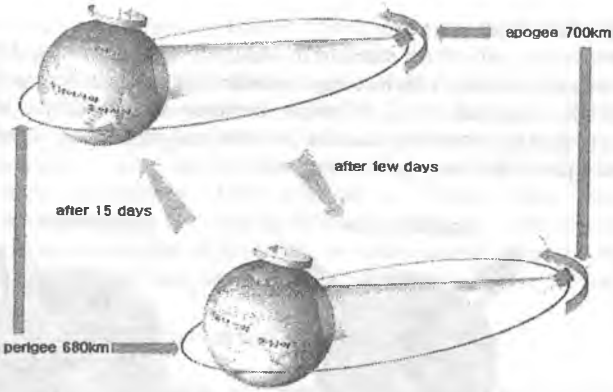

[Figure 6] Possible and Impossible orbits




[Figure 7] Factors of satellite orbit

### 4.1 LEO (Law Earth Orbit)

Altitude is from 500km to 1,500km. Period is usually short around 90 minutes or 2 hours so that it rotates the earth 10s times in a day. There are elliptical or circular orbit and inclination angle between orbital and equatorial plane is from 0° to more than 90°. More than 90° means that it rotates the earth in the reverse way of rotation of the earth. Most of earth observation satellite and mobile-telecommunication satellite use this orbit.




**[Figure 8]** LEO satellite (Lansat)

**2 MEO (Medium Earth Orbit)**

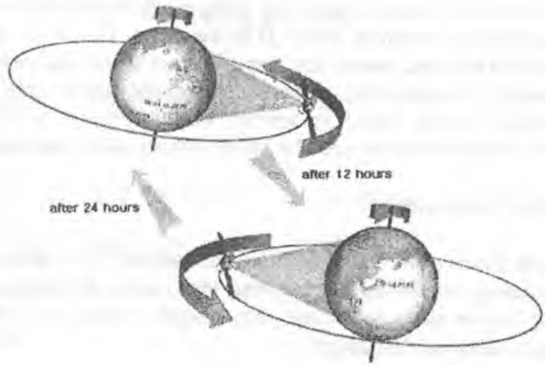
from 1,500km to 5,000km and from 15,000 to 30,000km, the altitude between these belts is MEO. As 'Van Allen Radiation Belts' includes lots of high-energy particles, it make the problem of radio-telecommunication.

**4.3 GEO (Geostationary Earth Orbit)**

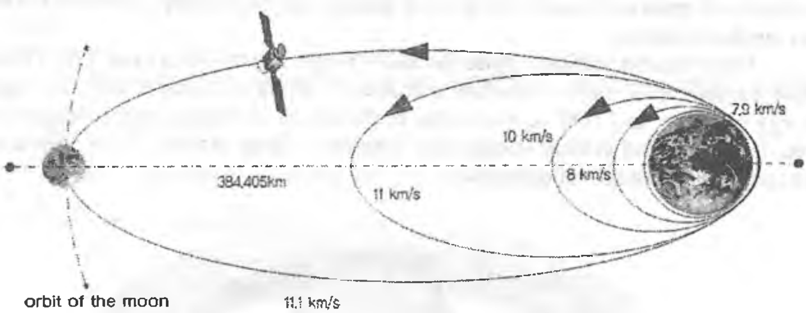
This orbit is equatorial orbit, which has altitude of 35,786km and inclination angle of 0°. Its period is 23h 56m 4s, which is same of period of the earth so the observer in the ground sees that satellite is fixed. Therefore, antenna in the ground does not have to change its angles. This orbit is usually used for telecommunication broadcasting satellite or meteorological satellite.

**4.4 Elliptical Orbit**

In circular orbit, satellite has steady velocity. If velocity of the satellite is increased, its shape of orbit becomes ellipse. With high velocity, the orbit has high curvature of ellipse. Molniya orbit which has high eccentricity, 40,000km of apogee and 600km of perigee can cover the high-latitude area, for example Russia, which geostationary orbit can't cover and survey for a long time. As this satellite moves slowly in the area of apogee by the second law of Kepler, it can survey the North Pole area for 10 hours in the period of 12 hours.




**[Figure 9]** Geostationary Earth Orbit




**[Figure 10]** Change of orbit by velocity of satellite

### *5 Composition of satellite*

To operate satellite, various kinds of equipments are needed. This can be widely classified like these; satellite itself, ground control system and launch vehicle system.

Satellite is consisted of main-body (bus) and payload, and main-body includes some part-bodies and supplies, which support payload. Payload operates telecommunication, surveillance of the earth, weather and science research and so on. Main-body has great role from launch to end of operation, for example protection of payloads and equipments from outer circumstance, control of position and path, heat control, propulsion, mechanical support, proper electricity supply and communication with ground control center to make the payload succeed its operation.

Ground control center means the equipments and software in the ground to telecommunicate with satellite, which is in the space. This is the station for test and surveillance with antenna, electricity amplifier and low-noise receiver.

As launch vehicle system is the carrier, which put the satellite into the orbit, getting minimum speed, which is desired, is very important. To select the most proper launch vehicle is influenced by many factors, mass, cost, and so on.

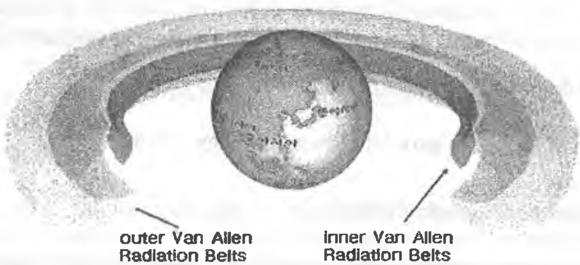
### *6. Space Environment*

In space environment, there are cosmic radiant light, micro gravity, vacuum condition, heating condition, cosmic dust, and so on. Therefore, these things are closely related to the altitude, seasons, time, sunspot cycle and orbit.

#### 6.1 Radiation environment

There are main three radiations in the space. Particles, which is consisted of photon and electron in Van Allen Radiation Belts, galaxy cosmic radiant light from explosion of nova and supernova in other galaxy, and high-energy particles for the sun are those things.

These radiant particles cause the SEE (Single Event Effect) and TDE (Total Dose Effect). SEE makes one-time mistakes in kinds of electric parts by high-energy particles, and TDE decreases the performance of satellite so that leakage of electric current and critical voltage can happen by long expose. These situations are connected to failure of operation.




**[Figure 11]** Van Allen Radiation Belts

#### 6.2 Neutral atmosphere in thermosphere

Thermosphere is from 90km to 600km. In thermosphere, there are various neutral gases, which are oxygen, helium and so on. Density of neutral gas is connected to altitude, life span and motion of satellite, because of its resistance satellite needs additional thrust to go through this sphere.

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**CLAUS CORDES,**
**BOTIR USMONOV**

# BASICS OF PERFORMANCE

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| | |
|---|---|
| Muharrir: | M.Mirkomilov |
| Tex. muharrir: | M.Holmuhamedov |
| Musavvir: | D.Azizov |
| Musahhih: | N.Hasanova |
| Kompyuterda sahifalovchi: | Sh.Mirqosimova |

**E-mail: tipografiyacnt@mail.ru Tel: 245-57-63, 245-61-61.**
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