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**R. Turgunbayev, Sh.Ismailov,
O.Abdullayev**

DIFFERENSIAL TENGLAMALAR KURSIDAN MISOL VA MASALALAR TO'PLAMI

(o'quv qo'llanma)

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R.Turgunbayev, Sh.Ismailov, O. Abdullayev. Differensial tenglamalar kursidan misol va masalalar to'plami / Toshkent, TDPU, 2007 y.

Differensial tenglamalar nazariyasi amaliy matematika, fizika, biologiya iqtisod va h.k. larda uchraydigan ko'plab masalalarni tadqiq etishda muhim vosita hisoblanadi. Differensial tenglamalar ishlatilmaydigan fan tarmog'ni topish qiyin. Ushbu o'quv qo'llanma pedagogika oliy ta'lim muassasalari talabalariga differensial tenglamalarni tushunish, yechish va interpretasiya qilishda yordam beradi. Qo'lanmada oddiy differensial tenglamalarning asosiy turlariga oid nazariy ma'lumotlar va bunday tenglamalarni yechish usullari bayon qilingan. Maple® kompyuter sistemasiga tayangan *differensial tenglamalarni simvolik va sonli* yechish metodlari bayon qilingan.

Bu qo'llanmadan «Fizika va astronomiya» ta'lim yonalishidagi talabalar ham foydalanishi mumkin.

Тургунбаев Р., Исмаилов Ш., Абдуллаев О. Сборник примеров и задач по курсу дифференциальных уравнений / Ташкент, ТГПУ, 2007 г.

Теория дифференциальных уравнений является важным средством в исследовании многих задач, возникающих в прикладной математике, физике, биологии, экономике, и т.д. Фактически трудно найти *ветвь науки, где не используются дифференциальные* уравнения.

Это пособие призвано помочь студентам высших педагогических учебных заведений в понимании, решении и интерпретации дифференциальных уравнений.

В пособии даются необходимая теоретическая информация и методы решения важных классов обыкновенных дифференциальных уравнений. Приведено большое количество приложений в физике, геометрии и других наук. Описаны методы символьных и численных решений в компьютерной системе Maple®.

R.Turgunbayev, Sh.Ismailov, O.Abdullayev. The Collection of examples and problems in course of differential equations / Tashkent, TSPU, 2007.

Theory of differential equations is an important tool in the investigation of many problems in applied mathematics, physics, biology, economics, etc.. In fact, it is hard to find a branch in science where differential equations is not used.

This book will be used to help for students of higher pedagogical institutions in understanding, solving, and interpreting differential equations.

In this book the theoretical information and the methods of solution of important classes of ordinary differential equations are given. Examples of applications to physics, geometry and the other sciences abound. Methods of *symbolic and numerical solutions in Maple® computer system are described.*

Taqrizchilar: O'.Toshmetov, Nizomiy nomidagi TDPU, professor
A.Xashimov, O'zR FA MI, katta ilmiy hodim

Mas'ul muharrir:
B.Islomov, fizika-matematika fanlari doktori, professor

O'quv qo'llanma Nizomiy nomidagi Toshkent davlat pedagogika universiteti Ilmiy kengashida ko'rib chiqilgan va o'quv qo'llanma sifatida nashrga tavsiya qilingan.

2007 yil « 25 » yanvar 6 –sonli majlis bayoni.

SO'Z BOSHI

Ushbu o'quv qo'llanma pedagogika oliy ta'lim muassasalari «Matematika va informatika» ta'lim yonalishi uchun «Differensial tenglamalar» kursining dasturi asosida yozilgan bo'lib, uning asosiy qismi «Fizika va astronomiya" ta'lim yo'nalishida ham foydalanilishi mumkin.

Mustaqil o'rganuvchi talabalar uchun qo'llanmadan foydalanishni osonlashtirish maqsadida muhim nazariy ma'lumotlar keltirilgan, bu ma'lumotlarni bilish misol va masalalarni tushunib echish uchun zaruriy hisoblanadi. To'liq nazariy ma'lumotlarni qo'llanma so'ngida keltirilgan adabiyotlardan topish mumkin.

Qo'llanma uch bobdan iborat bo'lib, birinchi bobda birinchi tartibli oddiy differensial tenglamalar, ikkinchi bobda yuqori tartibli oddiy differensial tenglamalarga oid asosiy ma'lumotlar, ularga doir misol va masalalar yechish namunalari, amaliy mashg'ulotlarda hamda mustaqil ishlash uchun misol va masalalar keltirilgan. Qo'llanmada differensial tenglamalar yordamida fizik va geometrik masalalarni yechishga alohida e'tibor berilgan. Uchinchi bobda Maple® kompyuter algebrasi vositasiga tayangan masalalar yechish metodikasi bayon qilinib, bunda differensial tenglamalarni analitik hamda taqribiy yechish, grafiklarini chizish ko'rsatilgan. Shuningdek, mazkur qo'llanmada individual vazifalar to'plami ham berilgan.

Ushbu qo'llanmani o'qib chiqib, o'zining qimmatli fikrlarini bildirgan professor O'.Tosmetovga va fizika-matematika fanlari nomzodi, A.Xashimovga samimiy minnatdorchiligimizni bildiramiz.

Mualliflar.

I-BOB. BIRINCHI TARTIBLI DIFFERENSIAL TENGLAMALAR.

1-§. Asosiy tushunchalar. O'zgaruvchilari ajraladigan tenglamalar.

1. Asosiy tushunchalar.

x erkli o'zgaruvchi, shu o'zgaruvchining y funksiyasi va y' hosilani bog'lovchi

$$F(x,y,y')=0 \qquad (1)$$

munosabat 1- *tartibli differensial tenglama* deyiladi.

Agar (1) munosabatda y ni $\varphi(x)$ funksiya bilan almashtirish natijasida $F(x,\varphi(x),\varphi'(x))\equiv 0$ ayniyat hosil bo'lsa, $\varphi(x)$ funksiya (1) tenglamaning *yechimi* deyiladi.

Agar

$$\frac{\partial\Phi}{\partial x}+\frac{\partial\Phi}{\partial y}y'=0,$$

$$\Phi(x,y,C)=0$$

munosabatlardan C parametr yo'qotilgandan so'ng (1) tenglama hosil bo'lsa, u holda

$$\Phi(x,y,C)=0 \qquad (2)$$

oshkormas funksiya (1) tenglamaning *umumiy integrali* deyiladi.

Ixtiyoriy C o'zgarmasga ma'lum $C=C_0$ qiymat berish natijasida $\Phi(x,y,C)=0$ umumiy integraldan hosil qilingan $\Phi(x,y,C_0)=0$ oshkormas funksiya (1) differensial tenglamaning *xususiy integrali* deyiladi.

Geometrik nuqtai nazardan umumiy integral koordinatalar tekisligida C parametrga bog'liq bo'lgan va tenglamaning *integral egri chiziqlari* deb ataladigan egri chiziqlar oilasini ifodalaydi. Xususiy integralga bu oilaning $C=C_0$ ga mos bo'lgan egri chizig'i mos keladi.

Ayrim hollarda (2) dan

$$y=\varphi(x,C) \qquad (3)$$

ko'rinishdagi (1) tenglamaning *umumiy yechimini* hosil qilish mumkin.

Umumiy integralni, shuningdek umumiy yechimni topish jarayoni (1) tenglamani *integrallash* deb yuritiladi.

Izoh. Ayrim hollarda qulaylik tug'dirish maqsadida o'zgarmas C ning o'rniga kC yoki $k\ln C$ olinadi, bu yerda k – ixtiyoriy son.

C o'zgarmasga ma'lum $C=C_0$ qiymat berish natijasida $y=\varphi(x,C)$ umumiy yechimdan hosil qilingan har qanday $y=\varphi(x,C_0)$ funksiya (1) differensial tenglamaning *xususiy yechimi* deyiladi.

Qulaylik uchun (1) differensial tenglama *hosilaga nisbatan yechilgan*

$$\frac{dy}{dx}=f(x,y) \qquad (4)$$

tenglama shaklida yoki simvolik ravishda *differensiallar ishtirok etgan*

$$M(x,y)dx+N(x,y)dy=0 \qquad (5)$$

tenglama shaklida ifodalashga harakat qilinadi.

Izoh. Ayrim hollarda (4) o'rniga y ni erkli o'zgaruvchi deb, shu o'zgaruvchining $x(y)$ funksiyasiga mos $\frac{dx}{dy}=\frac{1}{f(x,y)}$ tenglama ham qaraladi.

(1) tenglamaning *boshlang'ich shart* deb nomlanadigan

$$y(x_0)=y_0 \qquad (6)$$

ko'rinishdagi shartni qanoatlantiradigan yechimlarini topish masalasi *Koshi*[1] *masalasi* yoki *boshlang'ich masala* deyiladi.

(4) tenglama uchun Koshi masalasi qisqacha quyidagicha yoziladi :

$$\frac{dy}{dx}=f(x,y), \quad y|_{x=x_0}=y_0$$

Koshi masalasi geometrik nuqtai nazardan qaraganda barcha integral egri chiziqlar ichidan berilgan (x_0,y_0) nuqtadan o'tuvchi integral egri chiziqni topish masalasidir.

Agar (x_0,y_0) nuqtadan ikkita va undan ko'p integral chiziqlar o'tsa bu nuqtada *yagonalik sharti bajarilmagan* deb yuritiladi.

Agar (1) tenglamaning $\varphi(x)$ yechimi uchun ixtiyoriy $(x_0,\varphi(x_0))$ nuqtada yagonalik sharti bajarilmasa u holda $\varphi(x)$ *maxsus* yechim deyiladi.

Izoh. (1) differensial tenglamaning $\varphi(x)$ maxsus yechimi (agar mavjud bo'lsa) C ning hech qanday qiymatida (3) ni (shuningdek (2) ni) qanoatlantirmaydi.

Maxsus yechimlarni aniqlash uchun alohida usullar mavjud. Biz ularni 5-§ da bayon qilamiz.

Berilgan $y'=f(x,y)$ tenglama aniqlanish sohasining har bir nuqtasidan o'tuvchi va abssissa o'qi bilan $\alpha=arctgf(x,y)$ burchak tashkil qiluvchi to'g'ri chiziqlar oilasiga differensial tenglamaning *yo'nalishlar maydoni* deyiladi.

Har bir nuqtasida yo'nalishlar maydoni bir xil bŏlgan chiziq *izoklina* deyiladi. Izoklina tushunchasini yana quyidagicha izohlash mumkin:

Bir hil yo'nalishga ega bo'lgan integral egri chiziqga o'tkazilgan urinmalar urinish nuqtalarining geometrik ŏrni izoklina deyiladi.

$y'=f(x,y)$ tenglamaning izoklinalar oilasi $f(x,y)=k$ tenglamalar bilan aniqlanadi.

(4) tenglamaning (x_0,y_0) nuqtadan o'tuvchi integral chiziqni tasvirlash uchun k ning yetarlicha ko'p qiymatlariga mos izoklinalar chiziladi. Har bir izoklina bo'ylab mos burchak koeffitsienti k ga teng shtrixlar yasaladi.

(x_0,y_0) nuqtadan boshlab har bir izoklinani mazkur strixlarga parallel ravishda integral chiziq yasaladi.

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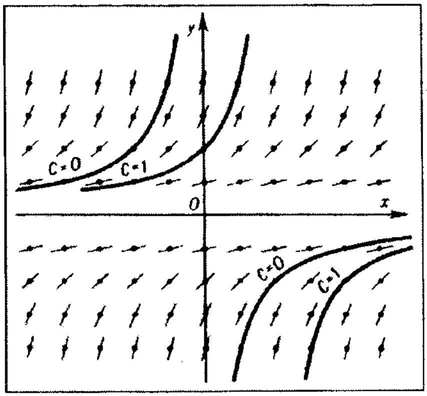
[1] Koshi Lui Ogyusten (1789-1857)- fransiyalik matematik.

1-rasmda mazkur yasashlar $\frac{dy}{dx}=y^2$ tenglama uchun amalga oshirilgan. Bu tenglamaning umumiy yechimi $y=\frac{1}{C-x}$ bo'lishini tekshirish qiyin emas.



1-rasm.

2. O'zgaruvchilari ajraladigan tenglamalar.

$$y'=f(x)g(y) \qquad (7)$$

ko'rinishdagi differensial tenglama *o'zgaruvchilari ajraladigan* tenglama deyiladi.

(7) tenglamani

$$y'-f(x)g(y)=0;$$
$$dy-f(x)g(y)dx=0;$$
$$\frac{dy}{g(y)}-f(x)dx=0 \quad (g(y)\neq 0);$$

ko'rinishlarga keltirsa bo'ladi.

$$f(x)=-X(x); \quad \frac{1}{g(y)}=Y(y);$$

belgilashlarni kiritsak, natijada *o'zgaruvchilari ajralgan*

$$X(x)dx+Y(y)dy=0$$

tenglamaga ega bo'lamiz.

Ravshanki, bu tenglama

$$\int X(x)dx+\int Y(y)dy=C$$

ko'rinishdagi umumiy integralga ega.

Izoh. (7) tenglama uchun mos bo'lgan $g(y)=0$ algebraik tenglamaning $y=a$ ko'rinishdagi yechimlari alohida tekshirilishi lozim, aks holda maxsus yechimlarni yo'qotish mumkin.

Misollar. Quyidagi differensial tenglamalarni yeching

a) $yy'=\dfrac{-2x}{\cos y}$.   b) $y'=y^{2/3}$.   c) $y'+\sin(x+y)=\sin(x-y)$.

Yechish. a) $yy'=\dfrac{-2x}{\cos y}$ tenglamani soddalashtiramiz:

$$y\cos y\cdot\frac{dy}{dx}=-2x \Leftrightarrow y\cos y dy=-2xdx$$

Oxirgi tenglama o'zgaruvchilari ajralgan, uni integrallaymiz:

$$\int y\cos ydy=-2\int xdx$$

Chap tarafdagi integral bo'laklab integrallash usuli yordamida hisoblanadi:

$$\int y\cos ydy=\begin{Bmatrix} u=y; & dv=\cos ydy; \\ du=dy; & v=\sin y \end{Bmatrix}=y\sin y-\int\sin ydy=y\sin y+\cos y$$

Natijada

$$y\sin y+\cos y+x^2=C$$

umumiy integralni hosil qilamiz .

Javob: $y\sin y+\cos y+x^2=C$.

b) Berilgan $y'=y^{2/3}$ tenglamadan o'zgaruvchilari ajralgan

$$y^{-2/3}dy=dx$$

tenglamani hosil qilamiz.

Bu tenglamani integrallaymiz:

$$\int y^{-2/3}dy=\int dx$$

Bundan $3y^{1/3}-x=C$ ko'rinishdagi umumiy integralga ega bo'lamiz.

Natijada $y=\dfrac{1}{27}(x+C)^3$ umumiy yechimni topamiz.

$y^{2/3}=0$ algebraik tenglamaning $y=0$ yechimi berilgan tenglamaning maxsus yechimi bo'lishini qayd etamiz.

Javob: $y=\dfrac{1}{27}(x+C)^3$, $y=0$.

c) $y'+\sin(x+y)=\sin(x-y)$ ifodani soddalashtiramiz:

$$y'+\sin(x+y)-\sin(x-y)=0 \Leftrightarrow y'-2\sin\frac{x-y-x-y}{2}\cos\frac{x-y+x+y}{2}=0 \Leftrightarrow$$

$$\Leftrightarrow y'-2\sin(-y)\cos x=0 \Leftrightarrow y'+2\sin y\cos x=0.$$

Oxirgi tenglamadan o'zgaruvchilari ajralgan

$$\frac{dy}{\sin y}=-2\cos xdx$$

tenglamani hosil qilamiz. Bu tenglamani integrallaymiz:

$$\int\frac{dy}{\sin y}=-2\int\cos xdx$$

Bunda integrallar jadvalidan foydalanib, $\ln\left|tg\frac{y}{2}\right|+2\sin x=C$ umumiy integralni topamiz.

$\sin y=0$ algebraik tenglamaning $y=\pi n, n\in Z$ yechimlaridan har biri berilgan tenglamaning maxsus yechimi bo'lishini qayd etamiz.

Javob: $\ln\left|tg\frac{y}{2}\right|+2\sin x=C$, $y=\pi n, n\in Z$.

Misollar. Differensial tenglamaning berilgan boshlang'ich shartni qanoatlantiradigan yechimlarini toping:

a) $\frac{y}{y'}=\ln y$, $y|_{x=2}=1$. b) $\frac{yy'}{x}+e^{y}=0$, $y|_{x=1}=0$.

c) $y'=x(y^2+1)$, $y|_{x=x_0}=y_0$ (bunda x_0, y_0 - ixtiyoriy sonlar)

Yechish. a) Berilgan $\frac{y}{y'}=\ln y$ tenglamani $\frac{ydx}{dy}=\ln y$ ko'rinishda yozib, undan o'zgaruvchilari ajralgan

$$dx=\frac{\ln ydy}{y}$$

tenglamani hosil qilamiz. Bu tenglamani integrallaymiz:

$$\int dx=\int\frac{\ln ydy}{y},\quad x+C=\int\ln yd(\ln y),\quad x+C=\frac{\ln^2 y}{2}.$$

Endi $y(2)=1$ boshlang'ich shartdan foydalanib, C ning qiymatini topamiz:

$$2+C=\frac{\ln^2 1}{2};\quad\Rightarrow\quad 2+C=0;\quad\Rightarrow\quad C=-2;$$

Bundan $2(x-2)=\ln^2 y$ yani $y=e^{\pm\sqrt{2x-4}}$ ko'rinishdagi xususiy yechimlarga ega bo'lamiz.

Javob: $y=e^{\pm\sqrt{2x-4}}$.

b) $\frac{yy'}{x}+e^{y}=0$ tenglamani o'zgaruvchilari ajraladigan tenglamaga olib kelamiz:

$$\frac{ydy}{dx}+xe^{y}=0\Rightarrow ydy+xe^{y}dx=0.$$

Bundan quyidagilarni hosil qilamiz:

$$\frac{y}{e^{y}}dy=-xdx;\quad\int\frac{y}{e^{y}}dy=-\int xdx;$$

Chap tarafdagi integralni bo'laklab integrallash usulida topamiz:

$$\int ye^{-y}dy=\left\{\begin{matrix}u=y; & e^{-y}dy=dv;\\ du=dy; & v=-e^{-y};\end{matrix}\right\}=-e^{-y}y-\int(-e^{-y})dy=-e^{-y}y-e^{-y}=-e^{-y}(y+1)$$

Bundan $e^{-y}(y+1)-\frac{x^2}{2}=C$ umumiy integrallarga ega bo'lamiz. C ning qiymatini aniqlash uchun $y(1)=0$ boshlang'ich shartdan foydalanamiz.

$$e^0(0+1)-\frac{1}{2}=C \quad \Rightarrow \quad C=\frac{1}{2}$$

Natijada $2e^{-y}(y+1)=x^2+1$ xususiy integralga ega bo'lamiz.

Javob: $2e^{-y}(y+1)=x^2+1$

c) $y'=x(y^2+1)$ tenglamani o'zgaruvchilari ajralgan tenglamaga olib kelamiz:

$$\frac{dy}{dx}=x(y^2+1) \Leftrightarrow \frac{dy}{y^2+1}=xdx$$

Bundan $\int\frac{dy}{y^2+1}=\int xdx$ kelib chiqadi va biz $arctgy-\frac{x^2}{2}=C$ umumiy integralga va

$$y=tg\left(\frac{x^2}{2}+C\right)$$

umumiy yechimga ega bo'lamiz.

C ning qiymatini aniqlash uchun $y(x_0)=y_0$ boshlang'ich shartdan foydalanamiz.

$$arctgy_0=\frac{x_0^2}{2}+C \quad \Rightarrow \quad C=arctgy_0-\frac{x_0^2}{2}.$$

Natijada $y=tg\left(\frac{x^2}{2}+arctgy_0-\frac{x_0^2}{2}\right)$ xususiy yechimga ega bo'lamiz.

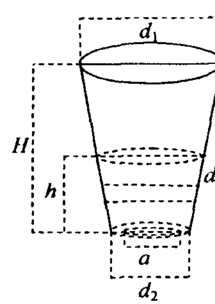
Javob: $y=tg\left(\frac{x^2}{2}+arctgy_0-\frac{x_0^2}{2}\right)$.

3. O'zgaruvchilari ajraladigan differensial tenglamalarga olib kelinadigan masalalarni ko'rib chiqamiz.

Masala. Ustki (katta) asosning diametri d_1, pastki asosining diametri d_2, balandlik H bo'lgan konussimon rezervuar suv bilan to'ldirilgan. Suv rezervuar tubidagi a diametrli teshik orqali oqizib yuborilganda rezervuar qancha vaqtda bo'shashini aniqlang. (2-rasm)

Masalani umumiy holda yechib, olingan natijani berilgan vaziyatga qo'llaymiz.

h balandlikka ($0\le h\le H$) mos bo'lgan idishning ko'ndalang kesim yuzi ma'lum $S=S(h)$ ko'rinishga ega bo'lib, u H sathgacha suyuqlik bilan to'ldirilgan bo'lsin. Idish tubida yuzi ω bo'lgan teshikdan suyuqlik oqib chiqmoqda. Suyuqlik sathi dastlabki H holatdan istalgan h gacha pasayish vaqti t ni va idishning to'la bo'shash vaqti T ni aniqlaymiz. Bunda idishdagi suyuqlik miqdorining o'zgarish tezligi v idishdagi suyuqlik sathi h ning ma'lum $v=v(h)$ funksiyasi deb faraz qilinadi.



2-rasm

Biror t vaqt momentida idishdagi suyuqlik balandligi h ga teng bo'lsin, t dan $t+dt$ gacha bo'lgan dt vaqt oralig'ida idishdan oqib chiqadigan suyuqlik miqdori dv ni asosning yuzi ω, balandligi $v(h)$ bo'lgan silindr hajmi sifatida hisoblab chiqish mumkin.

Shunday qilib

$$dv=\omega\, v(h)dt. \qquad (8)$$

Endi suyuqlikning ana shu hajmini boshqa usul bilan hisoblaymiz. Suyuqlik oqib chiqqanligi sababli idishdagi suyuqlikning h sathi $dh<0$ kattalikka o'zgaradi, demak

$$dv=-S(h)dh. \qquad (9)$$

(8) va (9) lardan ushbu o'zgaruvchilari ajraladigan differensial tenglamaga ega bo'lamiz:

$$\omega\, v(h)dt=-S(h)dh$$

O'zgaruvchilarni ajratamiz: $dt=-\dfrac{S(h)}{\omega\, v(h)}dh$

Oxirgi ifodaning chap tarafini 0 dan t gacha, o'ng tarafni esa mos bo'lgan H dan h gacha oraliqlarda integrallaymiz va natijada

$$t=-\frac{1}{\omega}\int_H^h \frac{S(h)}{v(h)}dh=\frac{1}{\omega}\int_h^H \frac{S(h)}{v(h)}dh$$

tenglikka ega bo'lamiz.

Idish batamom bo'shaganda $h=0$, shu sababli idishning to'la bo'shash vaqti T ushbu formula bo'yicha topiladi:

$$T=\frac{1}{\omega}\int_0^H \frac{S(h)}{v(h)}dh$$

Gidravlikadan ma'lumki, agar suyuqlik yetarlicha kichik teshikdan oqib chiqayotgan bo'lsa, u holda quyidagi Torrichelli qonuni o'rinli:

$$v(h)=\mu\sqrt{2gh},$$

bu yerda $g\approx 10$ m/s^2 -erkin tushish tezlanishi, μ - sarf bo'lish koeffitsienti (suv uchun $\mu\approx 0{,}6$). Bu holda hosil qilingan formulalar quyidagi ko'rinishda bo'ladi:

$$t=\frac{1}{\omega\mu\sqrt{2g}}\int_h^H \frac{S(h)}{\sqrt{h}}dh, \quad T=\frac{1}{\omega\mu\sqrt{2g}}\int_0^H \frac{S(h)}{\sqrt{h}}dh \qquad (10).$$

Ravshanki, berilgan konusning ko'ndalang kesim yuzi

$$S(h)=\frac{\pi}{4}\left(d_2+(d_1-d_2)\frac{h}{H}\right)^2$$

formula yordamida aniqlanadi.

Shu sababli T uchun hosil bo'lgan formulaga ko'ra:

$$T=\frac{1}{a^2\mu\sqrt{2g}}\int_0^H \frac{(d_2+(d_1-d_2)\frac{h}{H})^2}{\sqrt{h}}dh=\frac{2\sqrt{H}}{15a^2\mu\sqrt{2g}}\left(3d_1+4d_1d_2+8d_2^{\,2}\right)$$

$g \approx 10$ m/s^2 va $\mu \approx 0{,}6$ ni inobatga olsak,

$$T \approx \frac{0{,}05\sqrt{H}}{a^2}\left(3d_1 + 4d_1d_2 + 8d_2^2\right)$$

taqribiy formulaga ega bo'lamiz.

Javob: $T = \frac{2\sqrt{H}}{15a^2\mu\sqrt{2g}}\left(3d_1 + 4d_1d_2 + 8d_2^2\right) \approx \frac{0{,}05\sqrt{H}}{a^2}\left(3d_1 + 4d_1d_2 + 8d_2^2\right)$

Masala. Massasi m, issiqlik sig'imi c o'zgarmas bo'lgan jism boshlang'ich momentda T_0 temperaturaga ega bo'lsin. Havo temperaturasi o'zgarmas va τ ($T > \tau$) ga teng. Jismning cheksiz kichik dt vaqt ichida bergan issiqligi jism va havo temperaturalari orasidagi farqqa, shuningdek vaqtga proporsional ekanligini e'tiborga olgan holda jismning sovish qonunini toping.

Yechish. Sovish davomida jism temperaturasi T_0 dan τ gacha pasayadi. Vaqtning t momentida jism temperaturasi T ga teng bo'lsin. Cheksiz kichik dt vaqt oralig'ida jism bergan issiqlik miqdori masala shartiga ko'ra

$$dQ = -k(T - \tau)dt$$

ga teng, bu yerda $k = const$ - proporsionallik koeffitsienti.

Ikkinchi tomondan, jism T temperaturadan τ temperaturagacha soviganda beradigan issiqlik miqdori $Q = mc(T - \tau)$ ga teng. Demak, $Q = mcdT$.

dQ uchun topilgan har ikkala ifodani taqqoslab, $mcdT = -k(T - \tau)dt$ differensial tenglamani hosil qilamiz. O'zgaruvchilarni ajratish natijasida quyidagiga ega bo'lamiz:

$$\frac{dT}{T - \tau} = -\frac{k}{mc}dt.$$

Bu tenglamani integrallab, quyidagini topamiz:

$$\ln|T - \tau| = -\frac{k}{mc}t + \ln C, \text{ yoki } T - \tau = Ce^{-\frac{k}{mc}t}.$$

Boshlang'ich shart ($t = 0$ da $T = T_0$) C ni topishga imkon beradi: $C = T_0 - \tau$

Shuning uchun jismning sovish qonuni quyidagi ko'rinishda yoziladi:

$$T = \tau + (T_0 - \tau)e^{\frac{kt}{mc}}.$$

Javob: $T = \tau + (T_0 - \tau)e^{-\frac{kt}{mc}}$.

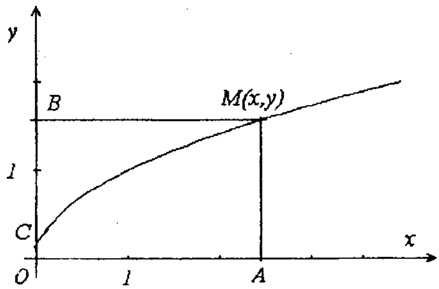
Masala. Egri chiziqning istalgan nuqtasidan koordinata o'qlariga parallel to'g'ri chiziqlar o'tkazishdan hosil bo'lgan to'g'ri to'rtburchak shu egri chiziq bilan ikki qismga bo'linadi. Bu bo'laklardan Ox o'qqa yopishganining yuzi ikkinchisinikidan ikki marta katta. Agar egri chiziq $M_0(1;1)$ nuqtadan o'tishi ma'lum bo'lsa, uni toping.

Yechish. Egri chiziqning $M(x,y)$ nuqtasi orqali Oy o'qqa parallel MA to'g'ri chiziq va Ox o'qqa parallel MB to'g'ri chiziq o'tkazamiz (3-rasm)

Masala shartiga ko'ra $S_{OCMA} = 2S_{CBM}$. Ma'lumki,

$$S_{OCMA} = \int_0^x ydx,\ S_{CBM} = S_{OBMA} - S_{OCMA} = xy - \int_0^x ydx$$

Noma'lum funksiya uchun $\int_0^x ydx = 2\left(xy - \int_0^x ydx\right)$ yoki $3\int_0^x ydx = 2xy$ munosabatlarni hosil qilamiz.



3-rasm

Oxirgi munosobatlarning ikkala tomonini x bo'yicha differensiallash natijasida $2xy' = y$ differensial tenglamani hosil qilamiz. Bu o'zgaruvchilari ajraladigan tenglamaning yechimi $y' = Cx$ ekanligini topish qiyin emas.

Boshlang'ich shartdan foydalanib, C=1 ni topamiz. Shunday qilib, izlanayotgan egri chiziq $y^2=x$ paraboladan iborat ekan.

Javob: $y^2=x$.

Quyidagi differensial tenglamalarni yeching (1.1-1.20):

1.1. $\left(1+y^2\right)dx+\left(1+x^2\right)dy=0.$ 1.2. $\left(1+y^2\right)dx=xydy.$

1.3. $x\sqrt{1+y^2}+yy'\sqrt{1+x^2}=0.$ 1.4. $(x+1)^3dy-(y-2)^2dx.$

1.5. $2x\sqrt{1-y^2}=y'\left(1+x^2\right).$ 1.6. $yy'=\dfrac{1-2x}{y}.$

1.7. $(x+2)\left(y^2+1\right)dx+\left(y^2-x^2y^2\right)dy=0.$ 1.8. $y^2\sin xdx+\cos^2 x\ln ydy=0.$

1.9. $\sec^2 x\sec ydx=-\operatorname{ctg}x\sin ydy$. 1.10. $x+xy+yy'(1+x)=0$

1.11. $(y+xy)dx+(x-xy)dy$. 1.12. $yy'+x=1$

1.13. $\sin x\cos ydx=\cos x\sin ydy$. 1.14. $1+(1+y')e^y=0$

1.15. $x \acute{o}dx+\sqrt{1+x^2}dy=0$. 1.16. $y'(x^2-4)=2xy$

1.17. $y'+y\operatorname{tg}x=0$. 1.18. $y'\sqrt{a^2+x^2}=y$

1.19. $(xy^2+y^2)dx+(x^2-x^2y)dy=0$. 1.20. $(xy^2+y^2)dx+(x-xy)dy=0$

Quyidagi Koshi masalalarini yeching (1.21-1.22):

1.21. $y^2+x^2y'=0, y(-1)=1$ 1.22. $2(1+e^x)yy'+e^x=0, y(0)=0$

1.23. $(1+x^2)y^3dx-(y^2-1)x^3dx=0,\ y(1)=-1$. 1.24. $2y'\sqrt{x}=y;\ y|_{x=4}=1\ \ y=e^{\sqrt{x}-2}$

1.25. $y' = (2y+1)ctgx;\ y|_{x=\frac{\pi}{4}} = \frac{1}{2}$.

1.26. $y' = 2\sqrt{y}\ln x;\ y|_{x=e} = 1$

1.27. $(1+y^2)dx = xydy;\ y|_{x=2} = 1$.

1.28. $y'\sin x - y\cos x = 0,\ y\left(\frac{\pi}{2}\right) = 1$.

1.29. Balandligi H=1,5m, asosining diametri D=1m bo'lgan silindrik idish suv bilan to'ldirilgan. Suv idish tubidagi diametri d=5sm bo'lgan teshik orqali oqizib yuborilganda idish qancha vaqtda bo'shashini aniqlang.

1.30. O'q v_0=200*m/s* tezlik bilan harakatlanib h=10 sm qalinlikdagi devorni teshib, undan v_1=80*m/s* tezlik bilan uchib chiqadi. Devorning qarshilik kuchi o'qning harakat tezligi kvadratiga proporsional. O'qning devor ichida harakatlanish T vaqtini toping.

1.31. $A(0;-2)$ nuqtadan o'tuvchi va ixtiyoriy nuqtasida o'tkazilgan urinmaning burchak koeffitsienti urinish nuqtasi ordinatasining uchlanganiga teng bo'lgan chiziqni toping.

1.32. Egri chiziqning istalgan nuqtasidagi urinmasining koordinatalar o'qlari orasidagi kesmasi urinish nuqtasida teng ikkiga bo'linadi. Shu egri chiziqni toping.

1.33. Jismning havoda sovish tezligi jism temperaturasi va havo temperaturasi ayirmasiga proportsional. Agar havo temperaturasi 20^0C bo'lganda jism 20 minutda 100^0C dan 60^0C gacha sovisa, uning temperaturasi necha minutda 30^0C gacha pasayadi?

2-§. Bir jinsli differensial tenglamalar.

Agar t parametrning ixtiyoriy noldan farqli qiymatida $f(tx,ty) = t^n f(x,y)$ ayniyat bajarilsa, $f(x,y)$ funksiya *n- tartibli bir jinsli* funksiya deyiladi.

Masalan, $f(x,y) = x^3 + 3x^2y$ funksiya uchun

$$f(tx,ty) = (tx)^3 + 3(tx)^2ty = t^3x^3 + 3t^3x^2y = t^3(x^3 + 3x^2y) = t^3 f(x,y).$$

Demak, bu funksiya 3- tartibli bir jinsli bo'ladi.

Agar $f(x,y)$ - nol - tartibli bir jinsli funksiya bo'lsa, u holda

$$y' = f(x,y) \qquad (1)$$

differensial tenglama *bir jinsli* deyiladi.

Ravshanki, bir xil tartibli bir jinsli $P(x,y)$ va $Q(x,y)$ funksiyalar qatnashgan

$$P(x,y)dx + Q(x,y)dy = 0 \qquad (2)$$

tenglama bevosita bir jinsli differensial tenglamaga olib kelinadi va shunung uchun u ham bir jinsli tenglama deb yuritiladi.

(1) tenglamani, shuningdek, (2) tenglamani o'zgaruvchilari ajraladigan tenglamaga keltirish mumkin.

$f(x,y)$ - nol - tartibli bir jinsli funksiya bo'lgani uchun quyidagi ayniyatga ega bo'lamiz:

$$f(tx,ty) = f(x,y).$$

t parametrni ixtiyoriy tanlab olishimiz mumkinligidan foydalanib, bu ayniyatda $t=\frac{1}{x}$ almashtirishni amalga oshirsak,

$$f(x,y)=f\left(1,\frac{y}{x}\right)$$

ayniyatni hosil qilamiz.

$y = ux$ formula orqali yangi izlanayotgan u funksiyani kiritib

$$y'=\varphi(u) \qquad (3)$$

ko'rinishdagi tenglamaga ega bo'lamiz, bu yerda

$$\varphi(u)=f(1,u).$$

$y = ux$ bo'lgani uchun,

$$y'=u'x+u.$$

bo'ladi. Buni (3) qo'yamiz:

$$u'x+u=\varphi(u).$$

Natijada u funksiyaga nisbatan

$$u'=\frac{\varphi(u)-u}{x}$$

ko'rinishdagi o'zgaruvchilari ajraladigan tenglamani hosil qilamiz.

Bu tenglamani integrallash quyidagicha amalga oshiriladi.

$$\frac{du}{\varphi(u)-u}=\frac{dx}{x}; \quad \int\frac{du}{\varphi(u)-u}=\int\frac{dx}{x}+C;$$

Bundan keyin hosil bo'lgan umumiy integralda yordamchi u funksiya o'rniga $\frac{y}{x}$ ifodani qo'yamiz.

Ushbu

$$\frac{dy}{dx}=f\left(\frac{ax+by+c}{a_1x+b_1y+c_1}\right)$$

ko'rinishdagi tenglama bir jinsli yoki o'zgaruvchilari ajraladigan tenglamaga keltiriladi.

Agar $\begin{vmatrix} a & b \\ a_1 & b_1 \end{vmatrix}\neq 0$ bo'lsa $x=u+\alpha$, $y=v+\beta$ almashtirish amalga oshiriladi, bu yerda α va β sonlar $\begin{cases} ax+by+c=0 \\ a_1x+b_1y+c_1=0 \end{cases}$ tenglamalar sistemasini qanoatlantiradi.

Natijada bir jinsli tenglamani hosil qilamiz.

Agar $\begin{vmatrix} a & b \\ a_1 & b_1 \end{vmatrix}=0$ bo'lsa, berilgan tenglama

$$\frac{dy}{dx}=f\left(\frac{k(a_1x+b_1y)+c}{a_1x+b_1y+c_1}\right)$$

ko'rinishda bo'ladi, bunda $k=\frac{a}{a_1}=\frac{b}{b_1}$. Bundan keyin

$$a_1x+b_1y=t \text{ yoki } ax+by=t$$

almashtirish berilgan tenglamani o'zgaruvchilari ajraladigan tenglamaga keltiradi.

Masala. Quyidagi tenglamarning umumiy yechimini toping:

a) $(y^2-2xy)dx+x^2dy=0$; *b*) $xy'=\sqrt{x^2-y^2}+y$;

c) $(x-2y+3)dy+(2x+y-1)dx=0$;

d) $2(x+y)dy+(3x+3y-1)dx=0$

Yechish. a) $(y^2-2xy)dx+x^2dy=0$ tenglama tarkibidagi $P=y^2-2xy$, $Q=x^2$ funksiyalar ikkalasi ham ikkinchi tartibli bir jinsli funksiyalar bo'lgani uchun bu tenglama bir jinsli tenglama bo'ladi.

Shuning uchun $y=xu$ almashtirishni qo'llaymiz. U holda $dy=xdu+udx$ va tenglama $x^2(u^2-2u)dx+x^2(xdu+udx)=0$ yoki $(u^2-u)dx+xdu=0$ ko'rinishda bo'ladi.

O'zgaruvchilarni ajratamiz: $\frac{dx}{x}=\frac{du}{u(1-u)}$ va hosil qilingan tenglamani integrallaymiz:

$$\int\frac{dx}{x}=\int\frac{du}{u(1-u)} \tag{4}$$

O'ng tomondagi integralni topamiz:

$$\int\frac{du}{u(1-u)}=\int\left(\frac{1}{u}+\frac{1}{1-u}\right)du=\int\frac{du}{u}+\int\frac{du}{1-u}=\ln|u|-\ln|1-u|+\ln|C|=\ln\left|\frac{Cu}{1-u}\right|.$$

Topilgan ifodani (4) ga qo'ysak,

$\ln|x|=\ln\left|\frac{Cu}{1-u}\right|$, yani $x=\frac{Cu}{1-u}$ yoki $u=\frac{x}{C+x}$ ga ega bo'lamiz.

So'ngi ifodadagi u o'rniga $\frac{y}{x}$ ni qo'yib, $y=\frac{x^2}{C+x}$ umumiy yechimni topamiz.

Javob: $y=\frac{x^2}{C+x}$.

b) Berilgan tenglamani $y'=\sqrt{1-\left(\frac{y}{x}\right)^2}+\frac{y}{x}$ ko'rinishda yozsak uni bir jinsli differensial tenglama ekanligiga ishonch hosil qilamiz.

$y=xu$ almashtirishni qo'llaymiz. U holda $y'=u+xu'$. Bu ifodalarni berilgan tenglamaga qo'ysak $x\frac{du}{dx}=\sqrt{1-u^2}$ bo'ladi. O'zgaruvchilarni ajratib, $\frac{du}{\sqrt{1-u^2}}=\frac{dx}{x}$ ni hosil qilamiz, bu yerdan $\arcsin u=\ln|Cx|$.

Bundan $u=y/x$ bo'lgani uchun, $\arcsin\frac{y}{x}=\ln C|x|$ umumiy integralni topamiz. Natijada $y=x\sin(\ln C|x|)$ umumiy yechim topiladi.

c) Berilgan tenglamani

$$\frac{dy}{dx}=\frac{-2x-y+1}{x-2y+3} \qquad (5)$$

ko'rinishda yozib olamiz.

$\begin{vmatrix} -2 & -1 \\ 1 & -2 \end{vmatrix}=4+1=5\neq 0$ bo'lgani uchun $x=u+\alpha, y=v+\beta$ almashtirishlarni amalga oshiramiz, bu yerda α va β parametrlar $\begin{cases} -2x-y+1=0 \\ x-2y+3=0 \end{cases}$ tenglamalar sistemasini qanoatlantiradi.

Bu sistemani yechamiz:

$$\begin{cases} -2x-y+1=0 \\ x-2y+3=0 \end{cases} \Leftrightarrow \begin{cases} y=1-2x \\ x-2+4x+3=0 \end{cases} \Leftrightarrow \begin{cases} \alpha=-1/5 \\ \beta=7/5 \end{cases}$$

Endilikda $x=u-1/5$; $y=v+7/5$ larni (5) ga qo'yamiz

$$(u-1/5-2v-14/5+3)dv+(2u-2/5+v+7/5-1)du=0;$$

$$(u-2v)dv+(2u+v)du=0;$$

$$\frac{dv}{du}=\frac{2u+v}{2v-u};$$

$$\frac{dv}{du}=\frac{2+v/u}{2v/u-1}.$$

Hosil bo'lgan bir jinsli tenglamani yechish uchun $\frac{v}{u}=t$ belgilash kiritamiz. U holda: $v=ut$; $v'=t'u+t$.

Natijada o'zgaruvchilari ajralgan

$$t'u+t=\frac{2+t}{2t-1}$$

tenglamani hosil qilamiz.

Uni integrallaymiz:

$$\frac{dt}{du}u=\frac{2+t}{2t-1}-t=\frac{2+t-2t^2+t}{2t-1}=\frac{2(1+t-t^2)}{2t-1};$$

$$\frac{du}{u}=-\frac{1}{2}\cdot\frac{1-2t}{1+t-t^2}dt; \qquad \int\frac{du}{u}=-\frac{1}{2}\int\frac{(1-2t)dt}{1+t-t^2}$$

$$-\frac{1}{2}\ln\left|1+t-t^2\right|=\ln|u|+\ln C_1$$

$$\ln\left|1+t-t^2\right|=-2\ln|C_1u|$$

$$\ln\left|1+t-t^2\right|=\ln\left|\frac{C_2}{u^2}\right|;\quad 1+t-t^2=\frac{C_2}{u^2}.$$

$t=\dfrac{v}{u}$, $u=x+1/5$; $v=y-7/5$ bo'lgani uchun

$$t=\frac{v}{u}=\frac{y-7/5}{x+1/5}=\frac{5y-7}{5x+1}$$

bo'ladi. Endi y va x larga qaytamiz:

$$1+t-t^2=\frac{C_2}{u^2}\Leftrightarrow\ 1+\frac{5y-7}{5x+1}-\left(\frac{5y-7}{5x+1}\right)^2=\frac{25C_2}{(5x+1)^2}\ \Leftrightarrow$$

$$\Leftrightarrow (5x+1)^2+(5y-7)(5x+1)-(5y-7)^2=25C_2\Leftrightarrow$$

$$\Leftrightarrow 25x^2+10x+1+25xy+5y-35x-7-25y^2+70y-49=25C_2\Leftrightarrow$$

$$\Leftrightarrow 25x^2-25x+25xy+75y-25y^2=25C_2+49-1+7\Leftrightarrow$$

$$\Leftrightarrow x^2-x+xy+3y-y^2=C_2+\frac{55}{25}=C$$

Bundan berilgan tenglamaning $x^2-x+xy+3y-y^2=C$ umumiy integralini hosil qilamiz.

Javob: $x^2-x+xy+3y-y^2=C$.

d) Berilgan tenglamani

$$\frac{dy}{dx}=-\frac{3x+3y-1}{2x+2y}$$

ko'rinishda yozib olamiz.

$\begin{vmatrix}-3 & -3\\ 2 & 2\end{vmatrix}=-6+6=0$ bo'lgani uchun $3x+3y=t$ almashtirishni amalga oshiramiz.

U holda

$$\frac{t'}{3}-1=-\frac{3(t-1)}{2t};\quad 2t(t'-3)=-9t+9;\quad 2tt'=6t-9t+9;\quad 2tt'=-3t+9$$

O'zgaruvchilarni ajratamiz: $\dfrac{2t}{-3t+9}dt=dx;\qquad \dfrac{t}{t-3}dt=-\dfrac{3}{2}dx$.

Oxirgi tenglamani integrallaymiz:

$$\int\left(1+\frac{3}{t-3}\right)dt=-\frac{3}{2}\int dx$$

$$t+3\ln|t-3|=-\frac{3}{2}x+C_1$$

Endi y va x larga qaytamiz:

$$2x+2y+2\ln|3(x+y-1)|=-x+C_2\Leftrightarrow$$

$$\Leftrightarrow 3x+2y+2\ln 3+2\ln|x+y-1|=C_2\Leftrightarrow$$

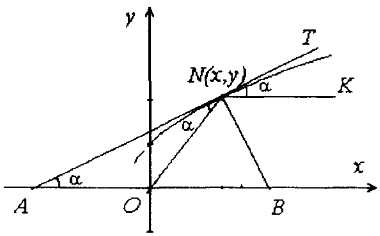
$$\Leftrightarrow 3x+2y+2\ln|x+y-1|=C.$$

Javob: $3x+2y+2\ln|x+y-1|=C$.

Masala. Ko'zguning shunday shaklini topingki, unga berilgan nuqtad tushgan hamma nurlar ko'zgudan qaytganda berilgan yo'nalishga parallel bo'lsin.

Yechish. Koordinata boshini berilgan nuqtaga deb olamiz va abssissalar o'q berilgan yo'nalishga parallel ravishda yo'naltiramiz.

Nur ko'zguning $N(x,y)$ nuqtasiga tushsin. Agar Ox o'q bilan egri chiziqni $N(x,y)$ nuqtasiga o'tkazilgan AN urinma orasidagi burchakni α orqali belgilasak, holda masala shartiga ko'ra: $\angle KNT = \alpha$. Ikkinchi tomondan nurning tushi burchagi $(\angle ONB)$ uning qaytish burchagi$(\angle BNK)$ ga teng bo'lganidan s burchaklarni $\frac{\pi}{2}$ ga to'ldiruvchi burchaklar sifatida$\angle ONA = \angle KNT$ va bund $\angle ONA = \alpha$. Shunday qilib, OAM uchburchak teng yonli va $AO=OM$ (4-rasm).



4-rasm

Bunda:

$$tg\alpha = y' = \frac{NP}{AP} = \frac{Y}{AP}, \quad AO = AP - OP = \frac{Y}{Y'} - x, \quad ON = \sqrt{x^2 + y^2}.$$

Natijada ushbu differensial tenglamani hosil qilamiz (bu yerda y – erkli o'zgaruvchi

$$\frac{y}{y'} - x = \sqrt{x^2 + y^2} \quad \text{yoki} \quad x' = \frac{x + \sqrt{x^2 + y^2}}{y}.$$

Bu tenglama birjinsli tenglama bo'ladi.

$x=yz$ almashtirishni bajarsak $yz' = \sqrt{1+z^2}$ hosil bo'ladi. Bundan

$$\frac{dz}{\sqrt{1+z^2}} = \frac{dy}{y} \quad \text{yoki} \quad \ln\left(z + \sqrt{1+z^2}\right) = \ln y + \ln C, \quad \text{ya'ni} \quad z + \sqrt{1+z^2} = Cy$$

z ni tenglamaning o'ng tomoniga o'tkazib, so'ngra hosil bo'lgan tenglikni ikkala tomonini kvadratga ko'tarsak, quyidagiga ega bo'lamiz:

$$C^2 y^2 = 1 + 2Cyz \quad \text{yoki } x \text{ ga qaytib, } y^2 = \frac{2}{C}\left(x + \frac{1}{2C}\right) \text{ ni hosil qilamiz.}$$

Demak, ko'zguning izlanayotgan shakli parabolalar oilasiga mansub.

Javob: $y^2 = \frac{2}{C}\left(x + \frac{1}{2C}\right).$

Masala. Istalgan $M(x,y)$ nuqtasida o'tkazilgan urinmaning ordinatalar o'qidan kesgan kesmaning OM vektorning uzunligiga nisbati o'zgarmas bo'lgan egri chiziqni toping.

Yechish. Izlanayotgan egri chiziqda ixtiyoriy $M(x,y)$ nuqta olamiz. (5-rasm). M nuqta orqali o'tkazilgan urinmaning tenglamasi:

$$Y - y = y'(X - x)$$

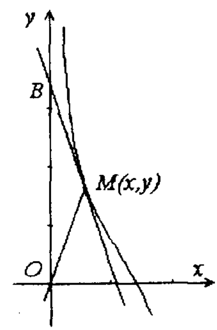
ko'rinishga ega bo'ladi, bu yerda X, Y - nuqtalarning o'zgaruvchi koordinatalari, y' - izlanayotgan funksiyaning berilgan nuqtadagi hosilasi. Urinmaning Oy o'qidan ajratgan OB kesmasini topish uchun $X=0$ deymiz. U holda $OB=Y=y-xy'$. Shartga ko'ra $\frac{OB}{OM} = a$, bu yerda $a=const$.

U holda $\frac{y-xy'}{\sqrt{x^2+y^2}} = a, \quad y' = \frac{y - a\sqrt{x^2+y^2}}{x}$

ko'rinishdagi bir jinsli tenglamaga ega bo'lamiz.

$y=xu$ almashtirishni bajarsak, $\frac{du}{\sqrt{1+u^2}} = -a\frac{dx}{x}$ tenglamani hosil qilamiz. Uni integrallaymiz: $u + \sqrt{1+u^2} = Cx^{-a}$.

Bu yerda u ni tenglikning o'ng tomoniga o'tkazib, so'ngra hosil bo'lgan tenglikning ikkala qismini kvadratga ko'tarsak, quyidagiga ega bo'lamiz: $1 = C^2x^{-2a} - 2Cux^{-a}$, eski y o'zgaruvchiga qaytsak, qo'yilgan masalaning yechimini hosil qilamiz:



5-rasm

$$y = \frac{1}{2}(Cx^{1-a} - \frac{1}{C}x^{1+a}).$$

Javob: $y = \frac{1}{2}(Cx^{1-a} - \frac{1}{C}x^{1+a})$.

Quyidagi differensial tenglamalarning umumiy yechimini toping (2.1-2.12).

2.1. $xy' = y + x\cos^2\frac{y}{x}$.	2.2. $2x^2y' = x^2 + y^2$.
2.3. $(4x-3y)dx + (2y-3x)dy = 0$.	2.4. $xy' = y(\ln y - \ln x)$.
2.5. $\frac{xy'-y}{x} = tg\frac{y}{x}$.	2.6. $x + y - 2 + (1-x)y' = 0$.
2.7. $(x-2y-1)dx+(3x-6y+2)dy=0$.	2.8. $(4x-3y)dx+(2y-3x)dy=0$.
2.9. $x^2 + y^2 - 2xyy' = 0$	2.10. $y' = e^{\frac{y}{x}} + \frac{y}{x}$
2.11. $(y^2 - 2xy)dx + x^2dy = 0$	2.12. $(x^2 - 3y^2)dx + 2xydy = 0$

Quyidagi differensial tenglamalarning boshlang'ich shartni qanoatlantiruvchi yechimlarini toping (2.13-2.17).

2.13. $xy'=y(1+\ln\frac{y}{x})$; $y\Big|_{x=1}=e^{-\frac{1}{2}}$. 2.14. $y-xy'=x\sec\frac{y}{x}$; $y\Big|_{x=1}=\pi$.

2.15. $y'=\frac{y^2-2xy-x^2}{y^2+2xy-x^2}$, $y\Big|_{x=1}=-1$. 2.16. $(xy'-y)arctg\frac{y}{x}=x$; $y\Big|_{x=1}=0$.

2.17. $y^2+x^2y'=xyy'$; $y\Big|_{x=3}=4$.

2.18. Ox o'qiga parallel hamma nurlar ko'zgudan qaytib bitta nuqtadan o'tadi. Shu ko'zguning shaklini toping.

2.19. $A(0,1)$ nuqtadan o'tuvchi shunday egri chiziqni topingki, uning istalgan M nuqtasining OM radius-vektori, shu nuqtadan o'tkazilgan urinma va Oy o'qi hosil qilgan uchburchak teng yonli bo'lsin.

2.20. Egri chiziqning ixtiyoriy nuqtasidan o'tkazilgan urinmasining burchak koeffitsienti urinish nuqtasi radius-vektori burchak koeffitsientining kvadratiga teng. Agar bu egri chiziq (2,-2) nuqtadan o'tsa, uning tenglamasini toping.

3-§. Chiziqli differensial tenglamalar va ularga keltiriladigan tenglamalar.

Noma'lum funksiya va uning hosilasiga nisbatan chiziqli bo'lgan

$$\frac{dy}{dx}+P(x)y=Q(x) \qquad (1)$$

ko'rinishdagi tenglama *chiziqli differensial tenglama* deyiladi. Bu yerda $P(x)$ va $Q(x)$ biror oraliqda berilgan uzluksiz funksiyalar. Agar $Q(x)=0$ bo'lsa, (1) tenglama *bir jinsli*, aks holda *bir jinsli bo'lmagan* chiziqli differensial tenglama deyiladi.

Dastlab

$$y'+P(x)y=0$$

bir jinsli chiziqli differensial tenglamani yechish bilan shug'ullanamiz.

Ravshanki, bu tenglama o'zgaruvchilari ajraladigan tenglama bo'ladi. Uni integrallaymiz:

$$\frac{dy}{y}=-P(x)dx \Leftrightarrow \ln|y|=-\int P(x)dx+\ln|C| \Leftrightarrow \ln\left|\frac{y}{C}\right|=-\int P(x)dx.$$

Bundan $y=Ce^{-\int P(x)dx}$ umumiy yechimga ega bo'lamiz.

Bir jinsli bo'lmagan chiziqli differensial tenglama asosan 2 ta usul bilan yechilishi mumkin. Bu usullar mos ravishda *Bernulli*[2] *va Lagranj*[3] *usullari* deb yurutiladi.

a) Bernulli usuli.

Bu usulda noma'lum funksiya $y=uv$ ko'rinishda ifodalaniladi, bu yerda u funksiya

$$\frac{du}{dx}+P(x)u=0 \qquad (2)$$

tenglamani qanoatlantiradi, ya'ni

[2] Yakob Bernulli (1654-1705) – sveytsariyalik matematik
[3] Lagranj Jozef Lui (1736-1813) – fransiyalik matematik

$$u = C_1 e^{-\int P(x)dx}. \qquad (3)$$

$y' = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ hosilani berilgan (1) tenglamaga qo'yib, quyidagilarga ega bo'lamiz:

$$u\frac{dv}{dx} + v\frac{du}{dx} + P(x)uv = Q(x)$$

$$u\frac{dv}{dx} + v\left(\frac{du}{dx} + P(x)u\right) = Q(x).$$

Bundan (2) va (3) ni inobatga olsak, noma'lum v funksiya uchun

$$u\frac{dv}{dx} = Q(x), \quad C_1 e^{-\int P(x)dx}\frac{dv}{dx} = Q(x); \qquad C_1 dv = Q(x)e^{\int P(x)dx}dx$$

munosabatlarga ega bo'lamiz.

Integrallab v ni topamiz :

$$C_1 v = \int Q(x)e^{\int P(x)dx}dx + C_2; \qquad v = \frac{1}{C_1}\left(\int Q(x)e^{\int P(x)dx}dx + C\right)$$

Natijada $y = uv = C_1 e^{-\int P(x)dx} \cdot \dfrac{1}{C_1}\left(\int Q(x)e^{\int P(x)dx}dx + C\right)$, yani

$$y = e^{-\int P(x)dx} \cdot \left(\int Q(x)e^{\int P(x)dx}dx + C\right).$$

b) Lagranj usuli.

Dastlab bir jinsli

$$y' + P(x)y = 0$$

tenglamaning $y = Ce^{-\int P(x)dx}$ yechimi topiladi.

Bundan keyin C parametrni x o'zgaruvchining funksiyasi deb o'linadi va (1) tenglamaning yechimi

$$y = C(x)e^{-\int P(x)dx} \qquad (4)$$

ko'rinishda qidiriladi.

Ravshanki,

$$y' = \frac{dy}{dx} = \frac{dC(x)}{dx}e^{-\int P(x)dx} + C(x)e^{-\int P(x)dx} \cdot (-P(x)).$$

(1) ga qo'yamiz:

$\dfrac{dC(x)}{dx}e^{-\int P(x)dx} - C(x)P(x)e^{-\int P(x)dx} + P(x)C(x)e^{-\int P(x)dx} = Q(x)$ va natijada $C(x)$ ga nisbatan tenglamaga kelamiz:

$$\frac{dC(x)}{dx}e^{-\int P(x)dx} = Q(x).$$

Bundan $dC(x) = Q(x)e^{\int P(x)dx}dx$ va $C(x) = \int Q(x)e^{\int P(x)dx}dx + C$ ni topamiz.

$C(x)$ ni (4) ga qo'yib

$$y = e^{-\int P(x)dx}\left(\int Q(x)e^{\int P(x)dx}dx + C\right)$$

umumiy yechimga ega bo'lamiz. Kutilganidek, ikkala usul ham bir xil natijaga olib keldi.

Endi biz chiziqli tenglamaga olib kelinadigan muhim tenglamani o'rganamiz. $n \neq 0$ va $n \neq 1$ bolsin

$$y' + P(x)y = Q(x) \cdot y^n, \quad n \neq 0, 1 \qquad (5)$$

qo'rinishdagi tenglama *Bernulli tenglamasi* deb yuritiladi.

$z = \frac{1}{y^{n-1}}$ almashtirish yordamida Bernulli tenglamasi chiziqli tenglamaga keltirishini ko'rsatamiz.

Buning uchun (5) tenglamaning ikkala tarafini y^n ga bo'lamiz:

$$\frac{y'}{y^n} + P\frac{1}{y^{n-1}} = Q \ .$$

Bundan $z' = -\frac{(n-1)y^{n-2}}{y^{2n-2}} \cdot y' = -\frac{(n-1)y'}{y^n}$ ni inobatga olib, z ga nisbatan chiziqli tenglamaga ega bo'lamiz:

$$-\frac{z'}{n-1} + Pz = Q, \quad z' - (n-1)Pz = -(n-1)Q$$

Misol. Quyidagi tenglamalarning umumiy yechimini toping.

$a)\quad y' + 2xy = 2xe^{-x^2};$ $\qquad c)\quad xy' + y = y^2 \ln x;$

$b)\quad xy' - 2y = x^3 \cos x;$ $\qquad d)\quad (2x - y^2)y' = 2y.$

Yechish. a) $y' + 2xy = 2xe^{-x^2}$ tenglama chiziqli differensial tenglama.

Bernulli usulidan foydalanamiz. $y = uv$ deylik. U holda $y' = vu' + uv'$ bo'ladi va bularni berilgan tenglamaga qo'ysak, u quyidagi

$vu' + u(v + 2xv) = 2xe^{-x^2}$ ko'rinishga keladi.

$v' + 2xv = 0$ bo'lishini talab qilamiz. O'zgaruvchilarni ajratib, $\frac{dv}{v} = -2xdx$ ni hosil qilamiz, bu yerdan $\ln|v| = -x^2 + \ln|C|$, $v = Ce^{-x^2}$. $C = 1$ deb $v = e^{-x^2}$. xususiy yechim bilan cheklanish mumkin. v ning ifodasini almashtirilgan $vu' = 2e^{-x^2}$ tenglamaga qo'yamiz:

$e^{-x^2}u' = 2xe^{-x^2}$, $\quad du = 2xdx$ Bu yerdan: $u = x^2 + C$ ma'lumki, $y = uv$, u holda umumiy yechim $y = e^{-x^2}(x^2 + C)$ ko'rinishda hosil bo'ladi.

Javob: $y = e^{-x^2}(x^2 + C)$

b) $xy' - 2y = x^3 \cos x$ tenglama

$$y' - \frac{2y}{x} = x^2 \cos x$$

chiziqli differensial tenglamaga olib kelinadi ($x \neq 0$).

Bu tenglamani Lagranj usuli yordamida yechamiz:

Dastlab bir jinsli

$$y' - \frac{2y}{x} = 0$$

tenglamaning yechimini topamiz.

$$\frac{dy}{dx} = \frac{2y}{x} \Leftrightarrow \frac{dy}{y} = \frac{2dx}{x} \Leftrightarrow y = Cx^2.$$

Bundan keyin C parametr x o'zgaruvchining funksiyasi deb o'linadi va (1) tenglamaning yechimi

$$y = C(x)x^2$$

ko'rinishda izlanadi.

Ravshanki,

$$y' = \frac{dC(x)}{dx}x^2 + 2xC(x).$$

$y' - \frac{2y}{x} = x^2 \cos x$ ga qo'yamiz:

$$y' - \frac{2y}{x} = \frac{dC(x)}{dx}x^2 + 2xC(x) - \frac{2C(x)x^2}{x} = x^2 \cos x$$ va natijada $C(x)$ ga nisbatan tenglamaga kelamiz:

$$\frac{dC(x)}{dx} = \cos x$$

Bundan $C(x) = \sin x + C$ ni topamiz.

$C(x)$ ni $y = C(x)x^2$ ga qo'yib

$$y = (\sin x + C)x^2$$

umumiy yechimga ega bo'lamiz.

Javob: $y = x^2(\sin x + C)$

c) $xy' + y = y^2 \ln x$ tenglama Bernulli tenglamasidir. Uning ikkala qismini y^2 ga bo'lib, $\frac{1}{y} = z$ deb olamiz, u holda

$$y = \frac{1}{z}, y' = -\frac{1}{z^2}z', -xz'\frac{1}{z^2} + \frac{1}{z} = \frac{1}{z^2}\ln x$$

Bundan $xz' - z = -\ln x$ ko'rinishdagi chiziqli tenglama hosil bo'ladi. Uning umumiy yechimi: $z = \ln x + 1 + Cx$ bo'ladi.

z ni $\frac{1}{y}$ bilan almashtirib, berilgan tenglamaning $y = \frac{1}{\ln x + 1 + Cx}$ umumiy yechimini hosil qilamiz.

Javob: $y = \frac{1}{\ln x + 1 + Cx}$.

d) Dastlab berilgan $(2x-y^2)\frac{dy}{dx}=2y$ tenglamani $2yx'-2x=-y^2$ ko'rinishda yozib olamiz. Bu tenglama $x=x(y)$ funksiyaga nisbatan chiziqli tenglamadir. Shu sababli $x=uv$ almashtirish bajaramiz. U holda $x'=u'v+uv'$. Olingan natijalarni so'ngi tenglamaga qo'ysak,

$$2yvu'+2u(yv'-v)=-y^2,\quad yv'-v=0, \frac{dv}{v}=\frac{dy}{y}, \ln v=\ln y,\ v=y,\ 2yvu'=-y^2$$

$u'=-\frac{1}{2}$, $u=-\frac{1}{2y}+C$, $x=-\frac{1}{2}y^2+Cy$ hosil bo'ladi.

Javob: $x=-\frac{1}{2}y^2+Cy$

Masala. Egri chiziqning istalgan $M(x,y)$ nuqtasi uchun OM kesma, shu nuqtadan o'tkazilgan MP urinma va Ox o'q hosil qilgan uchburchakning yuzi 4 ga teng. Egri chiziq $A(1,2)$ nuqtadan o'tadi. Uning tenglamasini toping. (6-rasm)

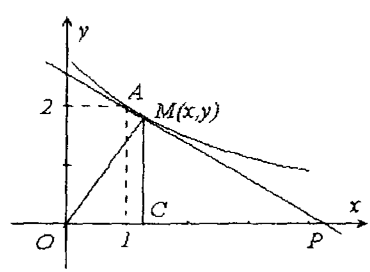
Yechish. Uchburchakning yuzi $S=\frac{1}{2}OP\cdot MC$ formula buyicha topiladi, bu yerda $MC=y$ son M nuqtaning ordinatasi. OP ni topishda uning MP urinmaning Ox o'q bilan kesishish nuqtasining abssissasi ekanligidan foydalanamiz, MP urinmaning tenglamasi ushbu ko'rinishda bo'ladi:
$Y-y=y'(X-x)$.

Bu tenglamada $Y=0$ desak, $X=x-\frac{y}{y'}$, $OP=x-\frac{y}{y'}$ ni hosil qilamiz.

Masalaning shartiga asosan $4=\frac{1}{2}(x-\frac{y}{y'})y$ yoki $\frac{dx}{dy}-\frac{1}{y}x=-\frac{8}{y^2}$ differensial tenglama hosil bo'ladi.

Bu y argumentning noma'lum x funksiyasiga nisbatan chiziqli differensial tenglama. $x=uv$ almashtirish bajargandan so'ng umumiy integral $x=y(\frac{4}{y^2}+C)$ ni hosil qilamiz.

$x=1$ da $y=2$ demak, $C=-\frac{1}{2}$.



6-rasm

Natijada egri chiziqning izlanayotgan tenglamasini ushbu ko'rinishda hosil qilamiz: $x=\frac{4}{y}-\frac{y}{2}$.

Javob: $x=\frac{4}{y}-\frac{y}{2}$.

Masala. m massali nuqta vaqtga proporsional bo'lgan kuch ta'sirida to'g'ri chiziqli harakat qilmoqda. Boshlangich $t=0$ vaqt momentida $v=0$ bo'lsin. Havo qarshiligi tezlikka proporsional bo'lgan holda tezlikni t ning funksiyasi sifatida aniqlang.

Yechish. t momentda nuqtaga ikkita kuch, yani vaqtga proporsional bo'lgan $F_1 = k_1 t$ kuch va $F_2 = -kv$ havoning qarshilik kuchi ta'sir etadi; ularning umumiy ta'sir etuvchisi quyidagicha:

$$F = F_1 + F_2 = k_1 t - kv$$

Ikkinchi tomondan, Nyutonning ikkinchi qonuniga binoan $F = m\frac{dv}{dt}$. F uchun topilgan ikkala ifodani taqqoslab, $\frac{dv}{dt} + \frac{k}{m}v = \frac{k_1}{m}t$ tenglamani hosil qilamiz.

Bu tenglama v ga nisbatan chiziqli differensial tenglamadir. Uni yechishda Bernulli usulini qo'llab, $v = u(t)w(t)$ almashtirish kiritamiz, u holda

$$v' = u'w + uw',\ u'w + uw' + \frac{k}{m}uw = \frac{k_1}{m}t,\quad u'w + u(w' + \frac{k}{m}w) = \frac{k_1}{m}t.$$

$$w' + \frac{k}{m}w = 0,\ \frac{dw}{w} = -\frac{k}{m}dt,\ \ln w = -\frac{k}{m}t,$$

$$w = e^{-\frac{k}{m}t},\ e^{\frac{k}{m}t}u' = \frac{k_1}{m}t,\ u' = \frac{k_1}{m}te^{\frac{k}{m}t}$$

$$u = \frac{k_1}{k}(t - \frac{m}{k})e^{\frac{k}{m}t} + C,\quad v = \frac{k_1}{k}(t - \frac{m}{k}) + Ce^{-\frac{k}{m}t}$$

Umumiy yechimga $v|_{t=0} = 0$ boshlang'ich shartdan foydalanib, $C = \frac{k_1}{k_2}m$ ni topamiz, u holda izlanayotgan tezlik ushbu ko'rinishda bo'ladi:

$$v = \frac{k_1}{m}(t - \frac{m}{k} + \frac{m}{k}e^{\frac{k}{m}t})$$

Javob: $v = \frac{k_1}{m}(t - \frac{m}{k} + \frac{m}{k}e^{\frac{k}{m}t})$

Quyidagi tenglamalarni umumiy yechimlarini toping (3.1-3.16.).

3.1. $y' + 2xy = x.$

3.2. $y' - ye^x = 2xe^{e^x}.$

3.3. $y'x\ln x - y = 3x^3\ln^2 x.$

3.4. $y' = \frac{1}{2x - y^2}.$

3.5. $y' - yctgx = \sin x$

3.6. $x^2y^2y' + xy^3 = 1$

3.7. $2xydy = (y^2 - x)dx.$

3.8. $(x^3 + e^y)y' = 3x^2.$

3.9. $xdx = (\frac{x^2}{y} - y^3)dy.$

3.10. $y' - y\cos x = y^2\cos x.$

3.11. $(a^2+x^2)y'+xy=1$.

3.12. $y'+\frac{2}{x}y=\frac{e^{-x^2}}{x}$

3.13. $xy'+y=-xy^2$.

3.14. $xy'+y=\ln x+1$.

3.15. $(2x+1)y'+y=x$.

3.16. $y'+xy=xy^3$.

Quyidagi differensial tenglamalarning boshlang'ich shartni qanoatlantiruvchi xususiy yechimlarini toping (3.17-3.22).

3.17. $x^2+xy'=y,\ y(1)=0$.

3.18. $y'-ytgx=\frac{1}{\cos^3 x},y(0)=0$.

3.19. $y'\cos x-y\sin x=2x,\ y(0)=0$.

3.20. $y'+y\cos x=\cos x, y(0)=1$.

3.21. $3y^2y'+y^3=x+1;\ y\ |_{x=1}=-1$.

3.22. $(1-x^2)y'-xy=xy^2,\ y|_{x=0}=\frac{1}{2}$

3.23. m massali moddiy nuqta vaqtning kubiga proporsional (k-proporsionallik koeffitsienti) kuch ta'sirida to'g'ri chiziqli harakat qilmoqda. Tezlik bilan vaqtning ko'paytmasiga proporsional (k_1 - proporsionallik koeffitsienti) bo'lgan havo qarshiligini hisobga olgan holda tezlikning t vaqtga bog'lanishini toping. Boshlang'ich tezlik v_0 ga teng.

3.24. Elektr yurituvchi kuchi $E(t)=E_0\sin wt$ ga, qarshiligi R ga o'zinduktivlik koeffitsienti L ga teng bo'lgan g'altakdagi I tok kuchini t vaqtning funksiyasi kabi toping. (Boshlang'ich tok kuchi $I_0=0$ ga teng)

3.25. m massali moddiy nuqtaga t vaqtga proporsional bo'lgan kuch ta'sir etadi (k_1-proporsionallik koeffitsienti). Tezlikka proporsional (k - proporsionallik koeffitsienti) bo'lgan havo qarshiligini hisobga olgan holda nuqtaning tezligini toping. (Boshlang'ich $t=0$ vaqt momentida $v_0=0$)

3.26. $(\frac{1}{2},1)$ nuqtadan o'tuvchi shunday egri chiziqni topingki, uning istalgan nuqtasining abssissasi va shu nuqtada o'tkazilgan urinmaning boshlang'ich ordinatasi yordamida qurilgan to'g'ri to'rtburchakning yuzi o'zgarmas bo'lib, a^2 ga teng bo'lsin.

3.27. $A(1,2)$ nuqtadan o'tadigan egri chiziqning istalgan urinmasining boshlang'ich ordinatasi urinish nuqtasining abssissasiga teng. Uning tenglamasini toping.

4-§. To'liq differensialli tenglamalar. Integrallovchi ko'paytuvchi.

Agar

$$P(x,y)dx+Q(x,y)dy=0 \qquad (1)$$

tenglamaning chap tomonini birorta $U(x,y)$ funksiyaning to'liq differensiali, ya'ni

$$P(x,y)dx+Q(x,y)dy=dU(x,y) \qquad (2)$$

bo'lsa, (1) tenglama *to'liq differensialli tenglama* deyiladi.

Bu holda uni $dU(x,y)=0$ ko'rinishda yozish mumkin va bu yerdan $U(x,y)=C$ umumiy integralga ega bo'lamiz.

Bu yerda $P(x,y)$ va $Q(x,y)$ funksiyalar D sohada aniqlangan va uzluksiz bo'lib, uzluksiz $\frac{\partial P(x,y)}{\partial y}$, $\frac{\partial P(x,y)}{\partial x}$ xususiy hosilalarga ega bulishi talab qilinadi.

U holda ushbu $P(x,y)dx+Q(x,y)dy$ differensial ifoda birorta $U(x,y)$ funksiyaning to'liq differensiali bo'lishi uchun D sohaning barcha nuqtalarida

$$\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x} \qquad (3)$$

tenglikning bajarilishi zarur va yetarlidir.

$$dU=\frac{\partial U}{\partial x}dx+\frac{\partial U}{\partial y}dy$$

ifodani (2) bilan solishtirsak

$$\frac{\partial U}{\partial x}=P(x,y) \qquad (4)$$

$$\frac{\partial U}{\partial y}=Q(x,y)$$

tengliklarga ega bo'lamiz.

Endi U funksiyani topish uchun y ni fiksirlab (4) ni integrallaymiz:

$$U=\int P(x,y)dx+C(y).$$

$C(y)$ ni topish uchun bu tenglikni y bo'yicha differensiallaymiz:

$$\frac{\partial U}{\partial y}=Q(x,y)=\frac{\partial}{\partial y}\int P(x,y)dx+C'(y).$$

Bu yerdan $C'(y)=Q(x,y)-\frac{\partial}{\partial y}\int P(x,y)dx.$

Demak, $C(y)=\int\left(Q(x,y)-\frac{\partial}{\partial y}\int P(x,y)dx\right)dy+C$

va

$$U=\int P(x,y)dx+\int\left(Q(x,y)-\int\frac{\partial}{\partial y}P(x,y)dx\right)dy+C.$$

Demak, berilgan tenglamaning umumiy integrali quyidagi ko'rinishda bo'ladi:

$$\int P(x,y)dx+\int\left(Q(x,y)-\frac{\partial}{\partial y}\int P(x,y)dx\right)dy=C. \qquad (5)$$

Aslini olganda konkret misollarni yechishda tayyor (5) formuladan foydalanmasdan, umumiy holdagi kabi yo'l tutish maqsadga muvofiq.

Izoh. Ayrim hollarda (1) tenglamani hadlarini guruhlash bilan $dU=0$ ko'rinishga keltirish mumkin. Buning uchun u

$$\left(M_1dx+N_1dy\right)+\left(M_2dx+N_2dy\right)+\ldots+\left(M_ndx+N_ndy\right)=0 \qquad (6)$$

ko'rinishga keltiriladi.

Bunda shunday $U_1(x,y),U_2(x,y),...,U_n(x,y)$ funksiyalar topiladiki, ular uchun

$$M_1dx+N_1dy=dU_1(x,y)$$
$$M_2dx+N_2dy=dU_2(x,y)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$M_ndx+N_ndy=dU_n(x,y)$$

munosabatlar bajariladi.

U holda (6) ning umumiy integrali $U_1(x,y)+U_2(x,y)+...+U_n(x,y)=C$ ko'rinishga ega.

Agar (3) shart bajarilmasa, u holda (1) differensial tenglama to'liq differensialli bo'lmaydi. Biroq bu tenglamani tegishli $\mu(x,y)$ funksiyaga ko'paytirish bilan to'liq differensialli tenglamaga keltirish mumkin. Bunday funksiya berilgan differensial tenglama uchun *integrallovchi ko'paytuvchi* nomi bilan yuritiladi.

$\mu(x,y)$ uchun (3) dan

$$\frac{\partial(\mu P)}{\partial y}=\frac{\partial(\mu Q)}{\partial x} \text{ yoki } Q\frac{\partial\mu}{\partial x}-P\frac{\partial\mu}{\partial y}=\mu\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right). \qquad (7)$$

shatni hosil qilamiz.

Faqat x ga bog'lik bo'lgan $\mu(x)$ integrallovchi ko'payruvchi uchun $\frac{\partial\mu}{\partial y}=0$ va (3) quyidagi ko'rinishni oladi :

$$Q\frac{d\mu}{dx}=\mu\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right) \text{ yoki } \frac{d\ln\mu}{dx}=\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}$$

Demak,

$$\mu(x)=e^{\int\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}dx} \qquad (8)$$

Faqat y ga bog'liq bo'lgan $\mu(y)$ integrallovchi ko'payruvchi uchun huddi shunday

$$\mu(y)=e^{\int\frac{\frac{dP}{dy}-\frac{dQ}{dx}}{P}dy}$$

ko'rinishni topamiz.

Misol. To'liq differensialli tenglamalarni yeching:

$a)$ $\frac{1}{x}dy-\frac{y}{x^2}dx=0;$

$b)$ $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy=0;$

$c)$ $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0;$

$d)$ $(\sin y+y\sin x+\frac{1}{x})dx+(x\cos y-\cos x+\frac{1}{y})dy=0.$

Yechish. a) $\frac{dy}{x}-\frac{y}{x^2}dx=0$ tenglamaning chap qismi $U=\frac{y}{x}$ funksiyaning to'liq differensiali ekanligini ko'rish oson. Shuning uchun tenglamani $d(\frac{y}{x})=0$ ko'rinishda qayta yozib olamiz, bu yerdan $y=Cx$ umumiy yechimni topamiz.

b) $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy=0$ tenglamani ham hadlarini guruhlash bilan $3x^2dx+6xy(ydx+xdy)+4y^3dy=0$ ko'rinishga keltirish mumkin. So'ngra

$$3x^2dx=d(x^3),\quad 6xy(ydx+xdy)=d(3(xy)^2),\quad 4y^3dy=dy^4$$

bo'lgani uchun $dx^3+d(3(xy)^2)+dy^4=0$ ni yoki $d(x^3+3(xy)^2+y^4)=0$ ni hosil qilamiz.

Bu yerdan $x^3+3(xy)^2+y^4=C$ umumiy integralni topamiz.

Javob: $x^3+3(xy)^2+y^4=C$.

c) $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4dy}=0$ tenglamada $P(x,y)=\frac{2x}{y^3}$, $Q(x,y)=\frac{y^2-3x^2}{y^4}$

$\frac{\partial P}{\partial y}=-\frac{6x}{y^4}$, $\frac{\partial Q}{\partial x}=-\frac{6x}{y^4}$. Demak, $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$ shart bajarildi. Bundan $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy$ ifodaning birorta $U(x,y)$ funksiyaning to'liq differensiali ekanligi kelib chiqadi.

Endi shu U funksiyani, ya'ni $\frac{\partial U}{\partial x}=\frac{2x}{y^3}$, $\frac{\partial U}{\partial y}=\frac{y^2-3x^2}{y^4}$ tenglamalarni qanoatlantiruvchi funksiyani topamiz.

$\frac{\partial U}{\partial x}=\frac{2x}{y^3}$ tenglamadan U funksiya

$$U(x,y)=\int\frac{2x}{y^3}dx+\varphi(y)=\frac{x}{y^3}+\varphi(y)\qquad(9)$$

ko'rinishda ekanligi kelib chiqadi, bu yerda $\varphi(y)$ - noma'lum funksiya.
(9) ni y bo'yicha differensiallaymiz

$$\frac{\partial U}{\partial y}=-\frac{3x^2}{y^4}+\varphi'(y).$$

Ammo $\frac{\partial U}{\partial y}=\frac{y^2-3x^2}{y^4}$, shuning uchun quyidagini hosil qilamiz:

$$\frac{y^2-3x^2}{y^4}=-\frac{3x^2}{y^4}+\varphi'(y),\quad \varphi'(y)=\frac{1}{y^2}.$$

Ravshanki, ohirgi tenglikni

$$\varphi(y)=-\frac{1}{y}\qquad(10)$$

funksiya qanoatlantiradi.

Natijada, (10) ni (9) ga qo'yib umumiy integralni topamiz:

$$\frac{x^2}{y^3}-\frac{1}{y}=C.$$

Javob: $\frac{x^2}{y^3}-\frac{1}{y}=C$

d) Berilgan tenglama uchun

$$P(x,y)=\sin y+y\sin x+\frac{1}{x},\quad Q(x,y)=x\cos y-\cos x+\frac{1}{y},$$

$$\frac{\partial P}{\partial y}=\cos y+\sin x,\quad \frac{\partial Q}{\partial x}=\cos y+\sin x.$$

Demak, $\frac{\partial P}{\partial x}=\frac{\partial Q}{\partial x}$ shart bajarilgan.

Umumiy integralni topishda tayyor (5) formuladan foydalanamiz:

$$\int P(x,y)dx=\int(\sin y+y\sin x+\frac{1}{x})dx=x\sin y-y\cos x+\ln x$$

$$\frac{\partial}{\partial y}\int P(x,y)dx=\frac{\partial}{\partial y}(x\sin y-y\cos x-\frac{1}{x^2})=x\cos y-\cos x$$

$$\int\left(Q(x,y)-\frac{\partial}{\partial y}\int P(x,y)dx\right)dy=\int(x\cos y-\cos x+\frac{1}{y}-(x\cos y-\cos x))dy=$$

$$=x\sin y-y\cos x+\ln y-x\sin y+y\cos x=\ln y.$$

Bu yerdan

$$x\sin y-y\cos x+\ln xy=C$$

ko'rinishdagi umumiy integralni topamiz.

Javob: $x\sin y-y\cos x+\ln xy=C$.

Misol. Quyidagi differensial tenglamalarning integrallovchi ko'paytuvchilarini toping va bu tenglamalarni integrallang.

a) $(1-x^2y)dx+x^2(y-x)dy=0;$

b) $(2xy^2-y)dx+(y^2+x+y)dy=0;$

c) $(x^2-y)dy+(x^2y^2+x)dy=0.$

Yechish. a) Bu holda $P(x,y)=1-x^2y, Q(x,y)=x^2(y-x)$,

$$\frac{\partial P}{\partial y}=-x^2,\ \frac{\partial Q}{\partial x}=2xy-3x^2,\ \frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}=\frac{-x^2-2xy+3x^2}{x^2(y-x)}=-\frac{2}{x}$$ nisbat x ga bog'liq.

Demak, $\mu=\mu(x)$ integrallovchi ko'paytuvchi (8) formula bo'yicha topiladi:

$$\mu(x)=e^{-\int\frac{2}{x}dx}=e^{-2\ln x}=\frac{1}{x^2}.$$

Tenglamaning ikkala tomonini $\frac{1}{x^2}$ ga ko'paytiramiz va quyidagicha almashtirishlar bajaramiz:

$(\frac{1}{x^2}-y)dx+(y-x)dy=0,\ \frac{1}{x^2}dx-(ydx+xdy)+ydy=0,$ va $\frac{1}{x^2}dx=d\left(-\frac{1}{x}\right),\ ydx+xdy=d(xy),$

$ydy=d\left(\frac{y^2}{2}\right)$ ekanligini e'tiborga olsak $d\left(-\frac{1}{x}-xy+\frac{y^2}{2}\right)=0$ bo'ladi. Bundan

$-\frac{1}{x}-xy+\frac{y^2}{2}=C$ hosil bo'ladi.

Javob: $-\frac{1}{x}-xy+\frac{y^2}{2}=C$

b) Bu holda $P(x,y)=2xy^2-y,\quad Q(x,y)=y^2+x+y,$

$\frac{\partial P}{\partial y}=4xy-1,\ \frac{\partial Q}{\partial x}=1,\ (\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x})/P=\frac{2(2xy-1)}{y(2xy-1)}=\frac{2}{y}$ nisbat faqat y ga bog'liq.

Demak, $\mu=\mu(y)$ integrallovchi ko'paytuvchi $\mu(y)=e^{-\int\frac{\frac{dP}{dy}-\frac{dQ}{dx}}{P}dy}=\frac{1}{y^2}$ formula bo'yicha topiladi.

Tenglamaning ikkala tomonini $\frac{1}{y^2}$ ga ko'paytiramiz:

$(2x-\frac{1}{y})dx+(1+\frac{x}{y^2}+\frac{1}{y})dy=0$ yoki $2xdx-(\frac{1}{y}dx-\frac{x}{y^2}dy)+(1+\frac{1}{y})dy=0$.

Bunda $\frac{1}{y}dx-\frac{x}{y^2}dy=d(\frac{x}{y})$ bo'lgani uchun $x^2-\frac{x}{y}+y+\ln y=C$

umumiy integralni hosil qilamiz.

c) Yuqoridagi misollarga o'xshash yechamiz:

$P(x,y)=x^2-y,\quad Q(x,y)=x^2y^2+x,\quad \frac{\partial P}{\partial y}=1,$

$\frac{\partial Q}{\partial x}=2xy^2+1,\quad \frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}=-2(1+x^2),$

$(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x})/Q=-\frac{2(1+xy^2)}{x(xy+1)}=-\frac{2}{x},\quad \mu(x)=e^{-2\ln x}=\frac{1}{x^2}.$

Berilgan tenglamaning ikkala tomonini $\frac{1}{x^2}$ ga ko'paytirib, hamda $d\left(\frac{y}{x}\right)=\frac{xdy-ydx}{x^2}$ ekanini e'tiborga olib, quyidagilarga ega bo'lamiz:

$(1-\frac{y}{x^2})dx+(y^2+\frac{1}{x})dy=0\quad dx+y^2dy+\frac{1}{x}dy-\frac{y}{x^2}dx=0$

$dx+y^2dy+d(\frac{y}{x})=0,\quad d(x+\frac{y^3}{3}+\frac{y}{x})=0$

bu yerdan berilgan tenglamaning umumiy integrali

$$x+\frac{y^3}{3}+\frac{y}{x}=C$$

ko'rinishda bo'lishi kelib chiqadi.

Quyidagi to'liq differensialli tenglamalarni yeching (67-71):

4.1. $(2x-y+1)dx+(2y-x-1)dy=0.$ 4.2. $(\frac{x}{\sqrt{x^2-y^2}}-1)dx-\frac{ydy}{\sqrt{x^2-y^2}}=0.$

4.3. $\frac{2x(1-e^y)}{(1+x^2)^2}dx+\frac{e^y}{1+x^2}dy=0.$ 4.4. $(3x^2-2x-y)dx+(2y-x+3y^2)dy=0.$

4.5. $(2x+\frac{x^2+y^2}{x^2y})dx=\frac{x^2+y^2}{xy^2}dy.$ 4.6. $(e^y+ye^x+3)dx=(2-xe^y-e^x)dy$

4.7. $(2x+ye^{xy})dx+(1+xe^{xy})dy=0.$ 4.8. $(3x^2+6xy^2)dx+(6x^2y+4y^3)dy=0.$

4.9. $e^ydx+(xe^y-2y)dy=0.$ 4.10. $xdx+ydy=\frac{xdy-ydx}{x^2+y^2}.$

4.11. $3x^2e^ydx+(x^3e^y-1)dy=0.$ 4.12. $2x\cos^2 ydx+(2x-x^2\sin 2y)dy=0.$

4.13. $(x\cos 2y+1)dx-x^2\sin 2ydy=0$. 4.14. $(12x+5y-9)dx+(5x+2y-4)dy=0.$

Quyidagi differensial tenglamalarning integrallovchi ko'paytuvchilarini toping va bu tenglamalarni integrallang (4.15.-4.18.).

4.15. $(\frac{x}{y}+1)dx+(\frac{x}{y}-1)dy=0.$ 4.16. $(xy^2+y)dx-xdy=0.$

4.17. $(x^4\ln x-2xy^3)dx+3x^2y^2dy=0.$ 4.18. $(2xy^2-3y^3)dx+(7-3xy^2)dy=0.$

5-§. Hosilaga nisbatan yechilmagan birinchi tartibli tenglamalar.

1-§ da aytganimizdek,

$$F(x,y,\ y')=0 \qquad (1)$$

differensial tenglamaning *maxsus* $\varphi(x)$ yechimi uchun ixtiyoriy $(x_0,\varphi(x_0))$ nuqtadan ikkita va undan ko'p integral chiziqlar o'tadi.

(1) tenglamaning

$$\Phi(x,y,C)=0 \qquad (2)$$

ko'rinishdagi umumiy integrali topilgan deb faraz qilamiz.

Maxsus yechimlarni aniqlash uchun alohida usullar mavjud. Biz ularni bayon qilamiz.

1-usul. Differensial geometriya kursidan ma'lumki, ixtiyoriy $\varphi(x)$ maxsus yechim *diskriminant egri chiziq* bo'ladi, yani

$$\begin{cases} F(x,y,y')=0 \\ F_{y'}(x,y,y')=0 \end{cases} \qquad (3)$$

tenglamalar sistemasini qanoatlantiradi.

Bundan keyin $\varphi(x)$ maxsus yechimi (agar mavjud bo'lsa) C ning hech qanday qiymatida (2) ni qanoatlantirmasligi tekshiriladi.

Masalan, $(y')^2 = y^2$ tenglama uchun (3) sistema $\begin{cases}(y')^2 = y^2\\ y' = 0\end{cases}$ ko'rinishda bo'lib, undan $y = 0$ diskriminant egri chiziqni topib olamiz. Tenglamani yechamiz: $y' = \pm y,\ y = Ce^{\pm x}$.

$y = 0$ yechim C ning $C=0$ qiymatida $y = Ce^{\pm x}$ ni qanoatlantirishini tekshirish oson. Demak, $y = 0$ maxsus yechim bo'lmaydi.

Ikkinchi tomondan, $y = 0$ $(y')^4 = y^2$ tenglama uchun diskriminant egri chiziq bo'ladi. Tenglamani yechamiz:

$$\left[\begin{array}{l} y' = \pm\sqrt{|y|} \\ y' = 0 \end{array}\right.;\ \pm\int\frac{dy}{\sqrt{y}} = \int dx;\ x = \pm 2\sqrt{y} + C, y \equiv 0.$$

$y = 0$ yechim C ning hech qanday qiymatida $x = \pm 2\sqrt{y} + C$ ni qanoatlantirmasligini tekshirish oson. Demak, $y = 0$ maxsus yechim.

2-usul. Bu usul bir parametrli $\Phi(x,y,C)=0$ egri chiziqlar oilasining *o'ramasini* hosil qilish qoidasiga asoslangan. Bu qoidaga muvofiq, $\varphi(x)$ maxsus yechim ushbu

$$\begin{cases}\Phi(x,y,C) = 0 \\ \dfrac{\partial}{\partial C}\Phi(x,y,C) = 0\end{cases} \qquad (4)$$

tenglamalar sistemasidan C ni yo'qotish orqali topiladi.

Umuman aytganda, (1) tenglamani y ga nisbatan har doim ham yechish mumkin bo'lavermaydi.

Shunday bo'lishiga qaramay, (1) tenglamani integrallash masalasini parametr kiritish yo'li bilan hosilaga nisbatan yechilgan tenglamani integrallash masalasiga keltirish mumkin.

Quyida (1) tenglamaning ayrim xususiy hollarini qarab chiqamiz.

1) y ga nisbatan yechilgan va x qatnashmagan $y = f(y')$ tenglama.

Bu holda $y' = p$ deb p parametrni kiritsak, quyidagilarni hosil qilamiz:

$$y = f(p); \qquad y' = f'(p)\frac{dp}{dx};$$

yani

$$p = f'(p)\frac{dp}{dx}.$$

Bu tenglamani integrallaymiz :

$$dx = \frac{f'(p)}{p}dp; \qquad x = \int\frac{f'(p)}{p}dp + C.$$

Bunda umumiy yechimning parametrik shaklini yozishimiz mo'mkin:

$$\begin{cases} x = \int \frac{f'(p)}{p} dp + C \\ y = f(p) \end{cases}$$

Ayrim hollarda umumiy yechim ushbu sistemadan p parametrni yo'qotish orqali topiladi.

2) x ga nisbatan yechilgan va x qatnashmagan $x = f(y')$ tenglama.

Huddi yuqoridagidek bu holda $y' = p$ deb p parametrni kiritib, umumiy yechimning parametrik shaklini hosil qilamiz:

$$\begin{cases} y = \int pf'(p)dp + C \\ x = f(p) \end{cases}$$

3) x (yoki y) qatnashmagan, biroq y (yoki x) ga nisbatan yechilgan bo'lishi shart bo'lmagan tenglama.

Bu holda tenglamani ushbu

$$F(y, y') = 0 \qquad (5)$$

yoki

$$F(x, y') = 0 \qquad (6)$$

ko'rinishda yozish mumkin.

Shu bilan birga tenglamadan y ni ((5) tenglamadan) yoki x ni ((6) tenglamada), shuningdek, $y' = p$ ni t parametr orqali ifodalash mumkin deb faraz qilamiz. 1) va 2) hollardagi kabi bu yerda ham tenglamaning umumiy yechimi parametrik shaklda hosil bo'ladi.

Masalan. $F(y, p)=0$ tenglama bo'lgan holni ko'raylik.

$y = \phi(t)$ deb, tenglamadan $p = \psi(t)$ ni yoki, aksincha, $p = \psi(t)$ tenglamadan $y = \varphi(t)$ ni topdik deb faraz qilaylik. U holda bir tomondan,

$dy = pdx = \psi(t)dx$ ikkinchi tomondan $dy = \phi'(t)dt$. Bu dy uchun ikkala ifodani taqqoslab, $\psi(t)dx = \varphi'(t)dt$ ni hosil qilamiz, bu yerdan

$$dx = \frac{\phi'(t)}{\varphi(t)}dt \quad \text{va} \quad x = \int \frac{\phi'(t)}{\varphi(t)}dt + C$$

Umumiy yechim parametrik shaklda quyidagicha yoziladi:

$$\begin{cases} x = \int \frac{\phi'(t)}{\varphi(t)} dt + C, \\ y = \phi(t). \end{cases}$$

4) x va y ga nisbatan chiziqli bo'lgan, ya'ni

$$P(y')x + Q(y')y + R(y') = 0$$

ko'rinishdagi tenglama *Lagranj tenglamasi* deyiladi. $y'=p$ deymiz.

$f(p) = -\frac{P(y')}{Q(y')}$, $\varphi(p) = -\frac{R(y')}{Q(y')}$ funksiyalarni kiritsak, bu holda tenglama

$$y = xf(p) + \varphi(p) \qquad (7)$$

ko'rinishda yoziladi.

$dy = pdx$ ni inobatga olib (7) ni ikkala tarafini x bo'yicha differensiallasak

$$pdx = f(p)dx + xf'(p)dp + \varphi'(p)dp \qquad (8)$$

ko'rinishdagi chiziqli tenglama hosil bo'ladi va (8) ning umumiy integrali

$$x = F(p,C) \qquad (9)$$

ko'rinishda bo'ladi. Natijada Lagranj tenglamasining

$$\begin{cases} x = F(p,C) \\ y = xf(p) + \varphi(p) = F(p,C)f(p) + \varphi(p) \end{cases}$$

parametrik shakldagi umumiy integralini hosil qilamiz.

5) Lagranj tenglamasining xususiy holi bo'lgan

$$y = xy' + \varphi(y') \qquad (10)$$

ko'rinishdagi tenglamaga *Klero*[4] *tenglamasi* deb aytiladi.

$y'=p$ deymiz va quyidagilarga ega bo'lamiz:

$$y = xp + \varphi(p)$$

$$y' = p + x\frac{dp}{dx} + \varphi'(p)\frac{dp}{dx}; \qquad p = p + x\frac{dp}{dx} + \varphi'(p)\frac{dp}{dx};$$

$$\left(x + \varphi'(p)\right)\frac{dp}{dx} = 0.$$

Quyidagi hollar vujudga kelishi mo'mkin:

$$p = C \text{ yoki } x + \varphi'(p) = 0$$

Birinchi holda bu tenglamaning umumiy yechimi bir parametrli integral egri chiziqlar oilasi $y=Cx+\varphi$ (C) dan iborat bo'ladi

Ikkinchi holda

$$\begin{cases} y = xp + \varphi(p) \\ x + \varphi'(p) = 0 \end{cases} \qquad (11)$$

parametrik ko'rinishdagi yechimni hosil qilamiz.

(11) sistema (3) sistemaning xususiy holi bo'lib, maxsus yechimni beradi.

Misol. Quydagi tenglamalarni integrallang.

a) $yy'^2+(x-y)y'-x=0$; b) $y=y'\ln y'$; c) $x=y'+\sin y'$;

d) $y'=e^{\frac{y'}{y}}$; e) $y=xy'^2+y'^2$; f) $y=xy'-y'^2$.

Yechish. a) Berilgan tenglamani y' ga nisbatan yechamiz:

$$y' = \frac{x - y \pm \sqrt{(x-y)^2 + 4xy}}{2y}, \quad y' = 1, \quad y' = -\frac{x}{y}$$

Bundan $y=x+C$, $y^2+x^2=C$.

Javob: $y=x+C$, $y^2+x^2=C$.

b) Berilgan tenglama - (3) ko'rinishdagi tenglama, shuning uchun $y'=p$ desak, $y=p\ln p$ ga ega bo'lamiz.

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[4] Aleksi Klod Klero (1713 – 1765) – fransiyalik matematik

Bu tenglamaning ikkala tomonini x bo'yicha differensiallasak, $y'=(\ln p+1)\frac{dp}{dx}$ yoki $y'=p$ bo'lgani uchun $p=(\ln p+1)\frac{dp}{dx}$ hosil bo'ladi.

Umumiy yechim bunday yoziladi:

$$\begin{cases} x+C=\dfrac{(\ln p+1)^2}{2} \\ y=p\ln p \end{cases}$$

Javob: $\begin{cases} x+C=\dfrac{(\ln p+1)^2}{2} \\ y=p\ln p \end{cases}$

c) $x=y'+\sin y'$ - (4) ko'rinishdagi tenglama. Bu yerda ham $y'=p$ deymiz, u holda $x=p+\sin p$. Endi $\frac{dy}{dx}=p$ tenglikni $dy=pdx$ kabi yozib olamiz.

So'ngra

$$\int dy=\int p(x)dx=\left|u=p(x),\ dv=dx,\ du=dp,\ v=x\right|=$$

$$=px-\int xdp=px-\int(p+\sin p)dp=px-\frac{p^2}{2}+\cos p+C$$

bo'lgani uchun $y=px-\frac{p^2}{2}+\cos p+C$.

Umumiy yechim quydagicha yoziladi:

$$\begin{cases} x=p+\sin p \\ y=\dfrac{1}{2}p^2+p\sin p+\cos p+C \end{cases}$$

Javob: $\begin{cases} x=p+\sin p \\ y=\dfrac{1}{2}p^2+p\sin p+\cos p+C \end{cases}$

d) $y'=e^{\frac{y'}{y}}$ tenglama (5) ko'rinishdagi tenglama.

Yuqoridagidek ish tutamiz:

$y'=p,\ p=e^{\frac{p}{y}},\ \ln p=\dfrac{p}{y},\ y=\dfrac{p}{\ln p},\ dy=\dfrac{\ln p-1}{\ln^2 p}dp,\ dx=\dfrac{1}{p}dy=\dfrac{dp}{p\ln p}-\dfrac{dp}{p\ln^2 p},$

$x=\ln\left|\ln p\right|+\dfrac{1}{\ln p}+C$

Umumiy yechim ushbu parametrik ko'rinishda bo'ladi:

$$x=\ln\left|\ln p\right|+\frac{1}{\ln p}+C;\ y=\frac{p}{\ln p}$$

Javob: $\begin{cases} x=\ln|\ln p|+\dfrac{1}{\ln p}+C \\ y=\dfrac{p}{\ln p} \end{cases}$

e) $y=xy'^2+y'^2$ tenglama Lagranj tenglamasidir. $y'=p$ bo'sin.

U holda $y=xp^2+p^2$ yoki $y=(x+1)p^2$.

Buni x bo'yicha differensiallaymiz:

$$y'=p^2+2(x+1)p\frac{dp}{dx}$$

$y'=p$ ekanini e'tiborga olib, so'ngra hosil bo'lgan tenglikning ikkala tomonini p ga qisqartirib, o'zgaruvchilarni ajratsak, quydagilarga ega bo'lamiz:

$p=p^2+2(x+1)p\dfrac{dp}{dx}$, $1-p=2(x+1)p\dfrac{dp}{dx}$, $\dfrac{dx}{x+1}=\dfrac{2dp}{1-p}$, bu yerdan

$\ln|x+1|=-2\ln|1-p|+2\ln C$.

Potensirlasak:

$$x+1=\frac{C^2}{(1-p)^2}$$

Demak, umumiy yechim parametrik shaklda ushbu ko'rinishda bo'ladi:

$$\begin{cases} x=\dfrac{C^2}{(1-p)^2}-1, \\ y=\dfrac{C^2p^2}{(1-p)^2} \end{cases} \qquad (13)$$

(13) dan p parametrni yo'qotamiz. Buning uchun

$p^2=(1-(1-p))^2=\left(1-\dfrac{C}{\sqrt{x+1}}\right)^2=\dfrac{\left(\sqrt{x+1}-C^2\right)^2}{x+1}$ ifodani topamiz va uni $y=(x+1)p^2$ tenglamaga qo'yamiz.

Shunday qilib, umumiy yechim quydagicha bo'ladi:

$$y=\left(\sqrt{x+1}-C\right)^2.$$

Javob: $y=\left(\sqrt{x+1}-C\right)^2$.

f) $y=xy'-y'^2$ tenglama - Klero tenglamasidir.

Umumiy yechimni bevosita tenglamadan y' ni C ga almashtirib topamiz:

$$y=Cx-C^2$$

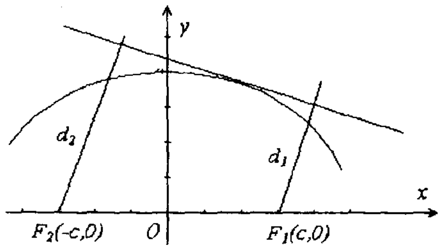
Bundan tashqari, bu to'g'ri chiziqlarning o'ramasi (11) ga asosan

$$\begin{cases} x=2C \\ y=Cx-C^2 \end{cases}$$

bo'lib, u ham Klero tenglamasining integrali bo'ladi. Bundan C ni yo'qotib maxsus yechim $y=\dfrac{x^2}{4}$ ni hosil qilamiz.

Javob: $\begin{cases} x = 2C \\ y = Cx - C^2 \end{cases}$; $y=\dfrac{x^2}{4}$.

Masala. Shunday egri chiziqlarni topingki, ular uchun berilgan ikkita nuqtadan istalgan urinmagacha bo'lgan masofalar ko'paytmasi o'zgarmas bo'lib, b^2 ga teng bo'lsin. Berilgan nuqtalar orasidagi masofa 2*s* ga teng.(7-rasm)



7-rasm

Yechish. Koordinata o'qlarini shunday tanlab olamizki, berilgan F_1 va F_2 nuqtalar Ox o'qda, koordinatalar boshi O esa bu nuqtalarninng o'rtasida joylashgan bo'lsin. $y=f(x)$ egri chiziqning istalgan $M(x,y)$ nuqtasidan o'tkazilgan urinma chiziq $Y-y=y'(X-x)$ tenglamasini $y'X-Y-(xy'-y)=0$ ko'rinishda yozib olamiz. Bu yerda X va Y urinma nuqtalarining o'zgaruvchi koordinatalari.

Urinma tenglamasini normal ko'rinishga keltirib, berilgan nuqtalardan urinmagacha bo'lgan d_1 va d_2 masofalarni topamiz:

$$d_{1,2} = \frac{\pm Cy' + (xy' - y)}{\sqrt{(y')^2 + 1}}$$

Shartga ko'ra $d_1 d_2=b^2$, shuning uchun

$(xy'-y)^2-C^2y'^2 = b^2(y'^2+1)$ yoki $y = xy' \pm \sqrt{a^2 y'^2 \pm b^2}$,

bu yerda $C^2 \pm b^2 = a^2$ deb olingan. Hosil qilingan tenglama Klero tenglamasidir. Uning

$y = Cx \pm \sqrt{a^2 C^2 \pm b^2}$ umumiy yechimi to'g'ri chiziqlar oilasidan iborat. Maxsus yechimni topamiz. Buning uchun umumiy yechimni C bo'yicha differensiallaymiz va ushbu tenglamalar sistemasini tuzamiz:

$$\begin{cases} x = \mp \dfrac{a^2 C}{\sqrt{a^2 C^2 \pm b^2}}, \\ y = \pm \dfrac{b^2}{\sqrt{a^2 C^2 \pm b^2}} \end{cases}$$

(ikkinchi tenglama x ning ifodasini umumiy yechimga qo'yish orqali hosil qilingan). Bu sistemani quydagicha qayta yozib olamiz:

$$\begin{cases} \dfrac{x}{a} = \mp \dfrac{aC}{\sqrt{a^2C^2 \pm b^2}} \\ \dfrac{y}{b} = \pm \dfrac{b}{\sqrt{a^2C^2 \pm b^2}} \end{cases}$$

Bundan C ni yuqotib $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ va $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ larni hosil qilamiz.

Shunday qilib, izlanayotgan egri chiziqlar ellipslar va giperbolalar ekan.

Javob: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Izoganal va ortegonal trayektoriyalar

Bir parametrli yassi silliq chiziqlar oilasi

$$\Phi(x,y,a)=0 \qquad (14)$$

a–parametr, tenglama bilan berilgan bo'lsin. Shu chiziqlar oilasining har bir chizig'ini o'zgarmas α burchak bilan kesib o'tuvchi chiziq berilgan oilaning *izogonal trayektoriyasi* deyiladi.

Hususan, $\alpha=\frac{\pi}{2}$ bo'lganda tegishli izogonal trayektoriya *ortogonal trayektoriya* deyiladi.

Egri chiziqlar oilasi o'zining

$$F(x,y,y')=0 \qquad (15)$$

differensial tenglamasi bilan berilganda izogonal trayektoriyalar oilasining differensial tenglamasini topish uchun (2) tenglamada y' ni $\frac{y'\mp k}{1\pm ky'}$ bilan almashtirish lozim, bu yerda k-egri chiziqlarning trayektoriyalar bilan kesishish burchagining tangensi. Hususan, ortogonal trayektoriyalar uchun y' ni $-\frac{1}{y'}$ ga almashtirish kerak.

Agar egri chiziqlar oilasining differensial tenglamasi qutb koordinatalar sistemasida

$$\Phi(r,\theta,r')=0 \qquad (16)$$

ko'rinishda berilsa, izogonal trayektoriyalar oilasining differensial tenglamasini topish uchun (16) da $r'=\frac{dr}{d\theta}$ ni $\frac{1+k\frac{r}{r'}}{\frac{r}{r'}-k}r$ bilan almashtiramiz.

Hususan, ortogonal trayektoriyalar uchun r' ni $-\frac{r^2}{r'}$ ga almashtirish kerak.

Masala. $x^2+y^2=2ax$ aylanalar oilasining ortogonal trayektoriyalarini toping.

Yechish. $x^2+y^2=2ax$ aylanalar oilasining differensial tenglamasini tuzamiz, buning uchun berilgan tenglamaning ikkala qismini x buyicha differensiallaymiz:

$$2x+2yy'=2a.$$

$x^2+y^2=2ax$ va $2x+2yy'=2a$ tenglamalardan a ni yo'qotsak, $2x+2yy'=\frac{x^2+y^2}{x}$ yoki $y'=\frac{2xy}{x^2-y^2}$ hosil bo'ladi. Ortogonal trayektoriyalar oilasining differensial tenglamasini topish uchun bu tenglamada y' ni $-\frac{1}{y'}$ ga almashtiramiz. Natijada $y'=\frac{2xy}{x^2-y^2}$ hosil bo'ladi. Hosil qilingan tenglama bir jinsli tenglama. Uni yechish uchun $y=xu$ almashtirishni qo'llaymiz. U holda $y'=u+xu'$ va tenglama $u'x+u=\frac{2u}{1-u^2}$ yoki $u'x=\frac{u+u^3}{1-u^2}$ ko'rinishda bo'ladi. O'zgaruvchilarni ajratib, so'ngra integrallaymiz:

$$\frac{dx}{x}=\frac{1-u^2}{u+u^3}du,\quad \ln|x|=\int\frac{1-u^2}{u+u^3}du$$

Bu tenglikning o'ng tomonidagi integralni topish uchun integral ostidagi to'g'ri kasr ratsional funksiyani oddiy kasrlarga ajratamiz:

$$\frac{1-u^2}{u+u^3}=\frac{1}{u}-\frac{2u}{1+u^2}$$

Bularni integrallab topamiz:

$$\int\frac{1-u^2}{u+u^3}du=\int\frac{du}{u}-\int\frac{2udu}{1+u^2}=\ln|u|-\ln\left|1+u^2\right|+\ln C=\ln\frac{|Cu|}{1+u^2}$$

Topilgan ifodani (3) ga qo'ysak, quydagiga ega bo'lamiz:

$$\ln|x|=\ln\frac{|Cu|}{1+u^2}\ \text{ yoki }\ x=\frac{Cu}{1+u^2}$$

Bu tenglikda u ni $\frac{y}{x}$ bilan almashtirsak, $x^2+y^2=Cy$ ga, ya'ni yana aylanalar oilasiga ega bo'lamiz.

Ikkala oilaning barcha aylanalari koordinatalar boshidan o'tadi, biroq berilgan oila aylanalarning markazlari Ox o'qda trayektoriyalarining markazlari esa Oy o'qda joylashgan.

Javob: $x^2+y^2=Cy$.

Masala. $r=2a\sin\theta$ egri chiziqlar oilasining ortogonal trayektoriyalarini toping.

Yechish. Avval $r=2a\sin\theta$ egri chiziqlar oilasining differensial tenglamasini topamiz:

$r=2a\sin\theta$, $r'=2a\cos\theta$

Bu tenglamalardan a ni yo'qotib,

$r'=r\operatorname{ctg}\theta$ ga ega bo'lamiz.

Ortogonal trayektoriyalar oilasining differensial tenglamasini topish uchun r' ni $-\frac{r^2}{r'}$ ga almashtirsak $\frac{r'}{r}=-tg\theta$ bo'ladi. Bu tenglamani integrallab, izlanayotgan egri chiziqlar oilasining ortogonal trayektoriyalarining

$r=2C\cos\theta$

ko'rinishda bo'lishini topamiz.

Javob: $r=2C\cos\theta$

Masala. $r=ae^{\theta}$ logarifmik spirallar oilasining har bir chizig'ini 45^0 burchak ostida kesuvchi egri chiziqlarni toping.

Yechish.

$$\begin{cases} r = ae^{\theta} \\ r' = ae^{\theta} \end{cases}$$

sistemadan $r'=r$ ko'rinishdagi tenglama kelib chiqadi. Izogonal trayektoriyalar oilasining differensial tenglamasini topish uchun bu tenglamadagi r' ni $\frac{r'+kr}{r-kr'}r$ bilan almashtiramiz, bu yerda masala shartiga ko'ra, $k=\text{tg}45^0=1$.

Demak, $\frac{r'+r}{r-r'}r=r$, bundan $2r'=0$ ekanligini ko'rish qiyin emas. Bundan $r=C$ ko'rinishdagi izogonal trayektoriyalar oilasini hosil qilamiz.

Javob: $r=C$.

Masala. $U=x^2+y^2$ ko'rinishdagi potensialga ega bo'lgan kuchlar hosil qilgan maydonning kuch chiziqlarini toping.

Yechish. Sath chiziqlari $U=C$ ko'rinishda bo'ladi. Maydonning kuch chiziqlari sath chiziqlar oilasining ortogonal trayektoriyalari bo'lishini ko'rsatish qiyin emas. Demak, izlanayotgan kuchlar $x^2+y^2=C$ aylanalar oilasining ortogonal trayektoriyalari bo'lar ekan. Bundan: $2x+2yy'=0$. y' ni $-\frac{1}{y'}$ bilan almashtirsak,

$$\ln y=\ln|x|+\ln C,\quad y=Cx,\quad x^2+\frac{y^2}{2}=a^2.$$

Javob: $y=Cx,\quad x^2+\frac{y^2}{2}=a^2.$

Birinchi tartibli differensial tenglamalarni yeching (5.1.-5.4.).

5.1. $y(y')^2-(xy+1)y'+x=0.$ 5.2. $(y)'^3-\frac{1}{4x}y'=0.$

5.3. $x^2(y')^2+3xyy'+2y^2=0.$ 5.4. $(y)'^3-y(y')^2-x^2y'+x^2y=0.$

Quyidagi tenglamalarni parametr kiritish yo'li bilan integrallang (5.5-5.8.).

5.5. $y=(y')^2e^{y'}$ 5.6. $\ln y'+\sin y'-x=0$

5.7. $y'\sin y'+\cos y'-y=0$ 5.8. $y=(y')^2+(x+a)y'-y=0$

Quyidagi Lagranj va Klero tenglamalarining yechimlarini toping (5.8-5.12.).

5.9. $y=xy'+\sqrt{1+y'^2}$.   5.10. $y=x(1+y')+(y')^2$.

5.11. $y=x(y')^2-y'$ 5.12. $(y')^2+4xy'-4y'=0$.

Quyidagi tenglamalarning maxsus yechimlarini toping (5.8-5.12.)

5.13. Ma'lumki, $y=Ce^x+\frac{4}{C}$ funksiyalar $(y')^2-yy'+4e^x=0$ tenglamaning yechimlari bo'ladi. Mazkur teng.amaning maxsus yechimlarini toping.

5.14. Ma'lumki, $x^2+C(x-3y)+C^2$ parabolalardan har biri $3x(y')^2-6yy'+x+2y=0$ tenglamaning integral egri chizig'i bo'ladi. Mazkur tenglamaning maxsus yechimlarini toping.

5.15. Istalgan nuqtasiga o'tkazilgan urinmasi koordinata o'qlaridan ajratgan kesmalari usinliklari yigindisi o'zgarmas $2a$ ga teng bo'lgan egri chiziqni toping.

5.16. Egri chiziqning istalgan nuqtasidagi normali va normalostisi yig'indisi shu nuqtaning abssissasiga proporsional. Shu egri chiziqni toping.

5.17. Istalgan nuqtasiga o'tkazilgan urinma va koordinata o'qlari hosil qilgan uchburchaklarning yuzi o'zgarmas $2a^2$ ga teng. Shu egri chiziqni toping.

5.18. Moddiy nuqtaning ixtiyoriy momentdagi tezligi harakat boshlangandan shu momentgacha bo'lgan o'rtacha tezlikdan nuqtaning kinetik energiyasiga proporsional va vaqtga teskari proporsional bo'lgan miqdorga farq qiladi. Yo'lning vaqtga bog'lanishini toping.

5.19. $y=Cx^2$ parabolalar oilasining ortogonal trayektoriyalarini toping.

5.20. $r=a(1+\cos\varphi)$ kardioidalar oilasining ortogonal trayektoriyalarini toping.

5.21. $y=Cx$ to'g'ri chiziqlar oilasining izogonal trayektoriyalarini toping

5.22. $x^2=2a(y-x\sqrt{3})$ egri chiziqlarni 60^0 burchak ostida kesuvchi izogonal trayektoriyalar oilasini toping.

5.23. $y^2=4Cx$ parabolalar oilasining izogonal trayektoriyalarini toping. Kesishish burchagi 45^0 ga teng.

5.24. $r^2=a^2\cos2\varphi$ lemniskatalar oilasining ortogonal trayektoriyalarini toping.

I – bobga doir misol va masalalarning javoblari

1.1. $arctgx+arctgy=C$. 1.2. $1+y^2=Cx^2$. 1.3. $\sqrt{1+x^2}+\sqrt{1+y^2}=C$.

1.4. $-\frac{1}{y-2}+\frac{1}{2(x+1)^2}=C$. 1.5. $y=\sin(C\cdot\ln(1+x^2))$; $y=1$. 1.6. $y=\sqrt[3]{3x-3x^2+C}$

1.7. $2y-2arctgy-3\ln|x-1|+\ln|x+1|=C$. 1.8. $y-x(\ln|y|+1)=Cy\cos x$, $y=0$.
1.9. $tg^2x+\sin^2y=C$. 1.10. $x+y=\ln(C(x+1)(y+1))$, $y=-1$. 1.11. $x-y+\ln|xy|=C$, $y=0$. 1.12. $(x-1)^2+y^2=C^2$. 1.13. $\cos y=C\cos x$. 1.14. $(1+e^y)e^x=C$.

1.15. $y=Ce^{-\sqrt{1+x^2}}$. 1.16. $y=C(x^2-4)$. 1.17. $y=C\cos x$. 1.18. $y=C(x+\sqrt{x^2+a^2})$.

1.19. $\ln\left|\frac{x}{y}\right|-\frac{x+y}{xy}=C$, $y=0$. 1.20. $\ln|xy|+xy=C$. 1.21. $x+y=0$. 1.22. $2e^{y^2}=e^x+1$.

1.23. $x^{-2}+y^{-2}=2\left(1+\ln\left|\frac{x}{y}\right|\right)$. 1.24. $y=e^{\sqrt{x}-2}$. 1.25. $y=2\sin^2 x-\frac{1}{2}$.

1.26. $\sqrt{y}=x\ln x-x+1$. 1.27. $x^2=2+2y^2$. 1.28. sinx. 1.29. 5 min 56s.

1.30. $\frac{3}{40\ln 2,5}s$. 1.31. $y=-2e^{3x}$. 1.32. $y=\frac{C}{x}$. 1.33. 60 min.

2.1. $tg\frac{y}{x}=\ln|Cx|$. 1.35. $x=Ce^{\frac{2x}{y-x}}$. 2.2. $y^2+3xy-2x^2=C$.

2.3. $x=C(\ln y-\ln x-1)$ 2.4. $x=\frac{Ctg(y/x)}{\sqrt{1+tg^2(y/x)}}$ 2.5. $y=-\frac{(x-2)^2+C}{2(x-1)}$

2.6. $3y+x-\ln(x-2y)=C$. 2.7. $y^2-3xy+2x^2=C$. 2.8. $x^2-y^2=Cx$.

2.9. $\ln Cx=-e^{-\frac{y}{x}}$. 2.10. $y=\frac{x^2}{C+x}$. 2.11. $y^2=x^2(1+Cx)$. 2.12. $y=xe^{-\frac{1}{2}x}$.

2.13. $y=xe^{-\frac{1}{2}x}$. 2.14. $\sin\frac{y}{x}+\ln|x|=0$. 2.15. $y=-x$. 2.16. $\sqrt{x^2+y^2}=e^{\frac{y}{x}arctg\frac{y}{x}}$.

2.17. $y=4e^{\frac{3y-4x}{3x}}$. 2.18. $y^2=2C\left(x+\frac{C}{2}\right)$. 2.19. $y=1-\frac{x^2}{4}$. 2.20. $y=\frac{x}{1-x}$.

3.1. $y=\frac{1}{2}+Ce^{-x^2}$. 3.2. $y=(x^2+C)e^{e^x}$. 3.3. $y=(x^3+C)\ln x$.

3.4. $x+\frac{1}{2}y^2+\frac{1}{2}y+\frac{1}{4}=Ce^{2y}$. 3.5. $y=(\tilde{o}+C)\sin x$. 3.6. $y=\sqrt[3]{\frac{3}{2x}+\frac{C}{x^3}}$

3.7. $y^2=x(C-\ln x)$. 3.8. $y-x^3e^{-y}=C$. 3.9. $x^2+y^4=Cy^2$. 3.10. $y=\frac{e^{\sin x}}{C-e^{\sin x}}$.

3.11. $y=\frac{\ln(x+\sqrt{a^2+x^2})+C}{\sqrt{a^2+x^2}}$. 3.12. $y=\frac{C-e^{-x^2}}{2x^2}$. 3.13. $y=\frac{1}{x\ln Cx}$.

3.14. $y=\ln x+\frac{C}{x}$. 3.15. $y=\frac{x-1}{3}+\frac{C}{\sqrt{2x+1}}$. 3.16. $y=\pm\frac{1}{\sqrt{1+Ce^{x^2}}}$.

3.17. $y=x-x^2$. 3.18. $\frac{\sin x}{\cos^2 x}$. 3.19. $y=\frac{x^2}{\cos x}$. 3.20. $y=1$. 3.21. $y^3=x-2e^{1-x}$.

3.22. $y=-\frac{1}{\sqrt{1-x^2}+1}$. 3.23. $v=(v_0+b)e^{-at^2}+b(at^2-1)$, bu yerda $a=\frac{k_1}{2m}$, $b=\frac{2km}{k_1^2}$

3.24. $I=\frac{E_0}{R^2+\omega L^2}\left(\omega Le^{-Rt/L}+R\sin\omega t-\omega L\cos\omega t\right)$, (zanjirdagi kuchlanish $L\frac{dI}{dt}+RI$ qonuniyat bilan o'zgaradi). 3.25. $v=\frac{k1}{k}\left(t-\frac{m}{k}+\frac{m}{k}e^{-kt/m}\right)$

3.26. $y=2(1\mp a^2)\pm\frac{a^2}{2x}$ (tenglamasi $|xy-x^2y'|=a^2$). 3.27. $y=2x-x\ln|x|$

4.1. $(x+1)(x-y)+y^2=C$. 4.2. $\sqrt{x^2-y^2}-x=C$. 4.3. $\frac{(1-e^y)}{1+x^2}=C$.

4.4. $x^3+y^3-x^2-xy+y^2=C$. 4.5. $x^3y+x^2-y^2=Cxy$. 4.6. $xe^y+ye^x+3x-2y=C$

4.7. $x^2+y+e^{xy}=C$. 4.8. $x^3+3\tilde{o}^2y^2+\acute{o}^4=C$. 4.9. $xe^y-y^2=C$.

4.10. $x^2+y^2-2arctg\frac{y}{x}=C$. 4.11. $x^3e^y-y=C$. 4.12. $x^2\cos^2 y+y=C$.

4.13. $\frac{x^2\cos 2y}{2}+x=C$. 4.14. $6x^2+5xy+y^2-9x-4y=C$.

4.15. $\frac{x^2-y^2}{2}+yx=C;\ \mu=y$. 4.16. $x-\frac{y}{x}=C;\mu=\frac{1}{x^2}$.

4.17. $y^3+x^3(\ln x-1)=Cx^2;\ \mu=\frac{1}{x^4}$ 4.18. $x^2-\frac{7}{y}-3xy=C;\ \mu=\frac{1}{y^2}$.

5.1. $y^2=2x+C;y=\frac{1}{2}x^2+C$. 5.2. $y=C;y=\pm\sqrt{x}+C$.

5.3. $y=\frac{C}{x^2};y=\frac{C}{x}$. 5.4. $y=\pm\frac{x^2}{2}+C;\quad y=Ce^x$

5.5. $y=0;\begin{cases}x=(p+1)e^p+C\\ y=p^2e^p\end{cases}$. 5.6. $\begin{cases}x=\ln p+\sin p\\ y=p+\cos p+p\sin p+C\end{cases}$

5.7. $y=1;\begin{cases}x=\sin p+C\\ y=p\sin p+\cos p\end{cases}$. 5.8. $y=C(x+a)+C^2,y=-\frac{(x+a)^2}{4}$

5.9. $y=Cx+\sqrt{1+C^2},x^2+y^2=1$ 5.10. $\begin{cases}x=2(1-p)+Ce^{-p}\sqrt{1+C^2}\\ y=2(p^2-1)^2+Ce^{-p}(1+p)+p^2\end{cases}$

5.11. $\begin{cases}x=\frac{p-\ln p+C}{(p-1)^2}\\ y=xp^2-p\end{cases}$. 5.12. $y=Cx+\frac{C^2}{4},\quad y=-x^2$. 5.13. $y=4e^{\frac{x}{2}},y=-4e^{\frac{x}{2}}$.

5.14. $y=-\frac{x}{3}$. 5.15. $(y-x-2a)^2=8ax$. 5.16. $y^2=Cx^{-1/k}+\frac{k^2x^2}{2k+1}$. 5.17. $xy=a^2$.

5.18. $S=at^2$, a-o'zgarmas son. 5.19. $\frac{x^2}{4}+\frac{y^2}{2}=C^2$. 5.20. $\rho=C\sin^2\frac{\varphi}{2}$.

5.21. $x^2+y^2=Ce^{\frac{1}{k}arctg\frac{y}{x}}$, $k=tg\alpha$. 5.22. $y^2=C(x-y\sqrt{3})$.

5.23. $y^2-xy+2x^2=Ce^{\frac{6}{\sqrt{7}}arctg\frac{2y-x}{x\sqrt{7}}}$. 5.24. $r^2=C\sin 2\varphi$.

II BOB. YUQORI TARTIBLI DIFFERENSIAL TENGLAMALAR.

1-§. Tartibini pasaytirish mumkin bo'lgan differensial tenglamalar.

n-tartibli differensial tenglamani simvolik ravishda

$$F(x, y, y', ..., y^{(n-1)}, y^{(n)})=0 \qquad (1)$$

ko'rinishda yoki bu tenglamani n-tartibli hosilaga nisbatan yechib bo'lsa,

$$y^{(n)}=f(x,y,y',...,y^{(n-1)}) \qquad (2)$$

ko'rinishda yozish mumkin.

n-tartibli differensial tenglamaning *umumiy yechimi* x ga va n-ta ixtiyoriy o'zgarmaslarga bog'liq bo'ladi: $y = g(x,C_1,C_2,...,C_n)$.

Shu sababli umumiy yechimdan xususiy yechimni ajratib olish uchun ixtiyoriy o'zgarmaslarni aniqlashga imkon beradigan ba'zi qo'shimcha shartlar ham berilgan bo'lishi kerak. Bu shartlarni izlanayotgan funksiyaning va uning (n-1)-tartibgacha (y ham kiradi) barcha hosilalarning biror nuqtadagi qiymatlarini, ya'ni $x=x_0$ da

$$y(x_0) = y_0,\ y'(x_0) = y_1,..., y^{(n-1)}(x_0) = y_{n-1} \qquad (3)$$

berish bilan hosil qilish mumkin. (3) sistema *boshlang'ich shartlar* sistemasi deyiladi. Berilgan (2) differensial tenglamaning (3) boshlang'ich shartlar sistemasini qanoatlantiruchi xususiy yechimini topish masalasi *Koshi masalasi* deyiladi.

Yuqori tartibli differensial tenglamalarni integrallash masalasi birinchi tartibli tenglamani integrallash masalasidan ancha qiyin bo'lib, har doim ham birinchi tartibli tenglamani integrallashga keltiraverilmaydi. Shunday bo'lsada chiziqli tenglamalardan tashqari barcha turdagi yuqori tartibli tenglamalar uchun integrallashning asosiy usuli tartibini pasaytirish, ya'ni berilgan tenglamani unda o'zgaruvchilarni almashtirish orqali tartibi pastroq tenglamaga keltirish bo'lib hisoblanadi. Biroq tenglamaning tartibini pasaytirishga har doim ham erishish mumkin emas. Biz bu yerda tenglama tartibini pasaytirishga imkon beradigan n-tartibli tenglamalarning eng sodda turlari bilan tanishamiz.

1. Ushbu

$$y^{(n)}=f(x) \qquad (4)$$

tenglamaning tartibini pasaytirish, ketma-ket integrallash yo'li bilan amalga oshiriladi:

$$y^{(n-1)} = \int f(x)dx + C_1;$$

$$y^{(n-2)} = \int\left(\int f(x)dx + C_1\right)dx + C_2 = \int dx\int f(x)dx + C_1x + C_2;$$

....

$$y = \int dx \int dx .... \int f(x)dx + C_1 \frac{x^{n-1}}{(n-1)!} + C_2 \frac{x^{n-2}}{(n-2)!} + ... + C_n;$$

2. Izlanayotgan y funksiya va uning $y', y'', ..., y^{(k-1)}$ hosilalalri oshkor holda ishtirok etmagan

$$F(x, y^{(k)}, y^{(k+1)}, ..., y^{(n)}) = 0 \qquad (5)$$

differensial tenglamaning tartibi

$$y^{(k)} = z; \quad y^{(k+1)} = z'; \quad ... \quad y^{(n)} = z^{(n-k)}$$

almashtirishlar yordamida k birlikka pasaytiriladi:

$$F(x, z, z', ..., z^{(n-k)}) = 0.$$

3. Erkli x o'zgaruvchi oshkor holda ishtirok etmagan

$$F(y, y', y'', ..., y^{(n)}) = 0 \qquad (6)$$

tenglamaning tartibi

$$y' = p, \; y'' = \frac{dy'}{dx} = \frac{dy'}{dy} \cdot \frac{dy}{dx} = \frac{dp}{dy} p,$$

$$y''' = \frac{dy''}{dx} = \frac{dy''}{dy} \cdot \frac{dy}{dx} = \frac{dy''}{dy} p = \frac{d\left(\frac{dp}{dy} p\right)}{dy} p = \frac{d^2 p}{dy^2} p^2 + \left(\frac{dp}{dy}\right)^2 p$$

almashtirishlar orqali bir birlikka pasaytiriladi.

4. $F(x, y, y', y'', ..., y^{(n)})$ funksiya $y, y', y'', ..., y^{(n)}$ larga nisbatan bir jinsli bo'lgan

$$F(x, y, y', y'', ..., y^{(n)}) = 0 \qquad (7)$$

tenglamaning tartibi $y = e^{\int z(x)dx}$ almashtirish orqali bittaga kamaytiriladi.

5. Tenglamaning chap tomoni aniq hosila bo'lgan hol. Bu holda tenglama tartibini bir birlikka pasaytirish bevosita integrallash yo'li bilan amalga oshiriladi.

Albatta, bunday hol kamdan-kam uchraydi. Ayrim hollarda tenglamani bunday ko'rinishga keltirishga ba'zi sun'iy shakl almashtirishlar orqali erishiladi, biroq bunday shakl almashtirishlarning biron-bir umumiy usulini bu yerda ko'rsata olmaymiz va misol keltirish bilan chegaralanamiz.

Masalan, $y''-xy'-y=0$ tenglamani qaraylik, tenglamaning chap tomonini $(y'-xy)'=0$ ko'rinishga egaligini ko'rish oson, hosil qilingan tenglamani integrallab, quydagiga ega bo'lamiz:

$$y' - xy = C \qquad (8)$$

Bu tenglama birinchi tartibli chiziqli tenglamadir. Shu sababli

$$y = uv \qquad (9)$$

almashtirish bajaramiz. Bu holda

$$y' = u'v + uv' \qquad (10)$$

(9) va (10) ni (8) ga qo'ysak,

$u'v + u(v' - xv) = C_1, \qquad v' - xv = 0, \qquad \frac{dv}{v} = xdx, \ln v = \frac{x^2}{2}, v = e^{\frac{x^2}{2}}, \; e^{\frac{x^2}{2}} u' = C_1, \quad u' = C_1 e^{-\frac{x^2}{2}},$

$u = C_1 \int e^{-\frac{x^2}{2}} dx + C_2, y = e^{\frac{x^2}{2}} (C_1 \int e^{-\frac{x^2}{2}} dx + C_2$ umumiy yechim hosil bo'ladi.

Bu yerda hosil bo'lgan $\int e^{-\frac{x^2}{2}}dx$ integral elementar funksiyalar bilan ifodalanmaydi, biroq bunday noelementar funksiya uchun to'liq jadvallar mavjud.

Misol. Quyidagi tenglamalarning umumiy yechimlarini toping:

a) $y'''=x+\cos x$, b) $xy''=y'\ln\frac{y'}{x}$, c) $y''+(y')^2=2e^{-y}$,

d) $x^2yy''=(y-xy')^2$, e) $yy''-(y')^2-y^2=0$.

Yechish. a) $y'''=x+\cos x$ tenglamaning ikkala tomonini x bo'yicha uch marta ketma-ket integrallab, quydagilarni $y''=\frac{x^2}{2}+\sin x+2C_1$; $y'=\frac{x^3}{6}-\cos x+2C_1x+C_2$;

$y=\frac{x^4}{24}-\sin x+C_1x^2+C_2x+C_3$ hosil qilamiz.

b) $xy''=y'\ln\frac{y'}{x}$ tenglamani (5) ko'rinishdagi tenglamadir. $y'=p$ birinchi tartibli bir jinsli $p'=\frac{p}{x}\ln\frac{p}{x}$ tenglamaga kelamiz. Shuning uchun $p=xu$ almashtirishdan foydalanib, $p'=u+xu'$ ni topamiz. p va p' ning bu ifodalarini hosil qilingan tenglamaga qo'yib, $u+xu'=u\ln u$ o'zgaruvchilari ajraladigan differensial tenglamani hosil qilamiz. O'zgaruvchilarni ajratib,

$$\frac{du}{u(\ln u-1)}=\frac{dx}{x}$$

ni hosil qilamiz. Bu tenglamani integrallab, quydagilarga ega bo'lamiz:

$$\ln|\ln u-1|=\ln|x|+\ln|C_1|=\ln|C_1x|,\ \ln u-1=C_1x,\quad \ln u=1+C_1x, u=e^{1+C_1x}$$

Bu yerda u ni $\frac{p}{x}$ ga, p ni esa y' ga almashtirsak, $y'=xe^{1+C_1x}$ trnglama hosil bo'ladi. Uni integrallaymiz:

$$y=\int xe^{1+C_1x}dx=\left\|\begin{matrix}u=x, dv=e^{1+C_1x}dx\\ du=dx, v=\frac{1}{C_1}e^{1+C_1x}\end{matrix}\right\|=\frac{x}{C_1}e^{1+C_1x}-\frac{1}{C_1^2}e^{1+C_1x}+C_2=e^{1+C_1x}\left(\frac{x}{C_1}-\frac{1}{C_1^2}\right)+C_2$$

c) $y''+y'^2=2e^{-y}$ tenglama (6) ko'rinishdagi tenglamadir.

$y'=p$ va $y''=p\frac{dp}{dy}$ deb, $p\frac{dp}{dy}+p^2=2e^{-p}$

Bernulli tenglamasini hosil qilamiz. $p^2=z$ deb olamiz, u holda

$$\frac{dz}{dy}+2z=4e^{-y} \qquad (2)$$

chiziqli tenglama hosil bo'ladi. Shu sababli

$$z=uv \qquad (3)$$

almashtirishdan foydalanish mumkin. Bu holda

$$z'=u'v+uv' \qquad (4)$$

(3) va (4) ni (2) ga qo'ysak.

$u'v+u(v'+2v)=4e^{-y}$, $v'+2v=0$, $\frac{dv}{v}=-2dy$, $\ln v=-2y$, $v=e^{-2y}$, $e^{-2y}u'=4e^{-y}$, $u'=4e^{y}$,

$u=4e^{y}+C_1$, $z=4e^{-y}+C_1e^{-2y}$ ni topamiz. Bu yerda z ni $p^2=u'^2$, $(p^2=z)$ ga almashtirib, $\frac{dy}{dx}=\pm\sqrt{4e^{-y}+C_1e^{-2y}}$ ni hosil qilamiz. O'zgaruvchilarni ajratib, so'ngra integrallasak, quydagilarni hosil qilamiz:

$$\frac{dy}{\sqrt{4e^{-y}+C_1e^{-2y}}}=\pm dx,\quad \frac{e^{y}dy}{\sqrt{4e^{y}+C_1}}=\pm dx,\quad \frac{1}{2}\sqrt{4e^{y}+C_1}=\pm x+c_2,$$

$$\frac{1}{4}(4e^{y}+C_1)=(\pm x+C_2)^2,\qquad e^{y}+\frac{C_1}{4}=(\pm x+C_2)^2.$$

d) Berilgan $x^2yy''=(y-xy')^2$ tenglama y, y', y'' larga nisbatan bir jinsli, demak $y=e^{\int z(x)dx}$ desak,

$$y'=ze^{\int z(x)dx},\quad y''=(z'+z^2)e^{\int zdx},\quad x^2(z'+z^2)e^{2\int zdx}=(e^{\int zdx}-xze^{\int zdx})^2 \text{ bu yerdan}$$

$$x^2z=x+C_1,\quad z=\frac{1}{x}+\frac{C_1}{x^2},\quad \int zdx=\int\left(\frac{1}{x}+\frac{C_1}{x^2}\right)dx=\ln|x|-\frac{C_1}{x}+\ln C_2.$$ Shuning uchun

$$y=e^{\int zdx}=e^{\ln|x|-\frac{C_1}{x}+\ln C_2}=C_2xe^{-\frac{C_1}{x}}.$$

e) $yy''y'^2-y^2=0$ tenglamani quyidagicha yozish mumkin: $\frac{yy''-y'^2}{y^2}=1$. Bu tenglamani $\frac{d(\frac{y'}{y})}{dx}=1$ ko'rinishda keltirib integrallasak, birinchi tartibli $\frac{y'}{y}=x+C_1$ tenglamani hosil qilamiz. Uni yechamiz:

$$\frac{dy}{y}=(x+C_1)dx,\ \ln|y|=\frac{(x+C_1)^2}{2}-\ln|C_2|,\text{ ya'ni } y=C_2e^{\frac{(x+C_1)^2}{2}}.$$

Misol. Koshi masalasini yeching: $y''=yy'$, $y(1)=2$, $y'(1)=2$.

Yechish. $p(y)=y'$, $y''=pp'$ almashtirishlar berilgan tenglamani $pp'=yp$ tenglamaga olib keladi. Bunda quyidagi ikkita hol qaralishi lozim :

a) $p=0, y'=0, y=C$. $y'(1)=2\neq 0$ bo'lgani uchun bu holda yechim yo'q;

b) $p'=y$, $\int dp=\int ydy$, $p=\frac{y^2}{2}+C_1$, $p(2)=2\Rightarrow 2=2+C_1\Rightarrow C_1=0\Rightarrow p=\frac{y^2}{2}$.

Demak,

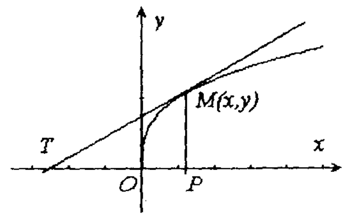
$$\frac{dy}{dx}=\frac{y^2}{2},\quad \int\frac{2dy}{y^2}=\int dx,\quad -\frac{2}{y}=x+C_2,\quad y(1)=2\Rightarrow -1=1+C_2\Rightarrow C_2=-2.$$

Natijada yechim hosil bo'ladi: $y=\frac{2}{2-x}$.

Javob. $y = \frac{2}{2-x}$.

Masala. Koordinatalar boshidan o'tuvchi shunday egri chiziqni topingki, uning biror M nuqtasidan o'tkazilgan MT urinma, shu nuqtaning MP ordinatasi va Ox o'qi bilan hosil qilingan MTP uchburchakning yuzi egri chiziqli OMP uchburchakning yuziga proporsional bo'lsin. (9-rasm).

Yechish. MTP uchburchakning yuzi $S_\Delta = \frac{1}{2}MP \cdot PT$ formula bo'yicha topiladi. Bu yerda $MP=y$ son M nuqtaning ordinatasi, PT urinma ostining uzunligi $PT=\frac{y}{y'}$ ga teng. Demak, $S_\Delta = \frac{1}{2}y\cdot\frac{y}{y'} = \frac{1}{2}\cdot\frac{y^2}{y'}$. OMP egri chiziqli trapetsiyaning yuzi $S_1 = \int_0^x ydx$ ga teng.



9-rasm

Masalaning shartiga ko'ra $\frac{1}{2}\cdot\frac{y^2}{y'} = k\int_0^x ydx$. Bu tenglamaning ikkala tomonini x bo'yicha differensiallab, $2y'^2 - yy'' = 2ky'^2, (y \neq 0)$ ni hosil qilamiz. Hosil qilingan tenglama (6) ko'rinishdagi tenglamadir.

$y'=p$ va $y'' = p\frac{dp}{dy}$ deb o'zgaruvchilari ajraladigan $2(k-1)p^2 = -py\frac{dp}{dy}$ tenglamaga ega bo'lamiz. Integrallashdan so'ng,

$2(k-1)\ln y = -\ln p + \ln C_1$ yoki $y^{2k-2}p = C_1$ hosil bo'ladi. p o'rniga y' ni qo'yamiz: $y^{2k-2}dy = \tilde{N}_1 dx$, $\frac{y^{2k-1}}{2k-1} = \tilde{N}_1 x + \tilde{N}_2$. $y(0)=0$ boshlang'ich shartdan $C_2=0$ kelib chiqadi. Demak, izlanayotgan egri chiziqning tenglamasini ushbu ko'rinishda hosil qilamiz: $y^{2k-1} = Cx$, bu yerda $C = C_1(2k-1)$.

Quyidagi tenglamalarning umumiy yechimlarini toping (1.1-1.6).

1.1. $xy'''=2$. 1.2. $y''=1+y'^2$.

1.3. $y'''+y''^2=0$. 1.4. $y''=a^2y$.

1.5. $2y'y''=1$. 1.6. $y'y'''-3y''^2=0$

Quyidagi tenglamalarning umumiy yechimlarini va $y(0)=-1, y'(0)=0$ boshlang'ich shartlarni qanoatlantirgan xususiy yechimlarini toping.

1.7. $xy'' - y' = x^2e^x$ 1.8. $yy'' - (y')^2 + (y')^3 = 0$

1.9. $y'' + y'tgx = \sin 2x$. 1.10. $(y'')^2 + (y')^2 = a^2$

1.11. Shunday egri chiziqni topingki, uning biror nuqtasidan boshlab hisoblangan yoy uzunligi shu yoyning oxirgi nuqtasida o'tkazilgan urinmaning burchak koeffitsientiga proporsional bo'lsin.

1.12. Egiluvchan bir jinsli cho'zilmaydigan ingichka ip uchlari bilan ikki nuqtada maxkamlangan va ipga uning gorizontal proeksiyasi bo'ylab bir xil taqsimlangan kuch ta'sir qiladi. Ipning og'irligini hisobga olmay, uning muvozanat holatdagi shaklini aniqlang.

1.13. m massali moddiy nuqta harakat bo'ylab yo'nalgan va yo'lga bog'liq bo'lgan kuch ta'sirida to'g'ri chiziqli harakat qilmoqda. Agar kuchning bajargan ishi harakat boshlangandan beri o'tilgan vaqtga proporsional va proporsionallik koeffitsienti k bo'lsa, nuqtaning harakat qonunini toping.

1.14. Boshlang'ich tezligi v_0 bo'lgan m massali moddiy nuqta vertikal tik yuqoriga otilgan. Havo qarshiligi kv^2 ga teng. Shu sababli, agar Oy o'qni vertikal yo'naltirsak, u holda yuqoriga harakat qilinganda $m\frac{d^2y}{dt^2}=-mg-kv^2$ ko'rinishdagi tenglamaga, pastga tushishda esa $m\frac{d^2y}{dt^2}=-mg+kv^2$ ko'rinishdagi tenglamaga ega bo'lamiz, bu yerda $v=\frac{dy}{dt}$. Nuqtaning yerga tushish paytdagi tezligini toping.

2-§. Yuqori tartibli chiziqli differensial tenglamalar.

n-tartibli chiziqli differensial tenglama deb,

$$y^{(n)}+p_1(x)y^{(n-1)}+p_2(x)y^{(n-2)}+\ldots+p_n(x)y'+p_n(x)y=f(x) \qquad (1)$$

ko'rinishdagi tenglamaga aytiladi. Bu yerda $p_1(x)$, $p_2(x)$,..., $p_n(x)$ va $f(x)$ lar biror $[a;b]$ kesmada uzluksiz funksiyalar.

Agar $f(x)\neq 0$ bo'lsa, (1) tenglama chiziqli *bir jinsli bo'lmagan* tenglama deyiladi. Aks holda, ya'ni $f(x)=0$ bo'lsa, (1) tenglama

$$y^{(n)}+p_1(x)y^{(n-1)}+p_2(x)y^{(n-2)}+\ldots+p_n(x)y'+p_n(x)y=0 \qquad (2)$$

ko'rinishga kelib, u chiziqli *bir jinsli* differensial tenglama deyiladi.

1. Agar n ta $\alpha_1,\alpha_2,...,\alpha_n$ bir vaqtda nolga teng bo'lmagan sonlar mavjud bo'lib, $[a;b]$ kesmada barcha x lar uchun

$$\alpha_1 y_1+\alpha_2 y_2+...+\alpha_n y_n=0 \qquad (3)$$

ayniy munosabat bajarilsa y_1, y_2,..., y_n funksiyalar sistemasi $[a;b]$ kesmada *chiziqli bog'liq* deyiladi.

Aks holda, ya'ni (3) ayniy munosabat faqat $\alpha_1=\alpha_2=...=\alpha_n=0$ bo'lganda bajarilsa, u holda $y_1, y_2,..., y_n$ funksiyalar sistemasi *chiziqli erkli* deyiladi.

Agar y_1, y_2,..., y_n funksiyalar $(n-1)$-marta differensiallanuvchi bo'lsa, u holda ulardan tuzilgan ushbu

$$W(y_1,y_2,...,y_n)=\begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \cdots & \cdots & \cdots & \cdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$$

determinant *Vronskiy*[5] *determinanti* yoki *vronskian* deyiladi. Vronskian funksiyalar sistemasining chiziqli bog'liqligi yoki chiziqli erkliligini tekshirish vositasi hisoblanadi. Uning qo'llanishi quydagi ikkita teoremaga asoslangan.

1-teorema. Agar y_1, y_2, ..., y_n funksiyalar chiziqli bog'liq bo'lsa, u holda sistemaning vronskiani aynan nolga teng bo'ladi.

2-teorema. Agar y_1, y_2 ,..., y_n chiziqli erkli funksiyalar bo'lib, ular birorta n-tartibli chiziqli bir jinsli differensial tenglamani qanoatlantirsa, u holda bunday sistemaning vronskiani hech bir nuqtada nolga aylanmaydi.

2. n-tartibli chiziqli bir jinsli differensial tenglamaning y_1, y_2, ..., y_n xususiy yechimlar sistemasi n ta chiziqli erkli funksiyadan iborat bo'lsa, bu sistemani *fundamental sistema* deymiz.

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[5] Yuzef Vronskiy (1776 – 1853) – polshalik matematik va faylasuf.

1-teorema. Agar $y_1, y_2, \ldots, y_n$ funksiyalar (2) tenglama yechimlarining fundamental sistemasini tashkil etsa, u holda ularning

$$y=C_1y_1+C_2y_2+\ldots+C_ny_n$$

chiziqli kombinatsiyasi bu tenglamaning umumiy yechimi bo'ladi.

2-teorema. Chiziqli bir jinsli bo'lmagan (1) differensial tenglamaning umumiy yechimi bu tenglamaning $\tilde{y}$ xususiy yechimi va unga mos bir jinsli (2) tenglamaning $\bar{y}$ umumiy yechimi yig'indisidan iborat, ya'ni

$$y=\bar{y}+\tilde{y}.$$

Agar (2) ning chiziqli erkli $y_1, y_2, \ldots, y_n$ yechimlari ma'lum bo'lsa, u holda *o'zgarmaslarni variatsiyalash usulini* qo'llab, (1) ning umumiy yechimini

$$y=C_1(x)y_1+C_2(x)y_2+\ldots+C_n(x)y_n$$

formula bo'yicha topish mumkin, bundagi $C_i(x)$ lar

$$\begin{cases}\sum_{i=1}^{n} C_i'(x)y_i^{(k)}=0, \quad (k=\overline{0,(n-2)}),\\ \sum_{i=1}^{n} C_i'(x)y_i^{(n-1)}=f(x)\end{cases} \qquad (3)$$

sistemadan topiladi.

Misol. Berilgan yechimlarning fundamental sistemalariga mos bir jinsli differensial tenglamalarni tuzing.

a) e^{-x}, e^{x}; *b)* x^3, x^4; *c)* e^x, x, x^3; *d)* $1, x, e^x$.

Yechish. a) Izlanayotgan tenglamaning ixtiyoriy yechimi (uni u deb belgilaymiz) e^{-x}, e^{x} larga chiziqli bog'liq bo'ladi. Shu sababli ularning Vronskiy determinanti

$$W(e^{-x},e^{x},y)=\begin{vmatrix} e^{-x} & e^{x} & y\\ -e^{-x} & e^{x} & y'\\ e^{-x} & e^{x} & y''\end{vmatrix}=0$$

Bundan $y''-y=0$ ko'rinishdagi izlanayotgan tenglama hosil bo'ladi.

b) Izlanayotgan tenglamani a) misoldagiga o'xshash tuzamiz:

$$W(x^3,x^4,y)=\begin{vmatrix} x^3 & x^4 & y\\ 3x^2 & 4x^3 & y'\\ 6x & 12x^2 & y''\end{vmatrix}=$$

$$=4x^6y''+36x^4y+6x^5y'-24x^4y-12x^5y'-3x^6y''=0$$

$$x^6y''-6x^5y'+12x^4y=0, \quad x^2y''-6xy'+12y=0.$$

c) Izlanayotgan tenglamaning istalgan yechimi e^x, x, x^3 larga chiziqli bog'liq bo'lgani uchun ularning Vronskiy determinanti $W(e^x,x,x^3,y)=0$ bo'ladi. Bu tenglamani ochib yozsak:

$$\begin{vmatrix} e^x & x & x^3 & y \\ e^x & 1 & 3x^2 & y' \\ e^x & 0 & 6x & y'' \\ e^x & 0 & 6 & y''' \end{vmatrix} = 0.$$

Chap tomondagi determinantdagi birinchi ustunda turgan e^x ni determinant belgisining oldiga chiqarib, so'ngra hosil qilingan determinantni oxirgi ustun elementlari bo'yicha yoysak, quyidagiga ega bo'lamiz:

$$e^x \begin{vmatrix} 1 & x & x^3 & y \\ 1 & 1 & 3x^2 & y' \\ 1 & 0 & 6x & y'' \\ 1 & 0 & 6 & y''' \end{vmatrix} =$$

$$= e^x \left((-1)^5 y \begin{vmatrix} 1 & 1 & 3x^2 \\ 1 & 0 & 6x \\ 1 & 0 & 6 \end{vmatrix} + (-1)^6 y' \begin{vmatrix} 1 & x & x^3 \\ 1 & 0 & 6x \\ 1 & 0 & 6 \end{vmatrix} + (-1)^7 y'' \begin{vmatrix} 1 & x & x^3 \\ 1 & 1 & 3x^2 \\ 1 & 0 & 6 \end{vmatrix} + (-1)^8 y''' \begin{vmatrix} 1 & x & x^3 \\ 1 & 1 & 3x^2 \\ 1 & 0 & 6x \end{vmatrix} \right) =$$

$$= e^x(-y(6x-6) + y'(6x^2-6x) - y''(6+3x^3-x^3-6x) + y'''(6x+3x^3-x^3-6x^2)) =$$

$$= e^x((2x^3-6x^2+6x)y''' - (2x^3-6x)y''' - (2x^3-6x+6)y'' + (6x^2-6x)y' -$$

$$-(6x-6)y) = 0$$

Hosil qilingan tenglamaning ikkala tomonini $2e^x$ga qisqartirsak, ushbu ko'rinishdagi

$$x(x^2-3x+3)y''' - (x^3-3x+3)y'' + 3x(x-1)y' - 3(x-1)y = 0$$

differensial tenglamaga ega bo'lamiz.

d) Izlanayotgan tenglama ushbu shaklda bo'ladi:

$$\begin{vmatrix} 1 & x & e^x & y \\ 0 & 1 & e^x & y' \\ 0 & 0 & e^x & y'' \\ 0 & 0 & e^x & y''' \end{vmatrix} = 0.$$

Bu tenglamaning chap tomonidagi determinantni c) misoldagiga o'xshash hisoblaymiz:

$$e^x \begin{vmatrix} 1 & x & 1 & y \\ 0 & 1 & 1 & y' \\ 0 & 0 & 1 & y'' \\ 0 & 0 & 1 & y''' \end{vmatrix} = e^x (-1)^5 y \begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} + (-1)^6 y' \begin{vmatrix} 1 & x & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} +$$

$$+(-1)^7 y'' \begin{vmatrix} 1 & x & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} + (-1)^8 y''' \begin{vmatrix} 1 & x & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = e^x(-y'' + y''') = 0$$

Hosil qilingan tenglamaning ikkala tomonini e^x ga qisqartirsak, quyidagiga ega bo'lamiz: $y''' - y'' = 0$.

Ushbu $y'' - p_1(x)y' + p_2(x)y = 0$ ikkinchi tartibli chiziqli differensial tenglamaning bitta yechimi $y_1 = y_1(x)$ ma'lum bo'lsa, uning umumiy yechimi

$$W(y_1, y) = \begin{vmatrix} y_1 & y \\ y_1' & y' \end{vmatrix} = Ce^{-\int p_1(x)dx}$$

ko'rinishdagi *Ostrogradskiy-Liuvill* formulasi yordamida topish mumkin. Bu formulaga asosan berilgan tenglamaning yechimi $y_1 y' - y_1' y = Ce^{-\int p_1(x)dx}$ tenglamaning yechimi bo'ladi.

Buni integrallash uchun uning har ikki tomonini $\frac{1}{y^2}$ ga ko'paytirib,

$\frac{d}{dx}\left(\frac{y}{y_1}\right) = \frac{y_1 y' - y_1' y}{y_1^2}$ tenglikni hisobga olsak, $\frac{y_1 y' - y_1' y}{y_1^2} = \frac{C}{y_1^2} e^{-\int p_1(x)dx}$ yoki $\frac{d}{dx}\left(\frac{y}{y_1}\right) = \frac{C}{y_1^2} e^{-\int p_1(x)dx}$ tenglamani hosil qilamiz. Bundan $\frac{y}{y_1} = \int \frac{Ce^{\int p_1(x)dx}}{y_1^2} dx + C_1$ yoki $y = C_1 y_1 + C_2 y_1 \int \frac{e^{-\int p_1(x)dx}}{y_1^2(x)} dx$ kelib chiqadi.

Misol. O'zgarmaslarni variatsiyalash usulidan foydalanib, ushbu $xy'' + (2x-1)y' = -4x^2$ (1) bir jinslimas tenglamaning umumiy yechimini toping.

Yechish. Avval berilgan tenglamani $y'' + \frac{2x-1}{x} y' = -4x \quad (x \neq 0)$

ko'rinishda yozib olamiz. Mos bir jinsli $y'' + \frac{2x-1}{x} y' = 0$ tenglamani $y' = p$ va $y'' = p'$ deb, o'zgaruvchilari ajraladigan

$$p' + \frac{2x-1}{x} p = 0$$

tenglamaga keltiriladi. O'zgaruvchilarni ajratib, so'ngra integrallasak, quyidagilarga ega bo'lamiz:

$$\frac{dp}{dx} = -\frac{2x-1}{x} p, \quad \frac{dp}{p} = \left(-2 + \frac{1}{x}\right) dx,$$

$$\ln|p| = -2x + \ln|x| + \ln|C_1|, \quad \ln\left|\frac{p}{C_1 x}\right| = -2x$$

$$p = C_1 x e^{-2x}$$

p ni y' ga almashtiramiz: $y' = C_1 x e^{-2x}$. Hosil qilingan tenglamani integrallasak, bir jinsli tenglamaning umumiy yechimi $y = C_1 e^{-2x}(2x+1) + C_2$ kelib chiqadi.

Berilgan tenglamaning umumiy yechimini $y = C_1(x) e^{-2x}(2x+1) + C_2(x)$ ko'rinishda izlaymiz. (3) ga ko'ra $C_1(x)$ va $C_2(x)$ funksiyalar

$$\begin{cases} C_1'(x)e^{-2x}(2x+1)+C_2'(x)=0 \\ C_1'(x)(-4x)e^{-2x}=-4x \end{cases}$$

sistemani qanoatlantiradi. Undan:

$$C_1'(x)=e^{2x}, \quad C_1(x)=\frac{1}{2}e^{2x}+C_1,$$

$$C_2'(x)=-2x-1, \quad C_2(x)=-x^2-x+C_2$$

Topilgan $C_1(x)$ va $C_2(x)$ funksiyalarni (2) ga qo'ysak berilgan (1) tenglamaning umumiy yechimi quyidagi

$$y=C_1\left(x+\frac{1}{2}\right)e^{-2x}+C_2-x^2-x$$

ko'rinishda bo'ladi.

Berilgan yechimlarning fundamental sistemalariga mos bir jinsli differensial tenglamalarni tuzing (2.1-2.8).

2.1. $y_1(x)=x, \quad y_2(x)=e^x.$ 2.2. $y_1(x)=1, \quad y_2(x)=\cos x.$

2.3. $y_1(x)=e^x, y_2(x)=x, y_3(x)=x^2.$ 2.4. $y_1(x)=e^x, \quad y_2(x)=shx, \; y_3(x)=chx$

2.5. $(2x+1)y''+(4x-2)y'-8y=0$ tenglamaning bitta $y_1=e^{-2x}$ xususiy yechimi ma'lum bo'lsa, uning umumiy yechimini toping.

2.6. $(4x^2-x)y''+2(2x-1)y'-4y=12x^2-6x$ tenglama $y_1=\frac{1}{x}$ xususiy yechimga ega. Bu tenglamaning umumiy yechimini toping.

2.7. $y''+tgxy'+\cos^2 xy=0$ tenglamaning bitta yechimi $y_1=\cos(\sin x)$ bo'lsa, uning $y(0)=0$, $y'(0)=1$ boshlang'ich shartlarini qanoatlantiradigan yechimini toping.

2.8. $x^3y'''-3x^2y''+6xy'-6y=0$ tenglamaning $y_1=x, y_2=x^2$ xususiy yechimlari yordamida uning umumiy yechimini toping.

O'zgarmaslarni variatsiyalash usulidan foydalanib, quyidagi bir jinslimas tenglamalarning umumiy yechimini toping (2.9-2.12).

2.9. $y''+y'tgx=\cos xctgx.$ 2.10. $x\ln xy''-y'=\ln^2 x$

2.11. $y''-y'=e^{2x}\cos e^x.$ 2.12. $xy''-(1+2x^2)y'=4x^3e^{x^2}.$

2.13. 6 *m* uzunlikdagi zanjir stol ustidan ishqalanishsiz sirpanib tushmoqda. Agar harakat zanjirning 1 *m* uzunlikdagi bo'lagi osilib turgan paytdan boshlansa, butun zanjir qancha vaqt ichida sirpanib tushadi?

2.14. Agar $t=0$ da $s=0$ va $t=5$ da $s=20$ bo'lsa va harakatning tezlanishi vaqtga bog'liq ravishda $a=1{,}2t$ formula bilan ifodalansa, nuqtaning harakat qonunini toping.

2.15. $m=1$ massali moddiy nuqta markaz tomon to'g'ri chiziqli harakat qilmoqda. Uni markazga k^2x teng bo'lgan kuch bilan itaradi. Bu yerda x-markazdan moddiy nuqtagacha bo'lgan oraliq. Agar $t=0$ bo'lganda $x=a$ va $\frac{dx}{dt}=ka$ bo'lsa, harakat qonunini toping.

3-§. O'zgarmas koeffitsientli chiziqli differensial tenglamalar.

n-tartibli o'zgarmas koeffitsientli chiziqli bir jinsli differensial tenglama

$$y^{(n)}+a_1y^{(n-1)}+...+a_ny=0 \qquad (1)$$

ko'rinishga ega. Bu yerda barcha $a_1,a_2,...,a_n$ koeffitsientlar haqiqiy o'zgarmas sonlardir. Bu holda xususiy yechimlarning fundamental sistemasini, binobarin, umumiy yechimini izlash sof algebraik amallarni bajarishga $-n$- darajali bitta algebraik tenglamani, ya'ni ushbu

$$r^n+ar^{n-1}+...+a_{n-1}r+a_n=0 \qquad (2)$$

xarakteristik tenglamani yechishga keltiriladi.

(2) tenglamaning har bir $m\geq 0$ karrali haqiqiy ildiziga umumiy yechimdagi

$$\left(C_1+C_2x+...+C_mx^{m-1}\right)e^{rx}$$

qo'shiluvchi mos keladi.

(2) tenglamaning har bir $m\geq 0$ karrali $\alpha\pm\beta i$ qo'shma kompleks ildizlar juftiga umumiy yechimda

$$e^{\alpha x}\left(\left(A_1+A_2x+...+A_{m-1}x^{m-1}\right)\cos\beta x+\left(B_1+B_2x+...+B_{m-1}x^{m-1}\right)\sin\beta x\right)$$

qo'shiluvchi mos keladi.

Bir jinslimas

$$y^{(n)}+a_1y^{(n-1)}+...+a_ny=f(x) \qquad (3)$$

tenglamaning y umumiy yechimini topish uchun, 2-§ dagi 2-teoremaga ko'ra uning birorta xususiy yechimini bilish yetarlidir, bunda unga mos bir jinsli (1) tenglamaning umumiy yechimi yuqorida keltirilgan 1) va 2) qoidalar bo'yicha topiladi.

Agar (3) ning o'ng tomonida ko'rsatkichli funksiyalar, sinuslar, kosinuslar va ko'phadlar yoki ularning butun ratsional kombinatsiyalari turgan bo'lsa, u holda uning xususiy yechimini topishda aniqmas koeffitsientlar usulini tatbiq qilish mumkin. Bu usul xususiy yechimning shaklini bilishga asoslangan. Tabiiyki, xususiy yechimning o'ng tomonning shakliga o'xshash shaklda izlash kerak. Biroq xususiy yechimning shakli tenglamaning chap tomoniga ham bog'liq bo'ladi.

a va b lar o'zgarmas sonlar, $P_n(x)$ va $Q_m(x)$ mos ravishda darajalari n va m bo'lgan ko'phadlar bo'lsin. (3) ning o'ng tomoni

$$f(x)=e^{ax}\left(P_n(x)\cos bx+Q_m(x)\sin bx\right) \qquad (4)$$

ko'rinishda bo'lsa, quyidagi hollar vujudga keladi:

1-hol. $a\pm ib$ (2) ning ildizi bo'lmaganda xususiy yechim

$$\tilde{y}=e^{ax}\left(\tilde{P}_l(x)\cdot\sin bx+\tilde{Q}_l(x)\cdot\cos bx\right) \qquad (5)$$

ko'rinishga ega, bu yerda $\widetilde{P}_l,\widetilde{Q}_l$ – l=max(n,m) darajali ko'phadlar.

2-hol. $a\pm ib$ (2) ning s karrali ildizi bo'lganida xususiy yechim

$$\tilde{y}=e^{ax}\cdot x^s\left(\tilde{P}_l(x)\cdot\sin bx+\tilde{Q}_l(x)\cdot\cos bx\right) \qquad (6)$$

ko'rinishga ega.

Har ikki holda ham $\tilde{P}_l, \tilde{Q}_l$ ko'phadlarning koeffitsientlari aniqmas koeffitsentlar usuli yordamida topiladi.

Misol. Quyidagi bir jinsli tenglamalarning umumiy yechimini toping.

a) $y''-5y'+6y=0$. *b*) $y'''+6y''+11y'+6y=0$.

c) $y''-10y'+25y=0$. d) $y''+2y'+5y=0$.

Yechish. a) Bu tenglama uchun $r^2-5r+6=0$ xarakteristik tenglama $r_1=2$, $r_2=3$ ildizlarga ega, shuning uchun umumiy yechim ushbu ko'rinishda bo'ladi: $y=C_1e^{2x}+C_2e^{3x}$

b) Berilgan tenglama uchun xarakteristik tenglama: $r^3+6r^2+11r+6=0$ ko'rinishda bo'ladi. Chap tomonini ko'paytuvchilarga ajratib, $(r+1)(r^2+5r+6)=0$ ni hosil qilamiz, bu yerdan $r_1=-1$, $r_2=-2$, $r_3=-3$.

Differensial tenglamaning umumiy yechimi:

$$y=C_1e^{-x}+C_2e^{-2x}+C_3e^{-3x}$$

c) $y''-10y'+25y=0$ tenglamaga mos xarakteristik tenglama $r^2-10r+25=0$ ikki karrali $r=5$ ildizga ega, binobarin, umumiy yechim quyidagicha bo'ladi:

$$y=(C_1+C_2x)e^{5x}$$

d) $y''+2y'+5y=0$ tenglamaga mos xarakteristik tenglama $r^2+2r+5=0$ ning ildizlari $r_{1,2}=-1\pm 2i$ demak, tenglamaning umumiy yechimi:

$$y=e^{-x}(C_1\cos 2x+C_2\sin 2x)$$

Masala. 1 g massali zarra A nuqta tomon shu nuqtadan zarracha bo'lgan qadar masofaga proporsional bo'lgan tortish kuchi ta'sirida to'g'ri chiziqli harakat qilmoqda. 1 sm masofada 0,1 Dina kuch ta'sir etadi. Muxit qarshiligi harakat tezligiga proporsional va u tezlik 1 sm/s bo'lganda 0,4 Dinaga teng. $t=0$ boshlang'ich momentda zarra *A* nuqtadan 10 sm o'ngroqda joylashgan va tezlik 0 ga teng. Yo'lning vaqtga bog'lanishini toping.

Yechish. Zarraga ikkita kuch ta'sir etadi: $F_1=k_1x$ va $F_2=k_2\frac{dx}{dt}$, bu yerda *x*-*t* momentda o'tilgan yo'l, $\frac{dx}{dt}$-tezlik. k_1 va k_2 larni

$$F_1\big|_{x=1}=0{,}1=k_1$$

$$F_2\big|_{x=1}=0{,}4=k_2$$

shartlardan topamiz: $k_1=0{,}1$; $k_2=0{,}4$.

F_1-tortish kuchi sifatida manfiy bo'ladi. U holda ushbu harakat tenglamasi:

$$m\frac{d^2x}{dt^2}=-F_1-F_2 \qquad m=1 \text{ da}$$

$\frac{d^2x}{dt^2}=-0{,}1x-0{,}4\frac{dx}{dt}$ yoki $\frac{d^2x}{dt^2}+0{,}4\frac{dx}{dt}+0{,}1x=0$ ko'rinishga ega bo'ladi.

Bu tenglamaga mos xarakteristik tenglama $r^2+0,4r+0,1=0$ bo'lib, uning ildizlari $r_{1,2}=-0,2\pm0,245i$ dan iborat. Demak, tenglamaning umumiy yechimi

$$x=e^{-0,2t}\left(C_1\cos0,245t+C_2\sin0,245t\right)\ \text{bo'ladi.}$$

$$x|_{t=0}=10,\quad \frac{dx}{dt}|_{t=0}=0\ \text{shartlar}$$

$$\begin{cases}C_1=10,\\ -0,2C_1+0,245C_2=0\end{cases}$$

tenglamalar sistemasiga olib keladi. Bu sistemadan $C_1=10,\quad C_2=8,16$ larni topamiz. Demak, izlangan yechim

$$x=e^{-0,2t}\left(10\cos0,245t+8,16\sin0,245t\right)$$

Misol. Quyidagi bir jinslimas tenglamalarning umumiy yechimini toping.

$a)\ y''-4y'+4y=x^2.$ $\qquad b)\ 7y''-y'=14x.$

$c)\ y''+4y'+3y=9e^{-3x}.$ $\qquad d)\ y''+4y'-2y=8\sin2x.$

$e)\ y''+y=4x\cos x.$ $\qquad f)\ y''+2y'+5y=e^{x}\cos2x.$

Yechish. a) Dastlab $y''-4y'+4y=x^2$ tenglamaga mos bir jinsli $y''-4y'+4y=0$ tenglamaning umumiy yechimini topamiz. Uning $r^2-4r+4=0$ xarakteristik tenglamasi $r_{1,2}=2$ karrali ildizga ega, shuning uchun umumiy yechim ushbu ko'rinishda yoziladi:

$$\bar{y}=e^{2x}\left(C_1+C_2x\right).$$

Agar a=0 va b=0 bo'lsa, (4) da $f(x)=P_n(x)$ ko'rinishda bo'ladi. Bu holda (5) ga ko'ra 0 soni xarakteristik tenglamaning ildizi bo'lmasa, xususiy yechimni $\tilde{y}=Q_n(x)$ ko'rinishda, 0 soni xarakteristik tenglamaning s karrali ildizi bo'lganida esa xususiy yechimni $\tilde{y}=x^sQ_n(x)$ ko'rinishda izlash kerak.

Berilgan tenglamaning o'ng tomoni 2-darajali ko'phad va 0 soni xarakteristik tenglamaning ildizi bo'lmagani sababli, xususiy yechimni $\tilde{y}=Ax^2+Bx+C$ ko'rinishda izlash lozim. Noma'lum A, B va C koeffitsientlarni topish uchun y_l ni va uning hosilalarini tenglamaga qo'yamiz hamda chap va o'ng tomondagi koeffitsientlarni taqqoslaymiz:

$$2A-4(2Ax+B)+4(Ax^2+Bx+C)=x^2,\quad A=\frac{1}{4},B=\frac{1}{2},C=\frac{3}{8}$$

Demak, xususiy yechim: $\tilde{y}=\frac{1}{8}\left(2x^2+4x+3\right)$.

Umumiy yechim: $y=\bar{y}+\tilde{y}=\left(C_1+C_2x\right)e^{2x}+\frac{1}{8}\left(2x^2+4x+3\right)$.

b) $7y''-y'=14x$ tenglamaga mos bir jinsli tenglamaning umumiy yechimi: $\bar{y}=C_1+C_2e^{\frac{x}{7}}$ chunki xarakteristik tenglamaning ildizlari $r_1=0,r_2=\frac{1}{7}$. 0 soni xarakteristik tenglamaning oddiy ildizi bo'lgani uchun xususiy yechimni

$\tilde{y}=x(Ax+B)$ ko'rinishda izlash kerak. Tegishli tenglamalardan A, B larni topamiz: A=-7, B=-98.

Demak, xususiy yechim: $\tilde{y}=C_1+C_2e^{\frac{x}{7}}-7x^2-98x$.

c) $y''+4y'+3y=9e^{-3x}$ tenglamaga mos bir jinsli tenglamaning umumiy yechimini osongina topamiz: $\bar{y}=C_1e^{-3x}+C_2e^{-x}$. Agar b=0 bo'lsa, (4) ifoda $f(x)=e^{ax}P_n(x)$ ko'rinishda bo'ladi. Bu holda a soni xarakteristik tenglamaning ildizi bo'lmasa, xususiy yechimni (5) formulaga ko'ra $\tilde{y}=e^{ax}Q_n(x)$ ko'rinishda, a soni xarakteristik tenglamaning s karrali ildizi bo'lganda esa xususiy yechimni (6) formulaga ko'ra $\tilde{y}=x^se^{ax}Q(x)$ ko'rinishda izlash kerak.

Berilgan tenglamaning o'ng tomoni $f(x)=9e^{-3x}$ ko'rinishida bo'lib, a=-3 xarakteristik tenglamaning oddiy ildizi bulgani uchun xususiy yechimni $\tilde{y}=Axe^{-3x}$ shaklda izlaymiz. Bu yechimni tenglamaga qo'yib, $-2Ae^{-3x}=9e^{-3x}$ni hosil qilamiz, bu yerdan $A=-\frac{9}{2}$. Demak, xususiy yechim: $\tilde{y}=-\frac{9}{2}xe^{-3x}$, umumiy yechim: $y=C_1e^{-3x}+C_2e^{-x}-\frac{9}{2}xe^{-3x}$.

d) $y''+4y'-2y=8\sin 2x$ tenglamaga mos bir jinsli tenglamaning umumiy yechimi:

$$\bar{y}=C_1e^{(-2-\sqrt{6})x}+C_2e^{(-2+\sqrt{6})x}$$

Berilgan tenglamaning o'ng tomoni $f(x)=e^{0x}P_0(x)\sin 2x$ ko'rinishida bo'lib, $a+bi=2i$ xarakteristik tenglamaning ildizi bo'lmagani uchun xususiy yechimni $\tilde{y}=A\cos 2x-B\sin 2x$ shaklda izlaymiz. Bu ifodani berilgan tenglamaga qo'ysak,

$$(-6A+8B)\cos 2x-(6B+8A)\sin 2x=8\sin 2x$$

cos2x va sin2x oldidagi koeffitsientlarni tenglab, A va B larni topamiz: $A=-\frac{16}{25}$, $B=-\frac{12}{25}$. Demak, xususiy yechim $\tilde{y}=-\frac{16}{25}\cos 2x-\frac{12}{25}\sin 2x$, umumiy yechim $y=C_1e^{-(\sqrt{6}+2)x}+C_2e^{(\sqrt{6}+2)x}-\frac{16\cos 2x+12\sin 2x}{25}$.

e) $y''+y=4x\cos x$ tenglamaga mos bir jinsli tenglamaning umumiy yechimi: $\bar{y}=C_1\cos x+C_2\sin x$. $a+bi=i$ xarakteristik tenglamaning oddiy ildizi bo'lgani uchun xususiy yechimni $\tilde{y}=x((Ax+B)\cos x+(Cx+D)\sin x)$ ko'rinishida izlaymiz. A, B, C, D lar uchun mos tenglamalarni yechib, A=0, B=1, C=1, D=1 larni topamiz. Demak, xususiy yechim: $\tilde{y}=x\cos x+x^2\sin x$, umumiy yechim:

$$y=C_1\cos x+C_2\sin x+x\cos x+x^2\sin x.$$

f) $y''+2y'+5y=e^{-x}\cos 2x$ tenglamaga mos $y''+2y'+5y=0$ tenglama uchun $r^2+2r+5=0$ xarakteristik tenglama $r_{1,2}=-1\pm 2i$ ildizlarga ega. Shuning uchun, mos bir jinsli tenglamaning umumiy yechimi:

$\bar{y}=(C_1\cos 2x+C_2\sin 2x)e^{-x}$, $a+bi=-1+2i$ son xarakteristik tenglamaning oddiy ildizi bo'lgani uchun xususiy yechimni $\tilde{y}=x(A\cos 2x+B\sin 2x)e^{-x}$ ko'rinishda izlaymiz. Noma'lum A va B koeffitsientlarni topish uchun $\tilde{y}$ ni va uning hosilalarini tenglamaga qo'yib va e^{-x} ga qisqartirib, bu yerdan A=0, $B=\frac{1}{4}$. Demak, $\tilde{y}=\frac{1}{4}xe^{-x}\sin 2x$. Shunday qilib, umumiy yechim:

$$y=(C_1\cos 2x+C_2\sin 2x)e^{-x}+\frac{1}{4}xe^{-x}\sin 2x.$$

Misol. Ixtiyoriy o'zgarmaslarni variatsiyalash usulini tatbiq etib, quyidagi tenglamalarni integrallang.

$a)\quad y''+y=\frac{1}{\cos^3 x};\qquad b)\quad y''-y'=e^{2x}\cos e^x;\qquad c)\quad y'''+y''=\frac{x-1}{x^2}.$

Yechish. a) Mos bir jinsli $y''+y=0$ tenglamaning umumiy yechimi: $\bar{y}=C_1(x)\cos x+C_2(x)\sin x$. O'zgarmaslarni variatsiyalab, xususiy yechimni $\tilde{y}=C_1(x)\cos x+C_2(x)\sin x$ ko'rinishda izlaymiz. $C_1(x)$ va $C_2(x)$ lar (II bob 2-§ dagi (3) ga ko'ra)

$$\begin{cases} C_1'(x)\cos x+C_2'(x)\sin x=0, \\ -C_1'(x)\sin x+C_2'(\delta)\cos x=\dfrac{1}{\cos^3 x}. \end{cases}$$

sistemani qanoatlantiradi. Bu sistemadan $C_2'(x)=\frac{1}{\cos^2 x}$ va $C_1'(x)=-\frac{\sin x}{\cos^3 x}$ kelib chiqadi. Integrallash ushbuni beradi:

$$C_1(x)=-\frac{1}{2\cos^2 x},\quad C_2(x)=tgx.$$

Demak, $\tilde{y}=-\dfrac{1}{2\cos x}+\dfrac{\sin^2 x}{\cos x}=\dfrac{-1+2\sin^2 x}{2\cos x}=-\dfrac{\cos 2x}{2\cos x}$.

Umumiy yechim: $y=\bar{y}+\tilde{y}=C_1\cos x+C_2\sin x-\dfrac{\cos 2x}{2\cos x}$.

b) $y''-y'=e^{2x}\cos e^x$. Eng oldin mos bir jinsli $y''-y'=0$ tenglamaning umumiy yechimini topamiz: $r^2-r=0, r_1=0, r_2=1, y=C_1(x)+C_2(x)e^x$. Xususiy yechimni $\tilde{y}=C_1(x)+C_2(x)e^x$ ko'rinishda izlaymiz. Bunda $C_1(x)$ va $C_2(x)$ lar ushbu

$$\begin{cases} C_1'(x)+C_2'(x)e^x=0 \\ C_1'(x)0+C_2'(x)e^x=e^{2x}\cos e^x \end{cases}$$

sistemadan aniqlanadi. Bu sistemadan quyidagilarga ega bo'lamiz:

$C_1'(x)=-e^{2x}\cos e^x$, $C_2'(x)=e^x\cos e^x$,

$C_1(x)=-e^x\sin e^x-\cos e^x, C_2(x)=\sin e^x$.

Bundan berilgan tenglamaning umumiy yechimini topamiz:

$$\tilde{y} = -e^x \sin e^x - \cos e^x + e^x \sin e^x = -\cos e^x,$$

$$y = \bar{y} + \tilde{y} = C_1 + C_2 e^x - \cos e^x.$$

c) $y''' + y'' = \dfrac{x-1}{x^2}$. Berilgan tenglamaga mos bir jinsli $y''' + y'' = 0$ tenglamaning umumiy yechimi:

$$r^3 + r^2 = 0, \quad r^2(r+1) = 0, r_{1,2} = 0. r_3 = -1, y = C_1 + C_2 x + C_3 e^{-x}.$$

Xususiy yechimni $\tilde{y} = C_1(x) + C_2(x)x + C_3(x)e^{-x}$ ko'rinishda izlaymiz. $C_1(x)$, $C_2(x)$, $C_3(x)$larni

$$\begin{cases} C_1'(x) + C_2'(x) + C_3'(x)e^{-x} = 0 \\ 0 + C_2'(x)1 - C_3'(x)e^{-x} = 0 \\ 0 + C_2'(x)0 + C_3'(x)e^{-x} = \dfrac{x-1}{x^2} \end{cases}$$ sistemadan topamiz:

$$C_1'(x) = -1 + \frac{1}{x^2}, \qquad C_1(x) = -x - \frac{1}{x} + C_1,$$

$$C_2'(x) = \frac{1}{x} - \frac{1}{x^2}, \qquad C_2(x) = \ln|x| + \frac{1}{x} + C_2,$$

$$C_3'(x) = \frac{x-1}{x^2} e^x, \qquad C_3(x) = \frac{1}{x} e^x + C_3.$$

Topilgan ifodalarni hisobga olib, umumiy yechimni yozamiz:

$$\tilde{y} = -x - \frac{1}{x} + x\ln|x| + 1 + \frac{1}{x} = 1 - x + x\ln|x|,$$

$$y = \bar{y} + \tilde{y} = C_1 + C_2 x + C_3 e^{-x} + 1 - x + x\ln|x|.$$

Quyidagi bir jinsli tenglamalarning umumiy yechimini toping (3.1-3.6).

3.1. $y'' + 3y' = 0.$ 3.2. $y'' + 4y' - 5y = 0.$

3.3. $y'' - 16y' + 64y = 0.$ 3.4. $y'' - 4y' + 5y = 0.$

3.5. $\dfrac{4y' - y}{y''} = 3.$ 3.6. $y''' + 8y = 0.$

3.7. $y'' + 4y = 0$ tenglamaning $M(0,1)$ nuqtadan o'tuvchi va shu nuqtada y-x=1 o'g'ri chiziqqa urinuvchi integral egri chizig'ini toping.

Quyidagi bir jinslimas tenglamalarning umumiy yechimini toping (3.8-3.19).

3.8. $y'' + 8y' = 8x.$ 3.9. $y'' + 2y' + y = -2.$

3.10. $y'' + y' + y = (x + x^2)e^x.$ 3.11. $y'' + 3y' = 3xe^{-3x}.$

3.12. $y'' + 4y' - 2y = 8\sin 2x.$ 3.13. $y'' + y = x^2 \sin x.$

3.14. $y'' - 4y' + 4y = 8e^{-2x}.$ 3.15. $y'' + 2y' + 5y = e^{x}\sin 2x.$

3.16. $y'' + 4y' + 3y = 9e^{3x}.$ 3.17. $2y'' + 5y' = 29\cos x.$

3.18. $y'' + 2y' = 4e^x(\sin x + \cos x)$ 3.19. $y'' + 4y' + 5y = 10e^{-2x}\cos x.$

Quyidagi tenglamalarning berilgan boshlang'ich shartlarni qanoatlantiruvchi yechimini toping (3.20-3.27).

3.20. $y''-4y'+3y=0,\ y(0)=6,\ y'(0)=10$. 3.21. $y''-2y'+2y=0,\ y(0)=0,\ y'(0)=1$.

3.22. $y''-2y'+3y=0,\ y(0)=1,\ y'(0)=3$. 3.23. $y''-5y'+4y=0,\ y(0)=1,\ y'(0)=1$.

3.24. $y''+9y'=6e^{3x},\ y(0)=0,\ y'(0)=0$. 3.25. $y''-4y'+5y=2x^2e^x,\ y(0)=2,\ y'(0)=3$.

3.26. $y''-2y'=2e^x,\ y(1)=-1,\ y'(1)=0$.

3.27. $y''+4y=4(\sin 2x+\cos 2x),\ y(\pi)=y'(\pi)=2\pi$

Ixtiyoriy o'zgarmaslarni variatsiyalash usulini tatbiq etib, quyidagi tenglamalarni integrallang (3.28-3.33).

3.28. $y''+y=\dfrac{1}{\cos x}$. 3.29. $y''+9y=\dfrac{1}{\sin 3x}$.

3.30. $y''-2y'+y=\dfrac{e^x}{x}$. 3.31. $y''+2y'+y=\dfrac{1}{xe^x}$.

3.32. $y''+y=ctgx$. 3.33. $y''+4y=\dfrac{1}{\sin^2 x}$.

3.34. Massasi 200 g bo'lgan yuk prujinaga osilgan. Yuk 2 sm pastga tortilib, keyin qo'yib yuborilgan. Agar yuk v=1sm/s tezlik bilan harakat qilsa, muxit unga $10^{-3}N$ qarshilik ko'rsatadi. Prujinaning qarshilik kuchi uni 2 sm cho'zganda 100N ga teng. Prujinaning massasini hisobga olmay, muxit qarshiligi harakat tezligiga proporsional bo'lgan holda yukning harakat qonunini toping.

3.35. 10 kg massali jismga uni muvozanat holatiga qaytarish uchun harakat qiluvchi elastik kuch ta'sir etadi. Kuch siljishga proporsional va u yuk 1 m siljishga 20N ga teng. Muhit qarshiligi harakat tezligiga proporsional uchta tebranishdan so'ng amplituda 10 baravar kamayadi. Tebranishlar davrini toping.

II – bobga doir misol va masalalarning javoblari

1.1. $y=x^2\ln x+C_1x^2+C_2x+C_3$. 1.2. $y=-\ln(1+\mathrm{tg}C_1\mathrm{tg}x)+\dfrac{1}{2}\ln(1+\mathrm{tg}^2x)+C_2$.

1.3. $y=(x+C_1)\ln(x+C_1)-x-C_1+C_2x+C_3$. 1.4. $y=C_1e^{ax}+C_2e^{-ax}$.

1.5. $y=\pm\dfrac{2}{3}(x+C_1)^{3/2}+C_2$. 1.6. $x=C_1+C_2y+C_3y^2$

1.7. $y=e^x(x-1)+C_1x+C_2;\ y=e^x(x-1)$. 1.8. $y+C_1\ln y=x+C_2;\ y=-1$.

1.9. $y=C_2+C_1\sin x-x-\dfrac{1}{2}\sin 2x;\ y=2\sin x-x-\dfrac{1}{2}\sin 2x-1$.

1.10. $y=C_2-a\cos(x+C_1);\ y=-1\pm a(1-\cos x)$.

1.11. Zanjir chiziq. 1.12. Parabola. 1.13. $S=\dfrac{m}{3k}\left(\sqrt{\left(\dfrac{2k}{m}t+C\right)^3}-\sqrt{C^3}\right)$.

1.14. $v=\sqrt{\dfrac{mgv_0^2}{mg+kv_0^2}}$.

2.1. $(x-1)y''-xy'+y=0$. 2.2. $y''-y'ctgx=0$. 2.3. $(x^2-2x+2)y'''-x^2y''+2xy'-2y=0$.

2.4. $y''-y=0$. 2.5. $y=C_1e^{-2x}+C_2(4x^2+1)$. 2.6. $y=C_1(2x-1)+\frac{C_2}{x}+x^2$.

2.7. $y=C_1\cos(\sin x)+C_2\sin(\sin x)$. 2.8. $y=C_1x+C_2x^2+C_3x^3$.

2.9. $y=C_1+C_2\sin x+\sin x\cdot\ln|\sin x|$. 2.10. $y=C_1(\ln x-1)+C_2+x(\ln^2 x-2\ln x-2)$.

2.11. $y=C_1e^x+C_2-\cos e^x$ 2.12. $y=C_1e^{x^2}+C_2+(x^2-1)e^{x^2}$.

2.13. $\frac{d^2x}{dt^2}=\frac{g}{m}x$, zanjirning osilgan bo'lagi, $t=\sqrt{0,6}\ln(6+\sqrt{35})$s. 2.14. $s=0,2t^3-t$.

2.15. $x=ae^{kt}$.

$y=C_1e^x+C_2e^x$

3.1. $y=C_1+C_2e^{-3x}$. 3.2. $y=C_1e^x+C_2e^{-5x}$. 3.3. $y=(C_1+C_2x)e^{8x}$.

3.4. $y=e^{2x}(C_1\cos x+C_2\sin x)$ 3.5. $y=C_1e^x+C_2e^{x/3}$.

3.6. $y=C_1e^{-2x}+e^x(C_2\cos 3x+C_3\sin 3x)$. 3.7. $y=\cos 2x+\frac{1}{2}\sin 2x$.

3.8. $y=C_1+C_2e^{-8x}+\frac{x^2}{2}-\frac{x}{8}$. 3.9. $y=(C_1+C_2x)e^{-x}-2$.

3.10. $y=e^{-\frac{x}{2}}\left(C_1\sin\frac{\sqrt{3}}{2}x+C_2\cos\frac{\sqrt{3}}{2}x\right)+\left(\frac{x^3}{2}-\frac{x}{3}+\frac{1}{3}\right)e^x$.

3.11. $y=C_1+C_2e^{-3x}-\left(\frac{x^2}{2}+\frac{x}{3}\right)e^{-3x}$.

3.12. $y=C_1e^{-(\sqrt{6}+2)x}+C_2e^{(\sqrt{6}-2)x}-\frac{12\sin 2x+16\cos 2x}{25}$.

3.13. $y=\left(C_1+\frac{x}{4}-\frac{x^3}{6}\right)\cos x+\left(C_2+\frac{x^2}{4}\right)\sin x$.

3.14. $y=(C_1+C_2x)e^{-2x}+4x^2e^{-2x}$. 3.15. $y=e^{-x}(C_1\cos 2x+C_2\sin 2x)-\frac{1}{4}xe^{-x}\cos 2x$.

3.16. $y=C_1e^{-3x}+C_2e^{-x}-\frac{9}{2}xe^{-3x}$. 3.17. $y=C_1+C_2e^{-5x/2}+5\sin x-2\cos x$.

3.18. $y=C_1+C_2e^{-2x}+\frac{1}{5}e^x(6\sin x-2\cos x)$.

3.19. $y=e^{-2x}(C_1\cos x+C_2\sin x)+5xe^{-2x}\sin x$.

3.20. $y=4e^x+2e^{3x}$. 3.21. $y=e^x\sin x$. 3.22. $y=e^x(\cos\sqrt{2}x+\sqrt{2}\sin\sqrt{2}x)$. 3.23. $y=e^x$

3.24. $y=-\frac{1}{3}(\cos 3x+\sin 3x-e^{3x})$. 3.25. $y=e^{2x}(\cos x-2\sin 2x)+(x+1)^2e^x$.

3.26. $y=e^{2x-1}-2e^x+e-1$. 3.27. $y=3\pi\cos 2x+\frac{1}{2}\sin 2x+x(\sin 2x-\cos 2x)$.

3.28. $y=C_1\cos x+C_2\sin x+x\sin x+\cos x\ln|\cos x|$.

3.29. $y=C_1\cos 3x+C_2\sin 3x-\frac{1}{9}x\cos x+\frac{1}{9}\sin x\ln|\sin 3x|$.

3.30. $y=C_1e^x+C_2xe^x+xe^x\ln|x|$. 3.31. $y=C_1e^{-x}+C_2xe^{-x}+xe^{-x}\ln|x|$.

3.32. $y=C_1\cos x+C_2\sin x+\sin x\ln|tg\frac{x}{2}|$.

3.33. $y = C_1 \cos 2x + C_2 \sin 2x - \cos 2x \ln|\sin x| - (x + 0,5ctgx)\sin 2x$.

3.34. $S = e^{-0,245t}(2\cos 156,6t + 0,00313\sin 156,6t)$.

3.35. $T = \frac{1}{3}\sqrt{\frac{5}{g}}\sqrt{(6\pi)^2 + \ln^2 10}$.

1-§. Differensial tenglamalarni analitik yechish

Differensial tenglamaning umumiy yechimini topishda *Maple* da `dsolve(de,y(x))` buyrug'i qo'llaniladi, bu yerda `de` – differensial tenglama, `y(x)` – noma'lum funksiya. Differensial tenglamada ishtirok etadigan hosilalalarni ifodalashda `diff` buyrug'idan foydalaniladi. Masalan, $y''+y=x$ tenglama `diff(y(x),x$2)+y(x)=x` ko'rinishda yoziladi.

Maple da umumiy yechimda ishtirok etadigan ixtiyoriy doimiylar $\_C1$, $\_C2$, ... kabi belgilanadi.

Misol. a) $y'+y\cos x=\sin x\cos x$; b) $y''-2y'+y=\sin x+e^{-x}$ tenglamalarning umumiy yechimlarini toping.

Yechim.

a)

```
> restart;
> de:=diff(y(x),x)+y(x)*cos(x)=sin(x)*cos(x);
```

$$de:=\left(\frac{\partial}{\partial x}y(x)\right)+y(x)\cos(x)=\sin(x)\cos(x)$$

```
> dsolve(de,y(x));
```

$$y(x)=\sin(x)-1+e^{(-\sin(x))}\_C1$$

Demak, umumiy yechim : $y(x)=\sin(x)-1+e^{(-\sin(x))}\_C1$.

b)

```
> restart;
> de:=diff(y(x),x$2)-2*diff(y(x),x)+y(x)=sin(x)+exp(-x);
```

$$de:=\left(\frac{\partial^2}{\partial x^2}y(x)\right)-2\left(\frac{\partial}{\partial x}y(x)\right)+y(x)=\sin(x)+e^{(-x)}$$

```
> dsolve(de,y(x));
```

$$y(x)=\_C1e^{x}+\_C2e^{x}x+\frac{1}{2}\cos(x)+\frac{1}{4}e^{(-x)}$$

Demak, umumiy yechim : $y(x)=\_C1e^{x}+\_C2e^{x}x+\frac{1}{2}\cos(x)+\frac{1}{4}e^{(-x)}$.

Misol. $y''+k^2y=\sin(qx)$ tenglamaning $q\neq k$ va $q=k$ (rezonans) hollarda umumiy yechimini toping.

Yechim.

```
> restart; de:=diff(y(x),x$2)+k^2*y(x)=sin(q*x);
```

$$de:=\left(\frac{\partial^2}{\partial x^2}y(x)\right)+k^2y(x)=\sin(qx)$$

```
> dsolve(de,y(x));
```

$$y(x)=\frac{\left(-\frac{1}{2}\frac{\cos((k+q)x)}{k+q}+\frac{1}{2}\frac{\cos((k-q)x)}{k-q}\right)\sin(kx)}{k}-$$

$$\frac{\left(\frac{1}{2}\frac{\sin((k-q)x)}{k-q}-\frac{1}{2}\frac{\sin((k+q)x)}{k+q}\right)\cos(kx)}{k}+\_C1\sin(kx)+\_C2\cos(kx)$$

Endi rezonans holini ko'ramiz:

```
> q:=k: dsolve(de,y(x));
```

$$y(x)=-\frac{1}{2}\frac{\cos(kx)^2\sin(kx)}{k^2}-\frac{\left(-\frac{1}{2}\cos(kx)\sin(kx)+\frac{1}{2}kx\right)\cos(kx)}{k^2}+$$

$$\_C1\sin(kx)+\_C2\cos(kx)$$

Differensial tenglamaning fundamental yechimlarini topishda *Maple* da **dsolve(de,y(x), output=basis)** buyrug'i qo'llaniladi.

Misol. $y^{(4)}+2y''+y=0$ tenglamaning fundamental yechimlarini topamiz:

Yechim.

```
> de:=diff(y(x),x$4)+2*diff(y(x),x$2)+y(x)=0;
```

$$de:=\left(\frac{\partial^4}{\partial x^4}y(x)\right)+2\left(\frac{\partial^2}{\partial x^2}y(x)\right)+y(x)=0$$

```
> dsolve(de, y(x), output=basis);
```

$$[\cos(x),\sin(x),x\cos(x),x\sin(x)]$$

Demak, fundamental yechimlar: $[\cos(x),\sin(x),x\cos(x),x\sin(x)]$.

Koshi masalasini yechishda **dsolve({de,cond}, y(x))** buyrug'i qullaniladi, bu yerda **cond** – boshlang'ich shartlar. Yuqori tartibli tenglamalar uchun boshlang'ich shartlarda ishtirok etgan hosilalalar uchun $D(y)$ (birinchi tartibli hosila uchun) va $(D@@n)(y)$ (n-chi tartibli hosila uchun) operatorlari qo'llaniladi. Masalan , $y'(1)=0$, $y''(0)=2$ shartlar mos ravishda $D(y)(1)=0$ va $(D@@2)(y)(0)=2$ kabi yoziladi.

Misol. Koshi masalasini yeching: $y^{(4)}+y''=2\cos x$, $y(0)=-2$, $y'(0)=1$, $y''(0)=0$, $y'''(0)=0$.

Yechim.

```
> de:=diff(y(x),x$4)+diff(y(x),x$2)=2*cos(x);
```

$$de:=\left(\frac{\partial^4}{\partial x^4}y(x)\right)+\left(\frac{\partial^2}{\partial x^2}y(x)\right)=2\cos(x)$$

```
> cond:=y(0)=-2, D(y)(0)=1, (D@@2)(y)(0)=0,(D@@3)(y)(0)=0;
```

$$cond:=y(0)=-2,\ D(y)(0)=1,\ (D^{(2)})(y)(0)=0,\ (D^{(3)})(y)(0)=0$$

```
> dsolve({de,cond},y(x));
```

$$y(x)=-2\cos(x)-x\sin(x)+x$$

Demak, Koshi masalasi $y(x)=-2\cos(x)-x\sin(x)+x$ yechimga ega.

2-§. Differensial tenglamalarni taqribiy yechish va tasvirlash

Ko'pincha differensial tenglamalarni yechimlarini analitik ko'rinishda topish imkoniyati bo'lmaydi. Bunday hollarda yechimlarni *Maple* dasturi Teylor formulasi shaklida aniqlashga imkon beradi.

Bunda *Maple* da `dsolve(de,y(x), series)` buyrug'i qullaniladi. Bundan oldin `Order:=n` buyrug'i yordamida ko'phadning darajasini belgillash mo'mkin.

Misol. $y' = y + xe^y$, $y(0) = 0$ Koshi masalasini taqribiy yeching .

Yechim. n =5 deb olamiz.

```
> restart; Order:=5:
> dsolve({diff(y(x),x)=y(x)+x*exp(y(x)),y(0)=0},y(x),
type=series);
```

$$y(x) = \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{6}x^4 + O(x^5)$$

Boshlang'ich shartlar berilmagan holna qaraylik.

Misol. $y''(x)-y^3(x)=e^{-x}\cos x$.

Yechim. n =4 deb olamiz.

```
> restart; Order:=4: de:=diff(y(x),x$2)- y(x)^3=
exp(-x)*cos(x):
> f:=dsolve(de,y(x),series);
```

$$f := y(x) = y(0) + D(y)(0)x + \left(\frac{1}{2}y(0)^3 + \frac{1}{2}\right)x^2 + \left(\frac{1}{2}y(0)^2 D(y)(0) - \frac{1}{6}\right)x^3 + O(x^4)$$

Endi $y(0)=1, y'(0)=0$ boshlang'ich shartlarni beramiz:

```
> y(0):=1: D(y)(0):=0:f;
```

$$y(x) = 1 + x^2 - \frac{1}{6}x^3 + O(x^4)$$

Qulaylik uchun taqribiy va aniq yechimlarni bitta chizmada bir-biri bilan solishtirish maqsadga muvofiq. Buni $y''' - y' = 3(2 - x^2)\sin x$, $y(0) = 1$, $y'(0) = 1$, $y''(0) = 1$ Koshi masalasida kuzataylik:

```
> restart; Order:=6:
> de:=diff(y(x),x$3)-diff(y(x),x)=3*(2-x^2)*sin(x);
```

$$de := \left(\frac{\partial^3}{\partial x^3}y(x)\right) - \left(\frac{\partial}{\partial x}y(x)\right) = 3(2 - x^2)\sin(x)$$

```
> cond:=y(0)=1, D(y)(0)=1, (D@@2)(y)(0)=1;
```

$$cond := y(0)=1,\ D(y)(0)=1,\ D^{(2)}(y)(0)=1$$

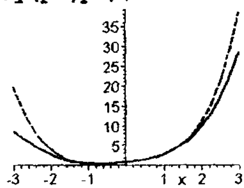
```
> dsolve({de,cond},y(x));
```

$$y(x) = \frac{21}{2}\cos(x) - \frac{3}{2}x^2\cos(x) + 6x\sin(x) - 12 + \frac{7}{4}e^x + \frac{3}{4}e^{(-x)}$$

```
> y1:=rhs(%):
> dsolve({de,cond},y(x), series);
```

$$y(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{7}{24}x^4 + \frac{1}{120}x^5 + O(x^6)$$

```
> convert(%,polynom): y2:=rhs(%):
> p1:=plot(y1,x=-3..3,thickness=2,color=black):
> p2:=plot(y2,x=-3..3, linestyle=3,thickness=2,
  color=blue):
> with(plots): display(p1,p2);
```

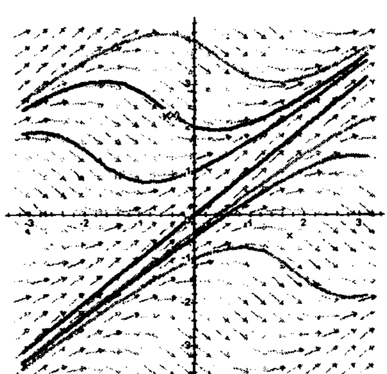


Maple izoklinalar yordamida bitta rasmda bir nechta Koshi masalalarning integral egri chiqlarini yasashga ham imkoniyat beradi.

Masalan, $y' = \cos(x - y)$ tenglama uchun $y(0)=0$, $y(0)=1$, $y(0)=-1$, $y(0)=-0.5$, $y(0)=4$, $y(0)=2$, $y(5)=2$ boshlang'ich shartlarga mos bo'lgan 7 ta integral chiziqlarni turli ranglarda (black, gold, red, green, blue, coral, magenta) tasvirlasa bo'ladi:

```
> restart:
> with(DEtools):
> diff(y(x),x) = cos(-y(x)+x);
> phaseportrait (D(y)(x)=cos(y(x)-x),y(x),x=-Pi..Pi,[[y(0)=0],
[y(0)=1], [y(0)=-1], [y(0)=-.5], [y(0)=4], [y(0)=2],[y(5)=2]],
>color=cos(y-x), linecolor=[black,gold,red,green,blue, coral,
magenta],arrows=medium);
```

$$\frac{\partial}{\partial x} y(x) = \cos(-y(x) + x)$$



Mustaqil ish uchun individual vazifalar.

1. Differensial tenglamaning umumiy integralini toping.

1.1. $4xdx-3ydy=3x^2ydy-2xy^2dx.$ 1.2. $x\sqrt{1+y^2}+yy'\sqrt{1+x^2}=0.$

1.3. $\sqrt{4+y^2}dx-ydy=x^2ydy.$ 1.4. $\sqrt{3+y^2}dx-ydy=x^2ydy.$

1.5. $6xdx-6ydy=2x^2ydy-3xy^2dx.$ 1.6. $x\sqrt{3+y^2}dx+y\sqrt{2+x^2}dy=0.$

1.7. $(e^{2x}+5)dy+ye^{2x}dx=0.$ 1.8. $y'y\sqrt{\frac{1-x^2}{1-y^2}}+1=0.$

1.9. $6xdx-6ydy=3x^2ydy-2xy^2dx.$ 1.10. $x\sqrt{5+y^2}dx+y\sqrt{4+x^2}dy=0.$

1.11. $y(4+e^x)dy-e^x dx=0.$ 1.12. $\sqrt{4-x^2}y'+xy^2+x=0.$

1.13. $2xdx-2ydy=x^2ydy-2xy^2dx.$ 1.14. $x\sqrt{4+y^2}dx+y\sqrt{1+x^2}dy=0.$

1.15. $(e^x+8)dy-ye^x dx=0.$ 1.16. $\sqrt{5+y^2}+y'y\sqrt{1-x^2}=0.$

1.17. $6xdx-ydy=yx^2dy-3xy^2dx.$ 1.18. $y\ln y+xy'=0.$

1.19. $(1+e^x)y'=ye^x.$ 1.20. $\sqrt{1-x^2}y'+xy^2+x=0.$

1.21. $6xdx-2ydy=2yx^2dy-3xy^2dx.$ 1.22. $y(1+\ln y)+xy'=0.$

1.23. $(3+e^x)yy'=e^x.$ 1.24. $\sqrt{3+y^2}+\sqrt{1-x^2}yy'=0.$

.25. $xdx-ydy=yx^2dy-xy^2dx.$ 1.26. $\sqrt{5+y^2}dx+4(x^2y+y)dy=0.$

.27. $(1+e^x)yy'=e^x.$ 1.28. $3(x^2y+y)dy+\sqrt{2+y^2}dx=0.$

.29. $2xdx-ydy=yx^2dy-xy^2dx.$ 1.30. $2x+2xy^2+\sqrt{2-x^2}y'=0.$

2. Differensial tenglamaning umumiy integralini toping.

2.1. $y' = \frac{y^2}{x^2} + 4\frac{y}{x} + 2.$

2.2. $xy' = \frac{3y^3 + 2yx^2}{2y^2 + x^2}.$

2.3. $y' = \frac{x+y}{x-y}.$

2.4. $xy' = \sqrt{x^2 + y^2} + y.$

2.5. $2y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 3.$

2.6. $xy' = \frac{3y^3 + 4yx^2}{2y^2 + 2x^2}.$

2.7. $y' = \frac{x+2y}{2x-y}.$

2.8. $xy' = 2\sqrt{x^2 + y^2} + y.$

2.9. $3y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4.$

2.10. $xy' = \frac{3y^3 + 6yx^2}{2y^2 + 3x^2}.$

2.11. $y' = \frac{x^2 + xy - y^2}{x^2 - 2xy}.$

2.12. $xy' = \sqrt{2x^2 + y^2} + y.$

2.13. $y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 6.$

2.14. $xy' = \frac{3y^3 + 8yx^2}{2y^2 + 4x^2}.$

2.15. $y' = \frac{x^2 + 2xy - y^2}{2x^2 - 2xy}.$

2.16. $xy' = 3\sqrt{x^2 + y^2} + y.$

2.17. $2y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 8.$

2.18. $xy' = \frac{3y^3 + 10yx^2}{2y^2 + 5x^2}.$

2.19. $y' = \frac{x^2 + 3xy - y^2}{3x^2 - 2xy}.$

2.20. $xy' = 3\sqrt{2x^2 + y^2} + y.$

2.21. $y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 12.$

2.22. $xy' = \frac{3y^3 + 12yx^2}{2y^2 + 6x^2}.$

2.23. $y' = \frac{x^2 + xy - 3y^2}{x^2 - 4xy}.$

2.24. $xy' = 2\sqrt{3x^2 + y^2} + y.$

2.25. $4y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 5.$

2.26. $xy' = \frac{3y^3 + 14yx^2}{2y^2 + 7x^2}.$

2.27. $y' = \frac{x^2 + xy - 5y^2}{x^2 - 6xy}.$

2.28. $xy' = 4\sqrt{x^2 + y^2} + y.$

2.29. $3y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 10.$

2.30. $xy' = 4\sqrt{2x^2 + y^2} + y.$

3. Differensial tenglamaning umumiy integralini toping.

3.1. $y' = \frac{x+2y-3}{2x-2}$.

3.2. $y' = \frac{x+y-2}{2x-2}$.

3.3. $y' = \frac{3y-x-4}{3x+3}$.

3.4. $y' = \frac{2y-2}{x+y-2}$.

3.5. $y' = \frac{x+y-2}{3x-y-2}$.

3.6. $y' = \frac{2x+y-3}{x-1}$.

3.7. $y' = \frac{x+y-8}{3x-y-8}$.

3.8. $y' = \frac{x+3y+4}{3x-6}$.

3.9. $y' = \frac{3y+3}{2x+y-1}$.

3.10. $y' = \frac{x+2y-3}{4x-y-3}$.

3.11. $y' = \frac{x-2y+3}{-2x-2}$.

3.12. $y' = \frac{x+8y-9}{10x-y-9}$.

3.13. $y' = \frac{2x+3y-5}{5x-5}$.

3.14. $y' = \frac{4y-8}{3x+2y-7}$.

3.15. $y' = \frac{x+3y-4}{5x-y-4}$.

3.16. $y' = \frac{y-2x+3}{x-1}$.

3.17. $y' = \frac{x+2y-3}{x-1}$.

3.18. $y' = \frac{3x+2y-1}{x+1}$.

3.19. $y' = \frac{5y+5}{4x+3y-1}$.

3.20. $y' = \frac{x+4y-5}{6x-y-5}$.

3.21. $y' = \frac{x+y+2}{x+1}$.

3.22. $y' = \frac{2x+y-3}{4x-4}$.

3.23. $y' = \frac{2x+y-3}{2x-2}$.

3.24. $y' = \frac{y}{2x+2y-2}$.

3.25. $y' = \frac{x+5y-6}{7x-y-6}$.

3.26. $y' = \frac{x+y-4}{x-2}$.

3.27. $y' = \frac{2x+y-1}{2x-2}$.

3.28. $y' = \frac{3y-2x+1}{3x+3}$.

3.29. $y' = \frac{6y-6}{5x+4y-9}$.

3.30. $y' = \frac{x+6y-7}{8x-y-7}$.

4. Koshi masalasining yechimini toping.

4.1. $y' - y/x = x^2, \quad y(1) = 0.$

4.2. $y' - y\operatorname{ctg} x = 2x\sin x, \quad y(\pi/2) = 0.$

4.3. $y' + y\cos x = \frac{1}{2}\sin 2x, \quad y(0) = 0.$

4.4. $y' + y\operatorname{tg} x = \cos^2 x, \quad y(\pi/4) = 1/2.$

4.5. $y' - \frac{y}{x+2} = x^2 + 2x, \quad y(-1) = 3/2.$

4.6. $y' - \frac{1}{x+1}y = e^x(x+1), \quad y(0) = 1.$

4.7. $y' - \frac{y}{x} = x\sin x, \quad y\left(\frac{\pi}{2}\right) = 1.$

4.8. $y' + \frac{y}{x} = \sin x, \quad y(\pi) = \frac{1}{\pi}.$

4.9. $y' + \frac{y}{2x} = x^2, \quad y(1) = 1.$

4.10. $y' + \frac{2x}{1+x^2}y = \frac{2x^2}{1+x^2}, \quad y(0) = \frac{2}{3}.$

4.11. $y' - \frac{2x-5}{x^2}y = 5, \quad y(2) = 4.$

4.12. $y' + \frac{y}{x} = \frac{x+1}{x}e^x, \quad y(1) = e.$

4.13. $y' - \frac{y}{x} = -2\frac{\ln x}{x}, \quad y(1) = 1.$

4.14. $y' - \frac{y}{x} = -\frac{12}{x^3}, \quad y(1) = 4.$

4.15. $y' + \frac{2}{x}y = x^3, \quad y(1) = -5/6.$

4.16. $y' + \frac{y}{x} = 3x, \quad y(1) = 1.$

4.17. $y' - \dfrac{2xy}{1+x^2} = 1 + x^2, \quad y(1) = 3.$

4.18. $y' + \dfrac{1-2x}{x^2} y = 1, \quad y(1) = 1.$

4.19. $y' + \dfrac{3y}{x} = \dfrac{2}{x^3}, \quad y(1) = 1.$

4.20. $y' + 2xy = -2x^3, \quad y(1) = e^{-1}.$

4.21. $y' + \dfrac{xy}{2(1-x^2)} = \dfrac{x}{2}, \quad y(0) = \dfrac{2}{3}.$

4.22. $y' + xy = -x^3, \quad y(0) = 3.$

4.23. $y' - \dfrac{2}{x+1} y = e^x (x+1)^2, \quad y(0) = 1.$

4.24. $y' + 2xy = x e^{-x^2} \sin x, \quad y(0) = 1.$

4.25. $y' - 2y/(x+1) = (x+1)^3, \ y(0) = 1/2.$

4.26. $y' - y\cos x = -\sin 2x, \quad y(0) = 3.$

4.27. $y' - 4xy = -4x^3, \quad y(0) = -1/2.$

4.28. $y' - \dfrac{y}{x} = -\dfrac{\ln x}{x}, \quad y(1) = 1.$

4.29. $y' - 3x^2 y = x^2(1 + x^3)/3, \quad y(0) = 0.$

4.30. $y' - y\cos x = \sin 2x, \quad y(0) = -1.$

5. Koshi masalasining yechimini toping.

5.1. $y^2 dx + (x + e^{2/y}) dy = 0, \quad y|_{x=e} = 2.$

5.2. $(y^4 e^y + 2x) y' = y, \quad y|_{x=0} = 1.$

5.3. $y^2 dx + (xy - 1) dy = 0, \quad y|_{x=1} = e.$

5.4. $2(4y^2 + 4y - x) y' = 1, \quad y|_{x=0} = 0.$

5.5. $(\cos 2y \cos^2 y - x) y' = \sin y \cos y, \quad y|_{x=1/4} = \pi/3.$

5.6. $(x\cos^2 y - y^2) y' = y\cos^2 y, \quad y|_{x=\pi} = \pi/4.$

5.7. $e^{y^2}(dx - 2xy dy) = y dy, \quad y|_{x=0} = 0.$

5.8. $(104y^3 - x) y' = 4y, \quad y|_{x=8} = 1.$

5.9. $dx+\left(xy-y^{3}\right)dy=0, \quad y|_{x=-1}=0.$

5.10. $\left(3y\cos 2y-2y^{2}\sin 2y-2x\right)y'=y, \quad y|_{x=16}=\pi/4.$

5.11. $8\left(4y^{3}+xy-y\right)y'=1, \quad y|_{x=0}=0.$

5.12. $\left(2\ln y-\ln^{2} y\right)dy=ydx-xdy, \quad y|_{x=4}=\mathrm{e}^{2}.$

5.13. $2\left(x+y^{4}\right)y'=y, \quad y|_{x=-2}=-1.$

5.14. $y^{3}(y-1)dx+3xy^{2}(y-1)dy=(y+2)dy, \quad y|_{x=1/4}=2.$

5.15. $2y^{2}dx+\left(x+\mathrm{e}^{1/y}\right)dy=0, \quad y|_{x=\mathrm{e}}=1.$

5.16. $\left(xy+\sqrt{y}\right)dy+y^{2}dx=0, \quad y|_{x=-1/2}=4.$

5.17. $\sin 2ydx=\left(\sin^{2}2y-2\sin^{2}y+2x\right)dy, \quad y|_{x=-1/2}=\pi/4.$

5.18. $\left(y^{2}+2y-x\right)y'=1, \quad y|_{x=2}=0.$

5.19. $2y\sqrt{y}dx-\left(6x\sqrt{y}+7\right)dy=0, \quad y|_{x=-4}=1.$

5.20. $dx=(\sin y+3\cos y+3x)dy, \quad y|_{x=\mathrm{e}^{\pi/2}}=\pi/2.$

5.21. $2\left(\cos^{2}y\cdot\cos 2y-x\right)y'=\sin 2y, \quad y|_{x=3/2}=5\pi/4.$

5.22. $\operatorname{ch} ydx=(1+x\operatorname{sh}x)dy, \quad y|_{x=1}=\ln 2.$

5.23. $\left(13y^{3}-x\right)y'=4y, \quad y|_{x=5}=1.$

5.24. $y^{2}\left(y^{2}+4\right)dx+2xy\left(y^{2}+4\right)dy=2dy, \quad y|_{x=\pi/8}=2.$

5.25. $\left(x+\ln^{2}y-\ln y\right)y'=y/2, \quad y|_{x=2}=1.$

5.26. $\left(2xy+\sqrt{y}\right)dy+2y^{2}dx=0, \quad y|_{x=-1/2}=1.$

5.27. $ydx+\left(2x-2\sin^{2}y-y\sin 2y\right)dy=0, \quad y|_{x=3/2}=\pi/4.$

5.28. $2\left(y^{3}-y+xy\right)dy=dx, \quad y|_{x=-2}=0.$

5.29. $\left(2y+x\operatorname{tg}y-y^{2}\operatorname{tg}y\right)dy=dx, \quad y|_{x=0}=\pi.$

5.30. $4y^{2}dx+\left(\mathrm{e}^{1/(2y)}+x\right)dy=0, \quad y|_{x=\mathrm{e}}=1/2.$

6. Koshi masalasining yechimini toping.

6.1. $y'+xy=(1+x)\mathrm{e}^{-x}y^{2}, \quad y(0)=1.$

6.2. $xy'+y=2y^{2}\ln x, \quad y(1)=1/2.$

6.3. $2(xy' + y) = xy^2, \quad y(1) = 2.$

6.4. $y' + 4x^3 y = 4(x^3 + 1)e^{-4x} y^2, \quad y(0) = 1.$

6.5. $xy' - y = -y^2(\ln x + 2)\ln x, \quad y(1) = 1.$

6.6. $2(y' + xy) = (1 + x)e^{-x} y^2, \quad y(0) = 2.$

6.7. $3(xy' + y) = y^2 \ln x, \quad y(1) = 3.$

6.8. $2y' + y\cos x = y^{-1}\cos x(1 + \sin x), \quad y(0) = 1.$

6.9. $y' + 4x^3 y = 4y^2 e^{4x}(1 - x^3), \quad y(0) = -1.$

6.10. $3y' + 2xy = 2xy^{-2} e^{-2x^2}, \quad y(0) = -1.$

6.11. $2xy' - 3y = -(5x^2 + 3)y^3, \quad y(1) = 1/\sqrt{2}.$

6.12. $3xy' + 5y = (4x - 5)y^4, \quad y(1) = 1.$

6.13. $2y' + 3y\cos x = e^{2x}(2 + 3\cos x)y^{-1}, \quad y(0) = 1.$

6.14. $3(xy' + y) = xy^2, \quad y(1) = 3.$

6.15. $y' - y = 2xy^2, \quad y(0) = 1/2.$

6.16. $2xy' - 3y = -(20x^2 + 12)y^3, \quad y(1) = 1/2\sqrt{2}.$

6.17. $y' + 2xy = 2x^3 y^3, \quad y(0) = \sqrt{2}.$

6.18. $xy' + y = y^2 \ln x, \quad y(1) = 1.$

6.19. $2y' + 3y\cos x = (8 + 12\cos x)e^{2x} y^{-1}, \quad y(0) = 2.$

6.20. $4y' + x^3 y = (x^3 + 8)e^{-2x} y^2, \quad y(0) = 1.$

6.21. $8xy' - 12y = -(5x^2 + 3)y^3, \quad y(1) = \sqrt{2}.$

6.22. $2(y' + y) = xy^2, \quad y(0) = 2.$

6.23. $y' + xy = (x - 1)e^x y^2, \quad y(0) = 1.$

6.24. $2y' + 3y\cos x = -e^{-2x}(2 + 3\cos x)y^{-1}, \quad y(0) = 1.$

6.25. $y' - y = xy^2, \quad y(0) = 1.$

6.26. $2(xy' + y) = y^2 \ln x, \quad y(1) = 2.$

6.27. $y' + y = xy^2, \quad y(0) = 1.$

6.28. $y' + 2y \operatorname{cth} x = y^2 \operatorname{ch} x, \quad y(1) = 1/\operatorname{sh} 1.$

6.29. $2(y' + xy) = (x - 1)e^x y^2, \quad y(0) = 2.$

6.30. $y' - y \operatorname{tg} x = -(2/3)y^4 \sin x, \quad y(0) = 1.$

7. Differensial tenglamaning umumiy integralini toping.

7.1. $3x^2 e^y dx + \left(x^3 e^y - 1\right)dy = 0.$

7.2. $\left(3x^2 + \frac{2}{y}\cos\frac{2x}{y}\right)dx - \frac{2x}{y^2}\cos\frac{2x}{y}dy = 0.$

7.3. $\left(3x^2 + 4y^2\right)dx + \left(8xy + e^y\right)dy = 0.$

7.4. $\left(2x - 1 - \frac{y}{x^2}\right)dx - \left(2y - \frac{1}{x}\right)dy = 0.$

7.5. $\left(y^2 + y\sec^2 x\right)dx + \left(2xy + \operatorname{tg} x\right)dy = 0.$

7.6. $\left(3x^2 y + 2y + 3\right)dx + \left(x^3 + 2x + 3y^2\right)dy = 0.$

7.7. $\left(\frac{x}{\sqrt{x^2 + y^2}} + \frac{1}{x} + \frac{1}{y}\right)dx + \left(\frac{y}{\sqrt{x^2 + y^2}} + \frac{1}{x} - \frac{x}{y^2}\right)dy = 0.$

7.8. $\left[\sin 2x - 2\cos(x + y)\right]dx - 2\cos(x + y)dy = 0.$

7.9. $\left(xy^2 + x/y^2\right)dx + \left(x^2 y - x^2/y^3\right)dy = 0.$

7.10. $\left(\frac{1}{x^2} + \frac{3y^2}{x^4}\right)dx - \frac{2y}{x^3}dy = 0.$

7.11. $\frac{y}{x^2}\cos\frac{y}{x}dx - \left(\frac{1}{x}\cos\frac{y}{x} + 2y\right)dy = 0.$

7.12. $\left(\frac{x}{\sqrt{x^2 + y^2}} + y\right)dx + \left(x + \frac{y}{\sqrt{x^2 + y^2}}\right)dy = 0.$

7.13. $\frac{1 + xy}{x^2 y}dx + \frac{1 - xy}{xy^2}dy = 0.$

7.14. $\frac{dx}{y} - \frac{x + y^2}{y^2}dy = 0.$

7.15. $\frac{y}{x^2}dx - \frac{xy + 1}{x}dy = 0.$

7.16. $\left(xe^x + \frac{y}{x^2}\right)dx - \frac{1}{x}dy = 0.$

7.17. $\left(10xy - \frac{1}{\sin y}\right)dx + \left(5x^2 + \frac{x\cos y}{\sin^2 y} - y^2 \sin y^3\right)dy = 0.$

7.18. $\left(\frac{y}{x^2+y^2}+e^x\right)dx-\frac{xdy}{x^2+y^2}=0.$

7.19. $e^y dx+\left(\cos y+xe^y\right)dy=0.$

7.20. $\left(y^3+\cos x\right)dx+\left(3xy^2+e^y\right)dy=0.$

7.21. $xe^{y^2}dx+\left(x^2ye^{y^2}+\operatorname{tg}^2 y\right)dy=0.$

7.22. $\left(5xy^2-x^3\right)dx+\left(5x^2y-y\right)dy=0.$

7.23. $\left[\cos\left(x+y^2\right)+\sin x\right]dx+2y\cos\left(x+y^2\right)dy=0.$

7.24. $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0.$

7.25. $\left(\sin y+y\sin y+\frac{1}{x}\right)dx+\left(x\cos y-\cos x+\frac{1}{y}\right)dy=0.$

7.26. $\left(1+\frac{1}{y}e^{x/y}\right)dx+\left(1-\frac{x}{y^2}e^{x/y}\right)dy=0.$

7.27. $\frac{(x-y)dx+(x+y)dy}{x^2+y^2}=0.$

7.28. $2\left(3xy^2+2x^3\right)dx+3\left(2x^2y+y^2\right)dy=0.$

7.29. $\left(3x^3+6x^2y+3xy^2\right)dx+\left(2x^3+3x^2y\right)dy=0.$

7.30. $xy^2dx+y\left(x^2+y^2\right)dy=0.$

8. Differensial tenglamaning umumiy yechimini toping.

8.1. $y'''x\ln x=y''.$

8.2. $xy'''+y''=1.$

8.3. $2xy'''=y''.$

8.4. $xy'''+y''=x+1.$

8.5. $\operatorname{tg}x\cdot y''-y'+\frac{1}{\sin x}=0.$

8.6. $x^2y''+xy'=1.$

8.7. $y'''\operatorname{ctg}2x+2y''=0.$

8.8. $x^3y'''+x^2y''=1.$

8.9. $\operatorname{tg}x\cdot y'''=2y''.$

8.10. $y'''\operatorname{cth}2x=2y''.$

8.11. $x^4y''+x^3y'=1.$

8.12. $xy'''+2y''=0.$

8.13. $\left(1+x^2\right)y''+2xy'=x^3.$

8.14. $x^5y'''+x^4y''=1.$

8.15. $xy'''-y''+\frac{1}{x}=0.$

8.16. $xy'''+y''+x=0.$

8.17. $\operatorname{th} x \cdot y^{IV} = y'''$.

8.18. $xy''' + y'' = \sqrt{x}$.

8.19. $y''' \operatorname{tg} x = y'' + 1$.

8.20. $y''' \operatorname{tg} 5x = 5y''$.

8.21. $y''' \operatorname{th} 7x = 7y''$.

8.22. $x^3 y''' + x^2 y'' = \sqrt{x}$.

8.23. $\operatorname{cth} x \cdot y'' - y' + \dfrac{1}{\operatorname{ch} x} = 0$.

8.24. $(x+1) y''' + y'' = (x+1)$.

8.25. $(1 + \sin x) y''' = \cos x \cdot y''$.

8.26. $xy''' + y'' = \dfrac{1}{\sqrt{x}}$.

8.27. $-xy''' + 2y'' = \dfrac{2}{x^2}$.

8.28. $\operatorname{cth} x y'' + y' = \operatorname{ch} x$.

8.29. $x^4 y'' + x^3 y' = 4$.

8.30. $y'' + \dfrac{2x}{x^2+1} y' = 2x$.

9. Koshi masalasining yechimini toping.

9.1. $4y^3 y'' = y^4 - 1,\quad y(0) = \sqrt{2},\quad y'(0) = 1/(2\sqrt{2})$.

9.2. $y'' = 128y^3,\quad y(0) = 1,\quad y'(0) = 8$.

9.3. $y''y^3 + 64 = 0,\quad y(0) = 4,\quad y'(0) = 2$.

9.4. $y'' + 2\sin y \cos^3 y = 0,\quad y(0) = 0,\quad y'(0) = 1$.

9.5. $y'' = 32\sin^3 y \cos y,\quad y(1) = \pi/2,\quad y'(1) = 4$.

9.6. $y'' = 98y^3,\quad y(1) = 1,\quad y'(1) = 7$.

9.7. $y''y^3 + 49 = 0,\quad y(3) = -7,\quad y'(3) = -1$.

9.8. $4y^3 y'' = 16y^4 - 1,\quad y(0) = \sqrt{2}/2,\quad y'(0) = 1/\sqrt{2}$.

9.9. $y'' + 8\sin y \cos^3 y = 0,\quad y(0) = 0,\quad y'(0) = 2$.

9.10. $y'' = 72y^3,\quad y(2) = 1,\quad y'(2) = 6$.

9.11. $y''y^3 + 36 = 0,\quad y(0) = 3,\quad y'(0) = 2$.

9.12. $y'' = 18\sin^3 y \cos y,\quad y(1) = \pi/2,\quad y'(1) = 3$.

9.13. $4y^3 y'' = y^4 - 16,\quad y(0) = 2\sqrt{2},\quad y'(0) = 1/\sqrt{2}$.

9.14. $y'' = 50y^3,\quad y(3) = 1,\quad y'(3) = 5$.

9.15. $y''y^3 + 25 = 0,\quad y(2) = -5,\quad y'(2) = -1$.

9.16. $y'' + 18\sin y \cos^3 y = 0,\quad y(0) = 0,\quad y'(0) = 3$.

9.17. $y'' = 8\sin^3 y \cos y,\quad y(1) = \pi/2,\quad y'(1) = 2$.

9.18. $y'' = 32y^3,\quad y(4) = 1,\quad y'(4) = 4$.

9.19. $y''y^3 + 16 = 0,\quad y(1) = 2,\quad y'(1) = 2$.

9.20. $y'' + 32\sin y \cos^3 y = 0,\quad y(0) = 0,\quad y'(0) = 4$.

9.21. $y'' = 50\sin^3 y\cos y,\quad y(1) = \pi/2,\quad y'(1) = 5.$

9.22. $y'' = 18y^3,\quad y(1) = 1,\quad y'(1) = 3.$

9.23. $y''y^3 + 9 = 0,\quad y(1) = 1,\quad y'(1) = 3.$

9.24. $y^3y'' = 4(y^4 - 1),\quad y(0) = \sqrt{2},\quad y'(0) = \sqrt{2}.$

9.25. $y'' + 50\sin y\cos^3 y = 0,\quad y(0) = 0,\quad y'(0) = 5.$

9.26. $y'' = 8y^3,\quad y(0) = 1,\quad y'(0) = 2.$

9.27. $y''y^3 + 4 = 0,\quad y(0) = -1,\quad y'(0) = -2.$

9.28. $y'' = 2\sin^3 y\cos y,\quad y(1) = \pi/2,\quad y'(1) = 1.$

9.29. $y^3y'' = y^4 - 16,\quad y(0) = 2\sqrt{2},\quad y'(0) = \sqrt{2}.$

9.30. $y'' = 2y^3,\quad y(-1) = 1,\quad y'(-1) = 1.$

10. Differensial tenglamaning umumiy yechimini toping.

10.1. $y''' + 3y'' + 2y' = 1 - x^2.$

10.2. $y''' - y'' = 6x^2 + 3x.$

10.3. $y''' - y' = x^2 + x.$

10.4. $y^{IV} - 3y''' + 3y'' - y' = 2x.$

10.5. $y^{IV} - y''' = 5(x+2)^2.$

10.6. $y^{IV} - 2y''' + y'' = 2x(1-x).$

10.7. $y^{IV} + 2y''' + y'' = x^2 + x - 1.$

10.8. $y^{V} - y^{IV} = 2x + 3.$

10.9. $3y^{IV} + y''' = 6x - 1.$

10.10. $y^{IV} + 2y''' + y'' = 4x^2.$

10.11. $y''' + y'' = 5x^2 - 1.$

10.12. $y^{IV} + 4y''' + 4y'' = x - x^2.$

10.13. $7y''' - y'' = 12x.$

10.14. $y''' + 3y'' + 2y' = 3x^2 + 2x.$

10.15. $y''' - y' = 3x^2 - 2x + 1.$

10.16. $y''' - y'' = 4x^2 - 3x + 2.$

10.17. $y^{IV} - 3y''' + 3y'' - y' = x - 3.$

10.18. $y^{IV} + 2y''' + y'' = 12x^2 - 6x.$

10.19. $y''' - 4y'' = 32 - 384x^2.$

10.20. $y^{IV} + 2y''' + y'' = 2 - 3x^2.$

10.21. $y''' + y'' = 49 - 24x^2.$

10.22. $y''' - 2y'' = 3x^2 + x - 4.$

10.23. $y''' - 13y'' + 12y' = x - 1.$

10.24. $y^{IV} + y''' = x.$

10.25. $y''' - y'' = 6x + 5.$

10.26. $y''' + 3y'' + 2y' = x^2 + 2x + 3.$

10.27. $y''' - 5y'' + 6y' = (x-1)^2.$

10.28. $y^{IV} - 6y''' + 9y'' = 3x - 1.$

10.29. $y''' - 13y'' + 12y' = 18x^2 - 39.$

10.30. $y^{IV} + y''' = 12x + 6.$

11. Differensial tenglamaning umumiy yechimini toping.

11.1. $y''' - 4y'' + 5y' - 2y = (16 - 12x)e^{-x}.$

11.2. $y''' - 3y'' + 2y' = (1 - 2x)e^{x}.$

11.3. $y''' - y'' - y' + y = (3x + 7)e^{2x}.$

11.4. $y''' - 2y'' + y' = (2x + 5)e^{2x}.$

11.5. $y''' - 3y'' + 4y = (18x - 21)e^{-x}$.

11.6. $y''' - 5y'' + 8y' - 4y = (2x - 5)e^{x}$.

11.7. $y''' - 4y'' + 4y' = (x - 1)e^{x}$.

11.8. $y''' + 2y'' + y' = (18x + 21)e^{2x}$.

11.9. $y''' + y'' - y' - y = (8x + 4)e^{x}$.

11.10. $y''' - 3y' - 2y = -4x \cdot e^{x}$.

11.11. $y''' - 3y' + 2y = (4x + 9)e^{2x}$.

11.12. $y''' + 4y'' + 5y' + 2y = (12x + 16)e^{x}$.

11.13. $y''' - y'' - 2y' = (6x - 11)e^{-x}$.

11.14. $y''' + y'' - 2y' = (6x + 5)e^{x}$.

11.15. $y''' + 4y'' + 4y' = (9x + 15)e^{x}$.

11.16. $y''' - 3y'' - y' + 3y = (4 - 8x)e^{x}$.

11.17. $y''' - y'' - 4y' + 4y = (7 - 6x)e^{x}$.

11.18. $y''' + 3y'' + 2y' = (1 - 2x)e^{-x}$.

11.19. $y''' - 5y'' + 7y' - 3y = (20 - 16x)e^{-x}$.

11.20. $y''' - 4y'' + 3y' = -4x \cdot e^{x}$.

11.21. $y''' - 5y'' + 3y' + 9y = (32x - 32)e^{-x}$.

11.22. $y''' - 6y'' + 9y' = 4x \cdot e^{x}$.

11.23. $y''' - 7y'' + 15y' - 9y = (8x - 12)e^{x}$.

11.24. $y''' - y'' - 5y' - 3y = -(8x + 4)e^{x}$.

11.25. $y''' + 5y'' + 7y' + 3y = (16x + 20)e^{x}$.

11.26. $y''' - 2y'' - 3y' = (8x - 14)e^{-x}$.

11.27. $y''' + 2y'' - 3y' = (8x + 6)e^{x}$.

11.28. $y''' + 6y'' + 9y' = (16x + 24)e^{x}$.

11.29. $y''' - y'' - 9y' + 9y = (12 - 16x)e^{x}$.

11.30. $y''' + 4y'' + 3y' = 4(1 - x)e^{-x}$.

12. Differensial tenglamaning umumiy yechimini toping.

12.1. $y'' + 2y' = 4e^{x}(\sin x + \cos x)$. 12.2. $y'' - 4y' + 4y = -e^{2x}\sin 6x$.

12.3. $y'' + 2y' = -2e^{x}(\sin x + \cos x)$. 12.4. $y'' + y = 2\cos 7x + 3\sin 7x$.

12.5. $y'' + 2y' + 5y = -\sin 2x$. 12.6. $y'' - 4y' + 8y = e^{x}(5\sin x - 3\cos x)$.

12.7. $y'' + 2y' = e^x(\sin x + \cos x)$. 12.8. $y'' - 4y' + 4y = e^{2x} \sin 3x$.

12.9. $y'' + 6y' + 13y = e^{-3x} \cos 4x$. 12.10. $y'' + y = 2\cos 3x - 3\sin 3x$.

12.11. $y'' + 2y' + 5y = -2\sin x$. 12.12. $y'' - 4y' + 8y = e^x(-3\sin x + 4\cos x)$.

12.13. $y'' + 2y' = 10e^x(\sin x + \cos x)$. 12.14. $y'' - 4y' + 4y = e^{2x} \sin 5x$.

12.15. $y'' + y = 2\cos 5x + 3\sin 5x$. 12.16. $y'' + 2y' + 5y = -17\sin 2x$.

12.17. $y'' + 6y' + 13y = e^{3x} \cos x$. 12.18. $y'' - 4y' + 8y = e^x(3\sin x + 5\cos x)$.

12.19. $y'' + 2y' = 6e^x(\sin x + \cos x)$. 12.20. $y'' - 4y' + 4y = -e^{2x} \sin 4x$.

12.21. $y'' + 6y' + 13y = -e^{3x} \cos 5x$. 12.22. $y'' + y = 2\cos 7x - 3\sin 7x$.

12.23. $y'' + 2y' + 5y = -\cos x$. 12.24. $y'' - 4y' + 8y = e^x(2\sin x - \cos x)$.

12.25. $y'' + 2y' = 3e^x(\sin x + \cos x)$. 12.26. $y'' - 4y' + 4y = e^{2x} \sin 4x$.

12.27. $y'' + 6y' + 13y = e^{-3x} \cos 8x$. 12.28. $y'' + 2y' + 5y = 10\cos x$.

12.29. $y'' + y = 2\cos 4x + 3\sin 4x$. 12.30. $y'' - 4y' + 8y = e^x(-\sin x + 2\cos x)$.

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Contents

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14 - buyurtma 280 nusxa. Hajmi 5,2 b.t.
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