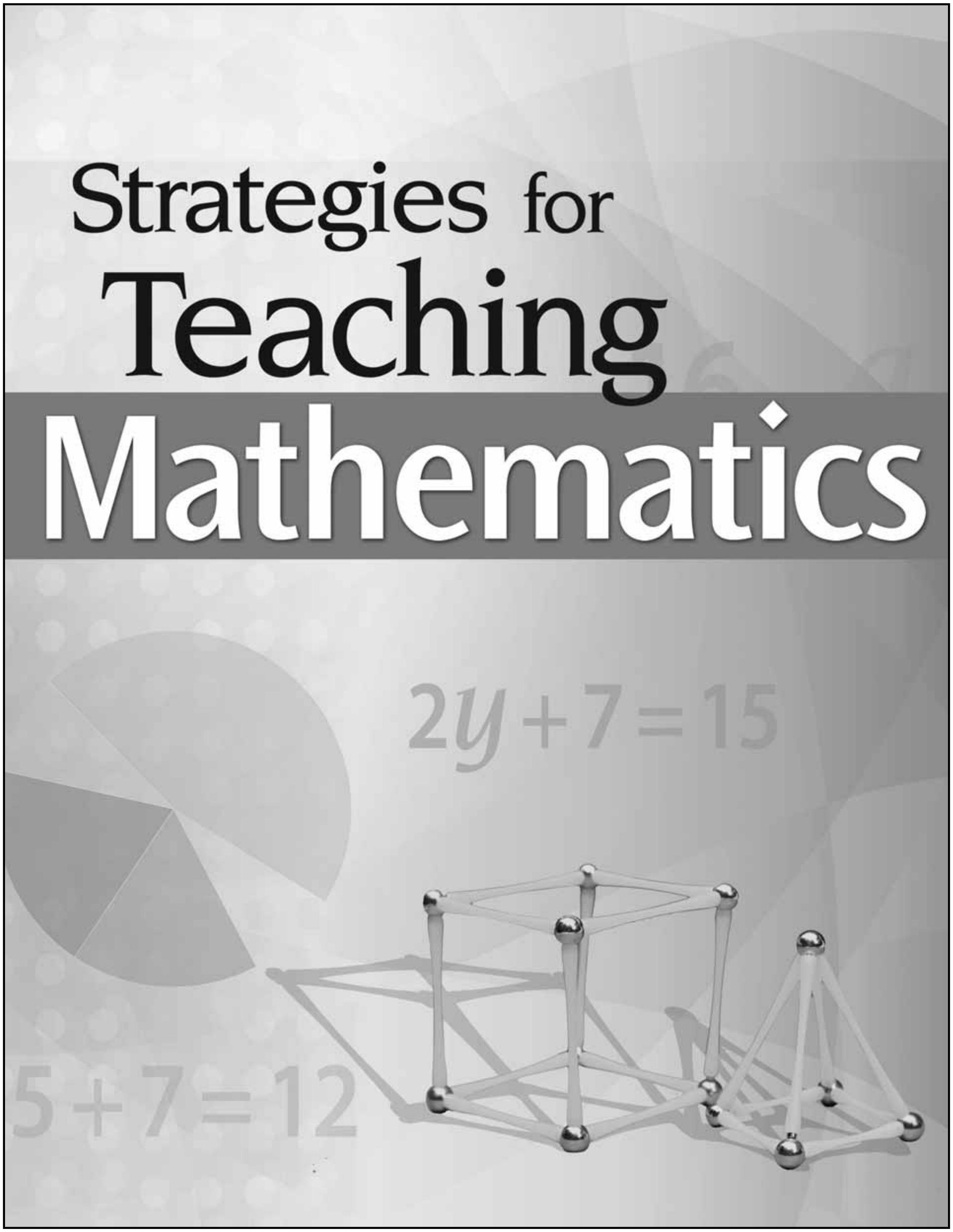

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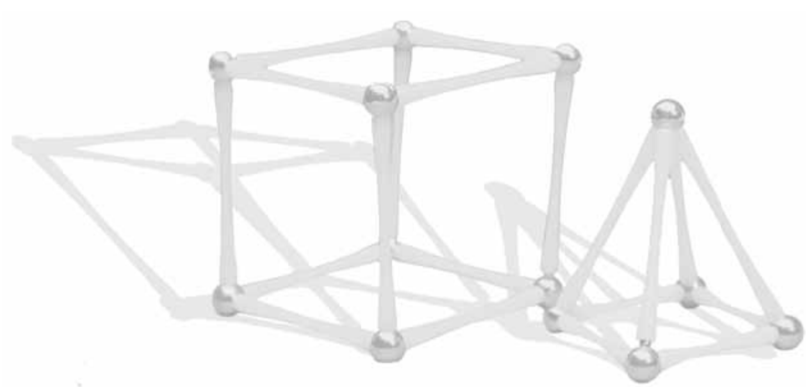

Foreword

Earlene J. Hall, Ed.D.
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"Tell me, and I'll forget. Show me, and I may not remember. Involve me, and I will understand."

—Native American proverb

Traveling throughout the country working with groups of teachers, my typical road trip often begins with the address of my ending destination. Sitting down with a road map to chart the route, I realize there are circumstances that will directly impact my arrival at the destination. This process mirrors the challenges of teachers when planning instruction.

Each state has developed grade-level standards, which are used to formulate grade-level content assessments. These objectives define the content of which students must demonstrate understanding. Having an expected outcome defined does not adequately prepare you for the journey to the destination. And although state assessments are not the curriculum, they certainly impact the design of instruction. The assessments require students to demonstrate their understanding by skillfully processing the mathematics content. Students are bombarded with numerous questions from various content strands in which they must demonstrate

- conceptual understanding of mathematics content through modeling or interpretation of representations,
- computational fluency,
- and problem solving through application of the content.

As teachers, we navigate our students through the maze of learning objectives in search of understanding—understanding that is measurable on state assessments. This can be challenging considering many of the circumstances that we encounter when planning instruction that is aligned to these standards. For example, our students are compilations of various skill levels and abilities. Each student represents a unique set of learning styles and prior experiences. Our challenge is in making the learning accessible for all students. The design of our daily instructional plan must be inclusive of strategies that develop student understanding.

Teachers often search for the answers to questions such as, "What does it mean for students to understand," and "How do we support the development of their understanding?" Most educators can agree that understanding is being able to carry out a variety of actions that demonstrate one's knowledge of a topic, and apply it in new ways. Survey responses by teachers also reflect a need for instructional resources for planning lessons.

Foreword *(cont.)*

Strategies for Teaching Mathematics is a toolkit that provides support much like that of a navigation system in your car for a trip as opposed to a traditional road map. An instructional resource for K–8 teachers, this resource provides step-by-step, research-based strategies that represent a balanced approach to learning mathematics. This is a resource that provides comprehensive instructional plans that can be used as "tools" for intervention and the development of critical supporting skills.

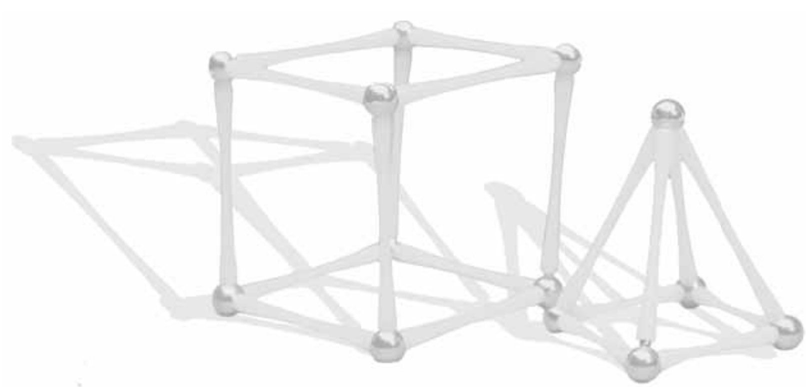
Sample lessons provide a sequential guide in developing student proficiency in the following:

- mathematical vocabulary
- conceptual understanding through the use of manipulatives
- procedural proficiency
- problem-solving strategies
- mathematical games
- assessing mathematical thinking

Each section establishes key instructional connections. Additionally, differentiated activities ensure adaptability for students of all ability levels. The strategies are structured in a format that provides teachers with the following tools:

- an alignment to mathematical standards
- vocabulary terms
- elementary and secondary applications
- step-by-step procedures
- differentiated instruction
- reproducible student pages

Mathematics is constantly developing and becoming ever more specialized. It is a challenge to effectively implement a curriculum that yields understanding for the wide range of student abilities represented in our classrooms today. As we move forward with our quest to increase student understanding, the learning experiences we provide must actively engage student thinking in the kind of rigorous, relevant learning experiences that are provided in this resource.

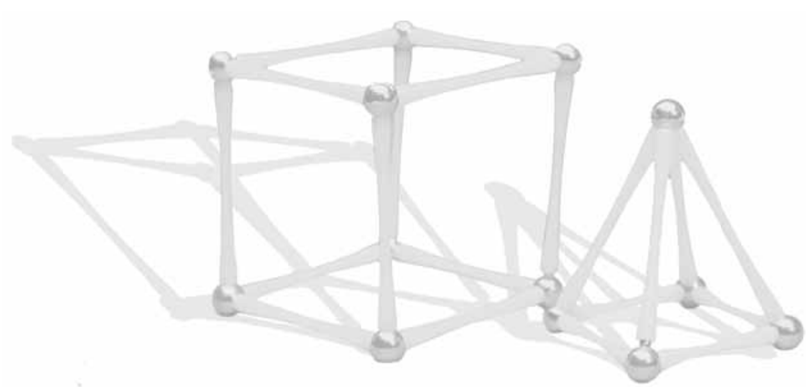


Research

Mathematics is one of the most feared subjects in school, yet it is a subject students will need for the rest of their lives. Students often struggle with learning mathematics, and teachers have long sought more effective methods for teaching it. In their landmark book, *Classroom Instruction that Works*, Robert Marzano, Debra Pickering, and Jane Pollock (2001) note that teaching has become more a science than an art. Research in the last two decades has helped educators develop a common understanding of effective instruction. This is especially evident in teaching mathematics. *Strategies for Teaching Mathematics* includes proven approaches to teaching mathematics at all levels.

Strategies for Teaching Mathematics provides educators with background information on effective instructional strategies with sample lesson plans and student reproducibles. These materials support students in truly understanding mathematical concepts, rather than just memorizing procedures. This kind of deep conceptual understanding has never been more important than it is today. Numerous educational leaders have stressed the importance of mastering 21st-century skills for today's students. Frank Levy and Richard Murnane, researchers at MIT and Harvard, found that the changing workplace has strong implications for students (2005). Computerization and globalization in today's world means that students need to be problem solvers who can think critically. No longer can students graduate from high school and expect to succeed without these qualities. Levy and Murnane note that occupations requiring higher-level problem solving have seen dramatic increases in average salary in the past 30 years, while more traditional blue-collar positions have seen even greater drops in average pay. Even more importantly, these blue-collar positions require more complex skills than similar positions of the past. Clearly, 21st-century skills need to be mastered by today's students.

So what are 21st-century skills, and what do they look like in mathematics? Students today need to understand both the procedural and conceptual foundations of mathematics. They also need to understand the "language" of mathematics. Mathematics is full of important vocabulary that has specific meanings in this context. Finally, students need to solve complex problems by making connections between prior learning and new situations. The need for students to learn these skills also means that teachers need to use new ways of teaching. Manipulatives are effective teaching tools that can be used throughout the K–12 curriculum to help students understand key mathematical concepts. New strategies for teaching essential mathematical procedures are also necessary for helping students identify real-world connections to mathematics. Finally, new methods of assessing student understanding help teachers form a stronger picture of students' skills and tailor instruction to their needs.



Research *(cont.)*

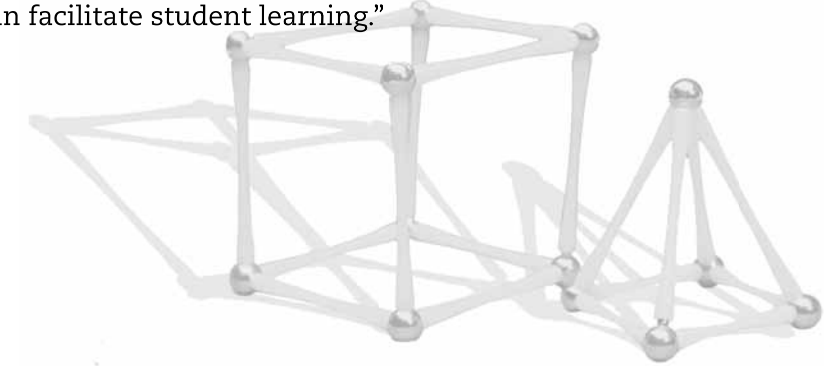
The Importance of Teaching Mathematics in a Balanced Approach

As early as 1989, educators identified the need for a balanced approach to teaching mathematics (Porter 1989). International comparisons of mathematics achievement were often made lamenting the performance of American students (Stigler and Hiebert 1997). As a result, the final decade of the 20th century was spent focusing on teaching problem-solving skills in addition to basic computation. However, new research has led to a revision of these earlier recommendations. The Mathematics Advisory Panel (U.S. Department of Education 2008), convened by the federal government, has reviewed the literature on mathematics instruction and sought the advice of key mathematics researchers. A representative study completed by Thomas Good (2008) at the University of Arizona revealed that the current mathematics curriculum in schools is too full, leading to a lack of depth of instruction and failure of students to master important concepts. Consequently, the Mathematics Advisory Panel recommended "the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts" (U.S. Department of Education 2008). This is different than earlier practices, because it focuses on all three areas of mathematics instruction without excluding any.

Balanced instruction includes more than just procedural and conceptual fluency. Instruction needs to be balanced to meet the needs of diverse learners as well. Phillip Schlechty (2002), in his book *Working on the Work*, suggests, "the key to school success is to be found in identifying or creating engaging schoolwork for students." Students become engaged in learning when they are taught using methods that motivate them. One way to differentiate instruction for learners is to teach with multiple intelligences in mind. Howard Gardner's (1983) groundbreaking work has helped teachers engage students who do not learn through traditional auditory methods. Strategies included in this resource such as using mathematical games and developing vocabulary meet the needs of kinesthetic and linguistic learners. Specific student activities such as Vocabulary Flip Books also engage artistic learners. Balancing mathematics instruction supports all learners in the classroom.

Differentiating Mathematics

As mathematical concepts are introduced, students learn them at different rates. Additionally, students bring different skill sets, learning styles, and prior knowledge with them to the classroom, which also cause differences in their learning. Because of these differences, a one-size-fits-all approach to mathematics instruction will not meet the needs of all students and ensure that they have conceptual understanding. However, using a differentiated approach to mathematics instruction will allow teachers to meet these needs. In their research, Strong, Thomas, Perini, and Silver (2004) note that "recognizing different mathematical learning styles and adapting differentiated teaching strategies can facilitate student learning."



Research *(cont.)*

Differentiating Mathematics *(cont.)*

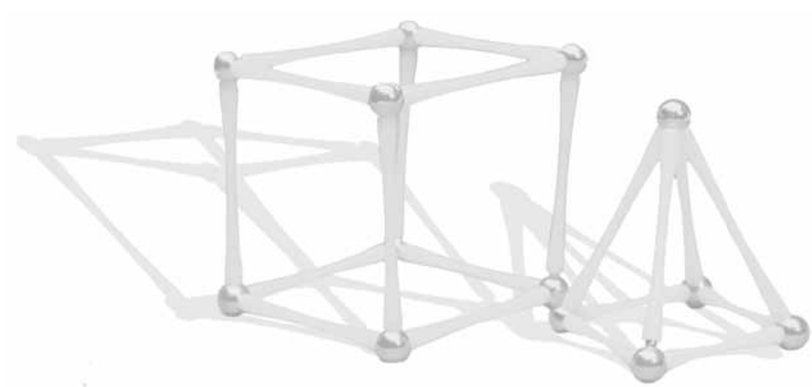
Differentiation is the modification of what is taught, how it is taught, and the product that students create based on instruction. This is commonly referred to as differentiating content, process, and product. However, differentiation can look very different depending on the learning outcome, the needs of the learners, and the structure of the classroom environment (Pettig 2000).

Strategies for Teaching Mathematics provides suggestions for differentiation throughout each section of the notebook. There are general suggestions for instructional techniques as well as specific suggestions that pertain to specific lessons. The differentiation suggestions will enable all students to connect with the content at a level that is appropriate for them.

Vocabulary Development

Vocabulary has long been overlooked in mathematics instruction. Yet, mathematics has more vocabulary and difficult text than other content areas (Schell as cited in Monroe 1998). Teaching vocabulary in mathematics is especially important for English language learners. Too often, students do not have an accurate understanding of mathematical terms. Researchers have identified several barriers that inhibit their learning as well (Thompson and Rubenstein 2007). Many vocabulary words have different meanings in other content areas. For example, the term *solution* has different meanings in mathematics and science. Other terms are also commonly used in everyday language, but they have more precise meanings in mathematics. Still other words are only used in mathematics. These terms must be explicitly taught to students. Vocabulary words, such as *quotient,* that are unique to the subject are difficult to learn. In this case, students are essentially learning a new language, and they need instructional strategies that will help them become aware of the new terms and apply them to problem-solving situations.

The National Reading Panel Report (2000) also identified academic vocabulary as essential in the development of students' reading skills. Academic vocabulary includes terms that are used throughout schooling. English teacher and author, Jim Burke (no date) identified almost 400 academic vocabulary words that students must know by the time they enter middle school. Terms such as *solve*, *quotient*, and *sum* are commonly understood by proficient students, but those who only have a limited understanding of these terms may not be able to complete the simplest mathematical problems. Further, a variety of teaching strategies is necessary to teach academic vocabulary. The National Reading Panel (2000) found that vocabulary is learned both indirectly and directly, and that dependence on only one instructional method does not result in optimal vocabulary growth.



Research *(cont.)*

Vocabulary Development *(cont.)*

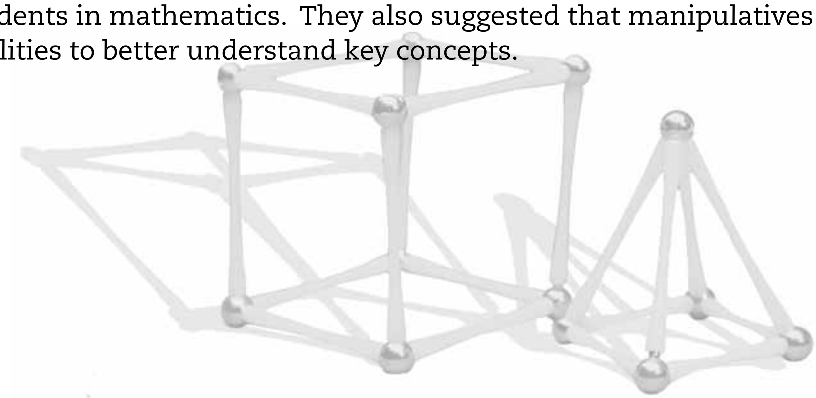
Content-area vocabulary is also highly specialized with words that are not typically used in everyday life. Therefore, all students need explicit introduction to those words to understand the text. The task is even more complicated for English language learners and struggling readers. According to Feldman and Kinsella (2005), "Developing readers cannot be expected to simply 'pick up' substantial vocabulary knowledge exclusively through reading exposure without guidance. Specifically, teachers must design tasks that will increase the effectiveness of vocabulary learning through reading practice."

While students' vocabulary skills may not be the primary concern in mathematics, the literature certainly suggests it must be an important consideration. It is not enough to give students a list of words and have them look up the definitions in dictionaries or glossaries. Students who are struggling with vocabulary are not going to find the process easier by simply being given more words to sort through (Echevarria, Vogt, and Short 2004). Struggling readers and English language learners need context-embedded activities that acquaint them with the necessary and most central words for comprehension of the content. This is why it is so important to use a variety of strategies to teach mathematical vocabulary.

Effective vocabulary development involves a rich contextual environment in which students learn terms as they read content-area text (Echevarria, Vogt, and Short 2004). *Strategies for Teaching Mathematics* includes a variety of strategies to teach vocabulary words and help students apply them in their work. Total Physical Response is one strategy that is often used in teaching English language learners, but it can be especially helpful for teaching vocabulary to all students. When using Total Physical Response, teachers connect a physical action with the teaching of a vocabulary word. Additionally, since many mathematical terms are only used in this subject area, Total Physical Response can be an especially useful strategy. Root Word Trees are another strategy found in *Strategies for Teaching Mathematics*. This strategy has students create a diagram of the root words of key mathematical terms. By linking vocabulary terms to known root words, students learn how terms are connected. This also helps students decode the meaning of unknown words.

Manipulatives

Manipulatives include almost any physical object used to represent an abstract concept. They are used to help students physically manipulate and act out mathematics algorithms. Manipulatives include items such as counters, base ten blocks, pattern blocks, calculators, and even students' own fingers. The benefits of using manipulatives in the mathematics classroom are well-established. Manipulatives provide concrete materials for students to understand concepts before making abstract connections. Lynne Marsh and Nancy Cooke (1996) found that manipulatives increased the conceptual understanding of students in mathematics. They also suggested that manipulatives helped students with learning disabilities to better understand key concepts.



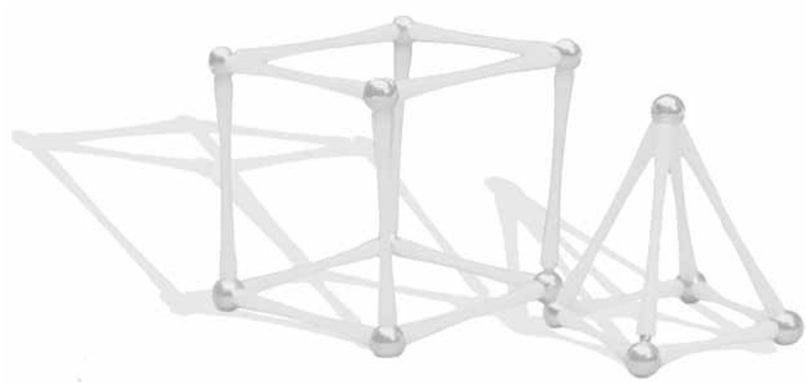
Research *(cont.)*

Manipulatives *(cont.)*

Research on general-education students has begun to look at ways in which these students learn when using manipulatives in mathematics. A recent study found that elementary teachers often use manipulatives throughout their instructional units, but that students can learn especially well when using manipulatives to solve problems (Kamii, Lewis, and Kirkland 2001). They further identified a connection between students' use of manipulatives in problem solving and Piaget's stages of child development. Learners benefit from using manipulatives when they are at the stage of expanding their understanding from the concrete to the abstract (Kamii, Lewis, and Kirkland 2001). This is Piaget's logico-mathematical stage of development. When faced with problem-solving tasks, students are making this shift in understanding. Therefore, using manipulatives helps them make this shift.

Manipulatives are most often used in elementary school classrooms. Because many middle school teachers question whether manipulatives are too childish for their students (Shrum 2005). However, research has shown that middle school students who use manipulatives make achievement gains in mathematics (Shrum 2005). When used in middle and high school, manipulatives often involve cooperative group work. One study found that students who learned mathematics concepts using manipulatives with a partner or in a small group performed better on standardized mathematics assessments than those who did not (Garrity 1998). This important finding suggests that manipulatives not only support the connection between concrete and abstract understanding but they also promote cooperative learning. A research team led by Adam Gamoran of the University of Wisconsin, Madison, also found that manipulatives are being used to prepare students for college. The study looked at tracked mathematics classrooms in high school. While the authors concluded that tracking should be eliminated, the results of the study showed that manipulatives are used in general and remedial mathematics classes. They further found that students in general education classes that used manipulatives typically went on to complete at least two college preparatory mathematics classes (Gamoran, Porter, Smithson, and White 1997).

Additionally, manipulatives need to be used appropriately in any classroom. Some methods for manipulative use are not effective in teaching mathematical concepts (Stein and Bovalino 2001). *Strategies for Teaching Mathematics* not only includes model lessons for using manipulatives in the mathematics classroom, but it also includes extensive suggestions for organizing and distributing manipulatives with students. Knowing how to effectively manage the use of manipulatives is especially important when beginning to use them with students.



Research *(cont.)*

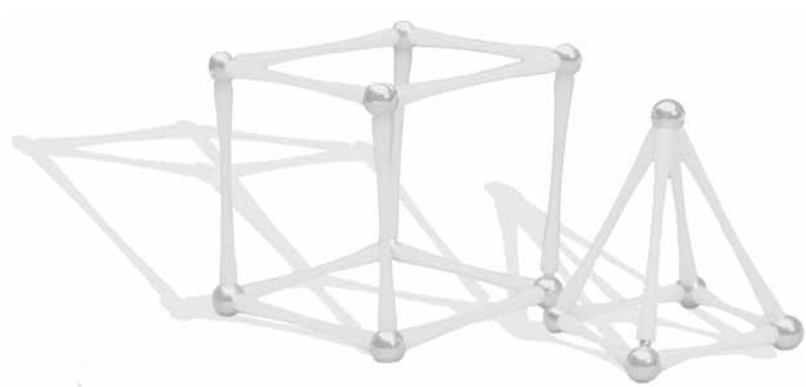
Manipulatives *(cont.)*

Manipulatives can also be effectively used to teach concepts as difficult as algebra. Algebra tiles are manipulatives that represent variables and constants in algebraic expressions. Students use algebra tiles to visually represent and solve equations. Students in middle school report that algebra is easier when they use algebra tiles (Sharp 1995). This finding is especially important since motivation has been established as such an important factor in middle school students. Providing students with their own set of manipulatives is yet another way to boost students' motivation in mathematics.

Teaching the Procedure

Mathematics instruction has not changed much in the last hundred years. It is a subject in which teachers use direct instruction to teach the core concepts. Mathematics lends itself to direct teaching because of its procedural nature. Teachers need to teach students the basic procedures of mathematics such as addition, subtraction, multiplication, and division. However, many students do not master the content after receiving direct instruction. The National Council of Teachers of Mathematics and the National Mathematics Advisory Panel have both supported balanced instruction that appropriately scaffolds learning for students. This more recent emphasis on scaffolding instruction and differentiation for various learners is the focus of the Teaching Procedures strategies found in this product.

In the past, teachers often provided direct instruction in the same format for all students in a class. A more effective strategy supported by research today is the use of scaffolded instruction. When instruction is scaffolded, teachers break instruction down into conceptual parts and provide additional explanation and support for students who need additional support in learning the material. When learning is scaffolded, topics often build off prior concepts, and the teacher makes this connection explicit for students rather than assuming they fully understand the connection. Additionally, scaffolded instruction often includes a focus on helping students think about their own thinking, or metacognition. Teaching mathematical procedures by having students think about their understanding helps them master concepts more easily. By helping students become comfortable with metacognition, the teacher encourages students to pause before rushing into computation.



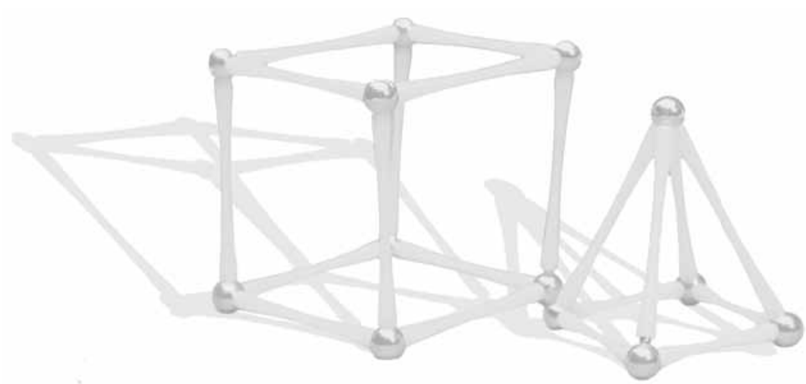
Research *(cont.)*

Teaching the Procedure *(cont.)*

The importance of connecting mathematics to everyday life when teaching procedures cannot be stressed enough. Nearly every mathematics teacher has heard a child ask, "When will I have to use this?" The answer is not obvious to many school children, and this question is sometimes difficult for teachers to answer. Putnam, Heaton, Prawat, and Remillard (1992), in their case study titled "Teaching the 'Hows' of Mathematics for Everyday Life," stresses the need for teachers to ensure that students consider real-world applications when learning mathematical procedures. Putnam et al. conducted an analytical case study of one fifth grade teacher's classroom and found that she made an effort to teach how to find averages using an everyday example, but she heavily emphasized computation. As a result, both the students and the teacher made calculation errors and did not realize it. Putnam et al. asserts that if the teacher had connected the procedure of teaching averaging to a real-world situation while stressing the importance of checking for realistic answers, she and her students would have realized that their answers did not make sense.

Putnam et al. (p. 176) noted, "Valerie omitted a number of questions in the student text that required reflection on the adequacy of averages for various purposes and the kinds of information that are lost with any average (e.g., change over time)." He goes on to state that if this teacher had included more reflection on averages while she taught the procedure, the students would have had a better understanding of the concept and been better able to apply it to new situations.

Focused practice is another essential strategy in teaching mathematical procedures to students. Students need opportunities to practice newly learned skills in order to master them. In *Mathematics Classrooms that Promote Understanding*, professors Thomas Carpenter and Richard Lehrer of the University of Wisconsin, Madison, differentiate between the purposes for mathematics practice. They note, "for understanding to develop on a widespread basis, tasks must be engaged in for the purposes of fostering understanding, not simply for the purpose of completing the task" (1999 p. 24). This distinction is evident in the focused practice strategies found in *Strategies for Teaching Mathematics*. Focused practice breaks down larger concepts into smaller units. Students have the opportunity to practice the smaller elements of larger concepts so that they can fully understand them. By providing more in-depth practice on these skills, students have an easier time putting the pieces together to deeply understand their related, larger concepts. For example, in algebra, focused practice in collecting like terms helps students understand this important procedure before proceeding to fully reducing linear equations.



Research *(cont.)*

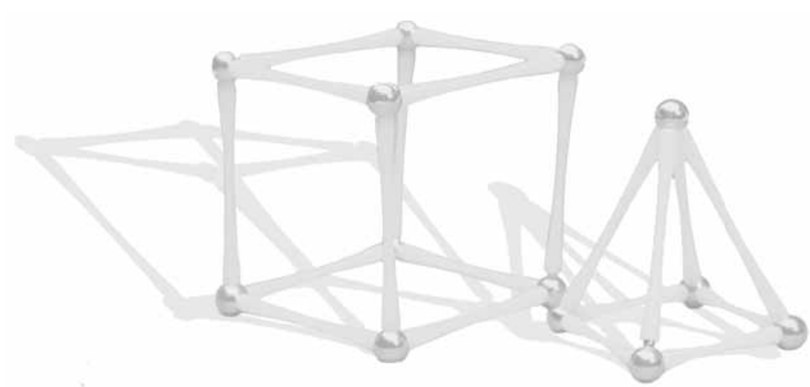
Teaching the Procedure *(cont.)*

In the last several years, research on mathematics instructional methods have found that more creative forms of instruction can be more effective than traditional direct instruction. A large amount of this research has studied the use of children's literature to teach mathematical skills. A controlled, experimental study found that kindergarten students who learned tasks such as number combination and classification through children's literature had better mastery of the procedures than those who did not (Hong 1996). Strategies for using children's literature in the classroom are also included in this book.

Understanding Problem Solving

Problem solving is one of the most difficult skills for students to learn in mathematics. Problem solving requires students to apply their knowledge to situations that require the student to identify the problem, draw on existing skills, and form a solution to the problem. This complex process can be quite difficult for many students. However, problem solving is the cornerstone of all mathematics. Throughout students' lives, they will need to use their understanding of mathematics to solve problems in their everyday lives. Many students look at problem solving as a mystery. In fact, problem solving can be made much simpler for students simply by teaching them strategies for solving a variety of problems. Strategies such as working backwards, looking for a pattern, and drawing a diagram are useful in solving problems at all levels. Twelve separate problem-solving strategies are included in this product. Each of these problem-solving strategies includes grade-level examples, making them easy to implement.

Although problem solving is a complex process, researchers argue that educational reform in mathematics should shift its focus to problem solving (Carpenter et al. 1999). These authors argue that problem solving in mathematics should involve more than the application of concepts. Carpenter et al. argue that problem solving should also include reflective learning of new concepts. By suggesting this, the authors conclude that problem solving is not just a method for application of previously learned skills. Rather, students can apply previous learning and even pursue new understandings as a result of problem solving. *Strategies for Teaching Mathematics* supports this method by showing teachers how they can help students learn to use multiple strategies to solve problems.



Research *(cont.)*

Understanding Problem Solving *(cont.)*

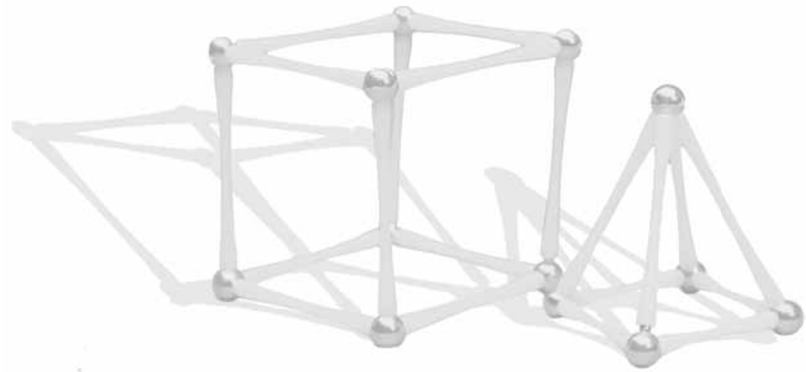
Acting It Out or Using Concrete Materials is one strategy found in this book. Using manipulatives to solve problems helps students understand concepts better. Researchers have conducted numerous studies on the use of manipulatives to help students with learning disabilities solve problems. One study conducted by Lynne Marsh and Nancy Cooke (1996) found that students with learning disabilities were more successful in solving problems when they were able to use manipulatives. These researchers propose that the visual and tactile natures of the manipulatives help students understand the abstract component of the problems.

Other problem-solving strategies help students use the foundation of our numeral system to make problems easier to solve. Using Simpler Numbers is a strategy that requires students to understand how they can recombine numbers to easily solve problems using difficult numbers. This is an especially useful strategy for doing mental math, which is stressed in today's schools. For example, many studies, such as the Trends in International Mathematics and Science Study (Mullis, Martin, and Foy 2007), require students to quickly solve problems. A sample fourth-grade problem is, "What is 3 times 23?" Many fourth graders would begin to solve this problem by writing it vertically on scratch paper. However, the strategy of Using Simpler Numbers found in this book would suggest that students multiply 3 times 20 mentally and then add 9 to it to get the correct answer of 69. When students have an understanding of how to use simpler numbers, they can solve problems more quickly.

Learning a variety of problem-solving strategies is also important for student success in mathematics. Students often rely on their familiarity with textbooks rather than their own understanding of problem solving to solve problems in mathematics (Glaser and Resnick 1989). The problem-solving strategies found in this book not only help students see problems in new ways, but they also provide teachers with the background information necessary to teach every step of the process to their students. Consequently, teachers can feel confident that they are using mathematically sound strategies that are also developmentally appropriate and research based.

Using Mathematical Games

Students need ample opportunities to practice in order to be able to execute procedures automatically without conscious thought (Kilpatrick, Swafford, and Findell 2003). Playing games with the goal of reinforcing skills, rehearsing information, and building retention of key concepts is one way to accomplish this goal. When students play games in mathematics, time on task is high, and students solidify their understanding and skills. In addition to being fun, games provide the added value of providing students with opportunities to rehearse and understand concepts (Sousa 2006).



Research *(cont.)*

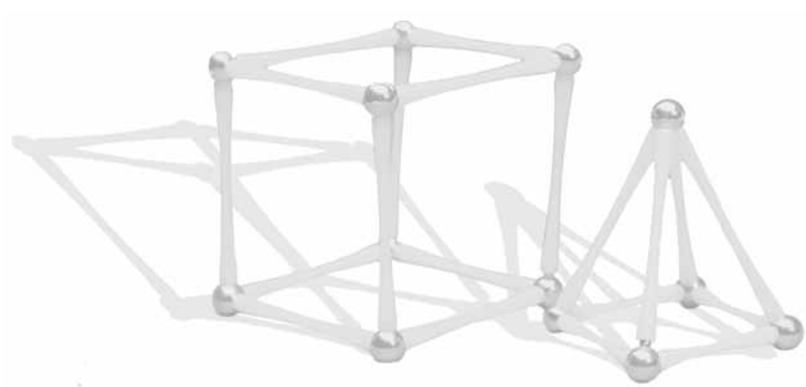
Using Mathematical Games *(cont.)*

Teachers know that using games in mathematics is motivating for their students. Often, ideas are shared between teachers of games that are successful. However, many teachers do not think games are "real math" (Moyer 2001). Mathematical games can be used in a more formal way in the classroom. They can be used to reinforce concepts, develop quick recall of facts, and provide targeted practice for students. Additionally, teachers can use games as an excellent observational assessment of students' mastery of skills.

Often, though, educators are unaware of the research literature surrounding the effect of mathematics games on students in the classroom. Teachers need to provide a variety of types of games in order to effectively engage both boys and girls in their classrooms. Research has shown that boys often prefer and learn more from competitive mathematics games, while girls engage more in less competitive games (Peterson and Fennema 1985). Additionally, all students respond well to games involving cooperative work. Therefore, teachers need to include a variety of games in the classroom and should include opportunities for teamwork.

Strategies for Teaching Mathematics includes a variety of games to address these needs. Physical games help kinesthetic learners remember the key concepts better and get students out of their seats. Playing board games or card games challenges the students and encourages learning. Playing games as a whole class on the board or overhead can serve as a form of informal assessment as the teacher checks for student understanding. These games are fun and educational and also help ensure that all students are successful. Large-group games provide a safe environment for lower-level students to take risks. These students are supported by their teammates and can stretch their thinking outside of their comfort zones.

Concept-based games are a category of games designed to reinforce newly learned topics. They are also excellent to use for standardized test preparation to remind students of key concepts. MATHO games are a classic twist on *BINGO*, but they can be adapted to a variety of situations. Furthermore, the game cards can be differentiated for different levels of learners in the same class. Category Quiz games are especially motivating for students, because most students are interested in competing to answer questions quickly. Matching games with mathematics skills is the perfect way to keep students engaged without them realizing they are even working.



Research *(cont.)*

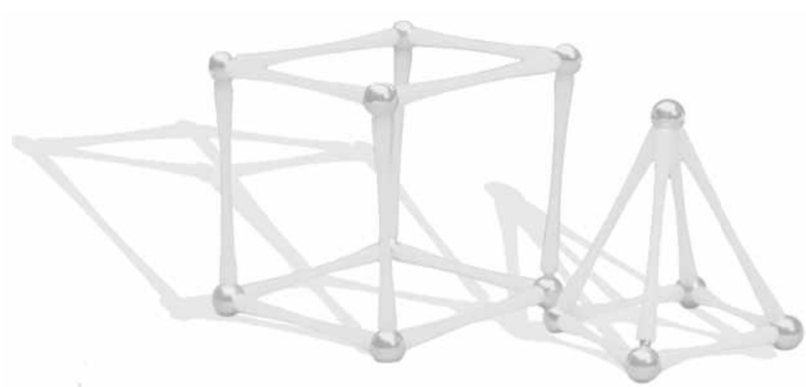
Assessing Mathematical Thinking

Assessment has become a major focus of educational reform since the authorization of the No Child Left Behind Act in 2001. Much of this emphasis has been placed on standardized assessment. However, this data is usually only available to teachers once per year and often does not arrive until students have advanced to the next grade. Standardized assessment data also does not provide insight into a student's actual mathematical thinking. As a result, educational reformers have supported the use of formative assessments by teachers. Formative assessments can include standardized tests, but usually include less formal tests such as observation, performance tasks, and student-work portfolios. When using formative assessment, teachers track students' progress periodically and adjust their instruction to meet the needs of the class.

A less common form of assessment included in *Strategies for Teaching Mathematics* is student interviewing. Interviewing students about their thinking in solving tasks can provide great insight into the student's thought process and depth of understanding of underlying concepts. Teachers have found interviews to be very successful in helping them understand their students' knowledge of mathematics (Clarke, Abrams, and Madaus 2001).

Performance tasks are another type of alternative assessment that can help teachers understand students' mathematical thinking. In this form of assessment, the teacher usually works with an individual student and asks him or her to complete a problem or mathematics-related activity. The teacher will observe the student and ask questions, while the problem is solved, to gain an understanding of what the student is thinking as he or she completes the problem. Performance tasks can also be used to assess students' creativity in solving problems (Haylock 1987). Students who creatively solve a problem or change approaches when they become stuck in completing a problem show that they can apply their knowledge in a different way to solve the problem (Haylock 1987). This view of assessment is non-traditional, but can be especially useful for challenging students in the mathematics classroom.

An additional strategy found in *Strategies for Teaching Mathematics* that is less commonly used is student self-assessment. Self-assessment can provide valuable information to mathematics teachers. The National Council of Teachers of Mathematics (1991) suggests that student self-assessment is essential to their learning. Research at the University of Michigan also shows that students need to be guided in completing self-assessments of their mathematics work and knowledge (MacIver 1987). Over 1,500 fifth graders were studied, and the researcher found that these students largely assessed their own math skills as high. He found that students often used their own criteria for determining their efficacy in mathematics rather than grades given by teachers. Criteria students used included their friends' perceptions and their own confidence.



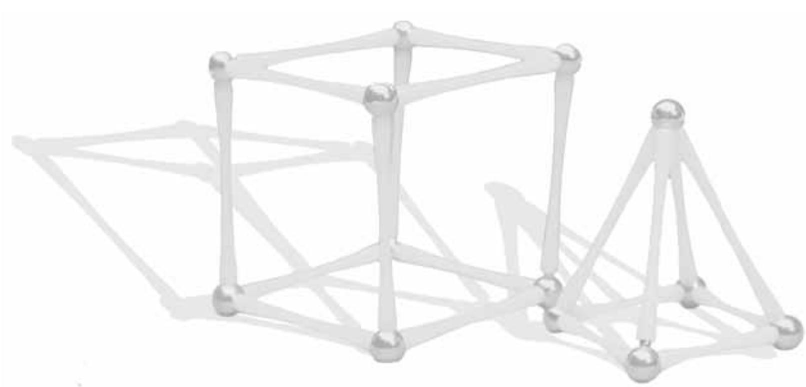
Research *(cont.)*

Assessing Mathematical Thinking *(cont.)*

This research suggests that students need to be provided with clear criteria for evaluating their mathematical understanding. *Strategies for Teaching Mathematics* includes templates for helping students conduct objective self-evaluations. In addition, similar peer evaluation forms can be used by students to compare their beliefs about their own skills to that of their peers. By better assessing their own understanding, students can begin to target their own areas for growth and also continue to develop their own sense of efficacy in mathematics as they see growth in assessments over time.

Graphic organizers have been used by teachers to help students organize their writing for years. Graphic organizers are used less often in mathematics instruction and very rarely to assess mathematical thinking. Graphic organizers may be a unique form of assessment, but they can help the teacher gain a quick view of students' skills. Using a K W L chart before introducing a new concept helps the teacher determine students' prior knowledge. This is a key way to also learn if students hold misconceptions. At the end of a unit, K W L charts help teachers judge the level of learning the group has achieved. Graphic organizers can also be used to assess students' problem-solving skills and to provide guidance for students as they solve mathematical problems (Braselton and Decker 1994).

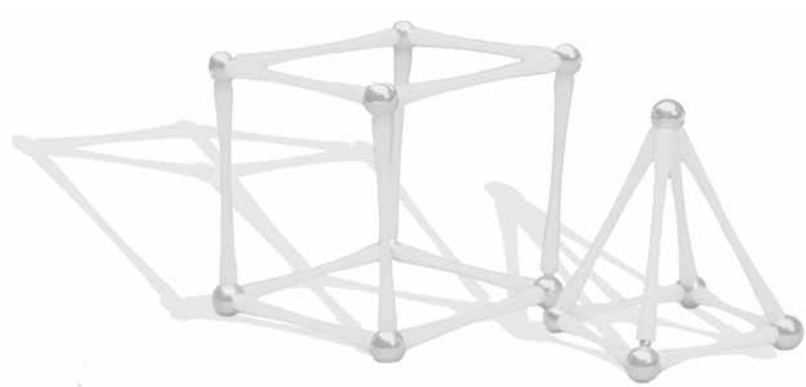
Journals are often used in subjects such as language arts and science, but they are rarely used in mathematics. Students are often surprised when they learn that some of the best mathematicians in history from Galileo to Einstein kept detailed journals of their work. Journals help the reader understand what the writer was thinking, but in the mathematics classroom, they also help students clarify their understanding. When students explain how they solved problems and connect current problems they have solved to previous ones, teachers know they have a solid understanding of the concepts. Journals also meet NCTM standards for communication. They also "give learners an opportunity to communicate their ideas and to clarify, refine, and consolidate their thinking" (McIntosh and Draper 2001).



Research *(cont.)*

Conclusion

The research on best practices in mathematics instruction has been expanded extensively in the last two decades. Our knowledge of the best ways to manage the mathematics classroom and how to present material to students has been enhanced with a new focus on meeting the needs of diverse learners. Additionally, the integration of content areas has also brought important insight into the use of academic vocabulary in schools. Marzano, Pickering, and Pollock (2001) note, “one of the most generalizable findings in the research is the strong relationship between vocabulary and important factors, such as intelligence, one’s ability to comprehend new information, and one’s level of income.” Most importantly, our understanding of teaching mathematical procedures has revealed that scaffolded instruction is essential to enable all students to learn. In *Working on the Work*, Phillip Schlechty (2002 p. xiv) concludes, “authentically engaged students see meaning in what they are doing.” Incorporating all of this new research into the classroom can be a daunting task. *Strategies for Teaching Mathematics* brings all of the research into a teacher-friendly guide that will support the goal of helping all students achieve the mathematical knowledge necessary to be successful in the 21st century.



How to Use This Book

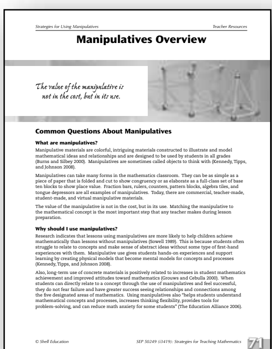
Integrating This Resource into Your Mathematics Curriculum

When planning instruction with *Strategies for Teaching Mathematics*, it is important to look ahead at the instructional standards you plan to teach to see where strategies and instructional tools from this resource can best be utilized. This resource can help you plan the strategies you will use in each component of your lesson plan and develop lessons that will engage your students and increase their conceptual understanding of mathematics.

The strategies in this resource are divided into the following sections:

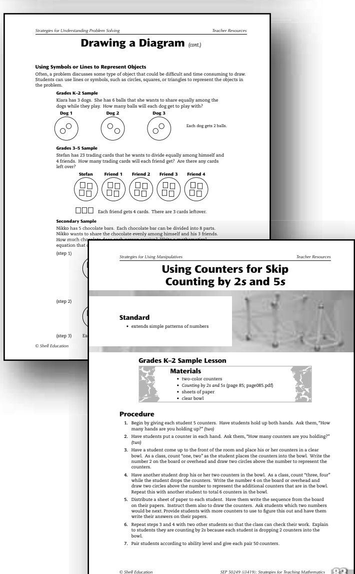
- Strategies for Vocabulary Development
- Strategies for Using Manipulatives
- Strategies for Teaching Procedures
- Strategies for Understanding Problem Solving
- Strategies for Using Mathematical Games
- Strategies for Assessing Mathematical Thinking

The information below describes the components within each section of *Strategies for Teaching Mathematics*.



Section Overview Pages

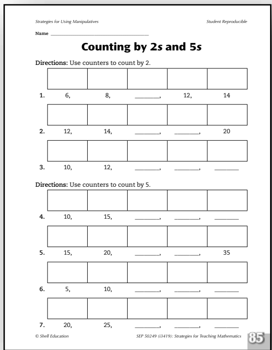
- Each section begins with an overview of the concept. Tips and strategies are described and supported with research.



Sample Problems and Lesson Plans

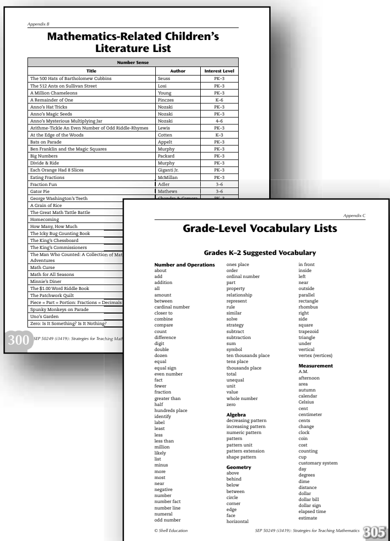
- Each section has sample problems or sample lessons to concretely illustrate how the strategies can be used with commonly taught mathematical topics. Teachers can use the lessons and problems as they are written, or they can reference these models as they begin creating their own lessons and problems.
- Each lesson includes a description of how the strategy benefits different kinds of learners as well as provides differentiation suggestions.

How to Use This Book *(cont.)*



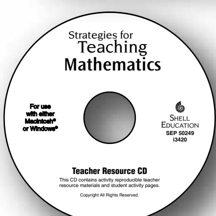
Student Reproducibles

- All the sample lessons have student pages included for teacher use. These pages are needed to fully implement the lessons. Many of these pages can also be used as teachers write their own lessons.
- The student reproducibles are included on the CD for easy printing or reference. A list of the filenames is included in Appendix E.



Appendices

- The Appendices provide great supplemental information, such as mathematics-related children's literature and suggested grade-level vocabulary words.



Teacher Resource CD

- The Teacher Resource CD is intended to allow for easy access to the resources in this book. PDFs of all the student reproducibles and additional resources are included on the CD.
- See Appendix E (pages 317–318) for a complete list of all the files on the Teacher Resource CD.

Correlation to Standards

The No Child Left Behind (NCLB) legislation mandates that all states adopt academic standards that identify the skills students will learn in kindergarten through grade 12. While many states had already adopted academic standards prior to NCLB, the legislation set requirements to ensure the standards were detailed and comprehensive. Standards are designed to focus instruction and guide adoption of curricula. Standards are statements that describe the criteria necessary for students to meet specific academic goals. They define the knowledge, skills, and content students should acquire at each level. Standards are also used to develop standardized tests to evaluate students' academic progress. In many states today, teachers are required to demonstrate how their lessons meet state standards. State standards are used in the development of Shell Education products, so educators can be assured that this product meets strict academic requirements.

How to Find Your State Correlations

Shell Education is committed to producing educational materials that are research and standards based. In this effort, all products are correlated to the academic standards of the 50 states, the District of Columbia, and the Department of Defense Dependent Schools. A correlation report customized for your state can be printed directly from the Shell Education website: **http://www.shelleducation.com**.
If you require assistance in printing correlation reports, please contact Customer Service at 1-800-877-3450.

McREL Compendium

Shell Education uses the Mid-continent Research for Education and Learning (McREL) Compendium to create standards correlations. Each year, McREL analyzes state standards and revises the compendium. By following this procedure, they are able to produce a general compilation of national standards. Each differentiation strategy in this book is based on one or more McREL content standards. This chart shows the McREL standards that correlate to each lesson used in the book.

NCTM Content Standard	Lesson Title and Page Number
Number and Operations Standard—Understand numbers, ways of representing numbers, relationships among numbers, and number systems	Alike and Different (p. 32); Total Physical Response (p. 39); Vocabulary Flip Book (p. 61); Content Links (p. 66)
Number and Operations Standard—Understand meanings of operations and how they relate to one another	Vocabulary Flip Book (p. 60); Missing Addends (p. 151)
Number and Operations Standard—Compute fluently and make reasonable estimates	Using Counters for Multiplications with Arrays (p. 86); Using Base Ten Blocks for Addition (p. 101); Using Base Ten Block for Division (p. 104); Using Algebra Tiles for Collecting Like Terms (p. 107); Alternative Algorithm for Addition (p. 136); Alternative Algorithm for Subtraction (p. 139); Alternative Algorithm for Multiplication (p. 142); Alternative Algorithm for Division (p. 145); Adding Fractions (p. 148)
Algebra Standard—Understand patterns, relations, and functions	Content Links (p. 68); Using Counters for Skip Counting by 2s and 5s (p. 83); Missing Addends (p. 151)
Algebra Standard—Represent and analyze mathematical situations and structures using algebraic symbols	Vocabulary Flip Book (p. 62); Using Algebra Tiles for Collecting Like Terms (p. 107); Colleting Like Terms (p. 154); Linear Equations (p. 157)

Standards are listed with permission of the National Council of Teachers of Mathematics (NCTM).
NCTM does not endorse the content or validity of these alignments.

Correlation to Standards *(cont.)*

NCTM Content Standard	Lesson Title and Page Number
Algebra Standard—Use mathematical models to represent and understand quantitative relationships	Using Counters for Solving Linear Equations (p. 89)
Measurement Standard—Understand measurable attributes of objects and the units, systems, and processes of measurement	Alike and Different (p. 33); Root Word Tree (p. 49); Sharing Mathematics (p. 55)
Measurement Standard—Apply appropriate techniques, tools, and formulas to determine measurements	Sharing Mathematics (p. 56); Using Linking Cubes for Understanding Area and Perimeter (p. 95); Calculating Area (p. 160); Calculating Perimeter (p. 163); Calculating Volume (p. 166); Measuring Length (p. 172); Converting Measurements (p. 175)
Geometry Standard—Analyze characteristics and properties of two- and three-dimensional geometric shapes	Total Physical Response (p. 38); Alike and Different (p. 34); Math Hunt (p. 43); Root Word Tree (p. 48); Root Word Tree (p. 51); Sharing Mathematics (p. 57); Finding Missing Angle Measures (p. 169)
Geometry Standard—Specify locations and describe spatial relationships using coordinate geometry and other representational systems	Total Physical Response (p. 40); Math Hunt (p. 44)
Geometry Standard—Apply transformations and use symmetry to analyze mathematical situations	Using Pattern Block for Transformation in Quadrant 1 (p. 114); Using Pattern Blocks for Rotational Symmetry (p. 119)
Geometry Standard—Use visualization, spatial reasoning, and geometric modeling to solve problems	Using Pattern Blocks for Spatial Visualization (p. 111)
Data Analysis and Probability Standard—Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	Using Linking Cubes for Data Analysis (p. 92); Creating Graphs (p. 178)
Data Analysis and Probability Standard—Select and use appropriate statistical methods to analyze data	Math Hunt (p. 45); Content Links (p. 67); Findign Central Tendencies (p. 181)
Data Analysis and Probability Standard—Understand and apply basic concepts of probability	Using Linking Cubes for understandign Probability (p. 98)

NCTM Process Standard	Lesson Title and Page Number
Communication Standard—Use the language of mathematics to express mathematical ideas precisely	Alike and Different (p. 30); Total Physical Response (p. 36); Math Hunt (p. 41) Root Word Tree (p. 46); Sharing Mathematics (p.53); Vocabulary Flip Book (p. 58); Content Links (p. 64)
Problem Solving Standard—Apply and adapt a variety of appropriate strategies to solve problems	Drawing a Diagram (p. 199); Acting It Out or Using Concrete Materials (p. 203); Creating a Table (p. 207); Looking for a Pattern (p. 210); Guessing and Checking (p. 214); Creating an Organized List (p. 217); Working Backwards (p. 220); Creating a Tree Diagram (p. 223); Using Simpler Numbers (p. 226); Using Logical Reasoning (p. 229); Analyzing and Investigating (p. 232); Solving Open-Ended Problems (p. 235)

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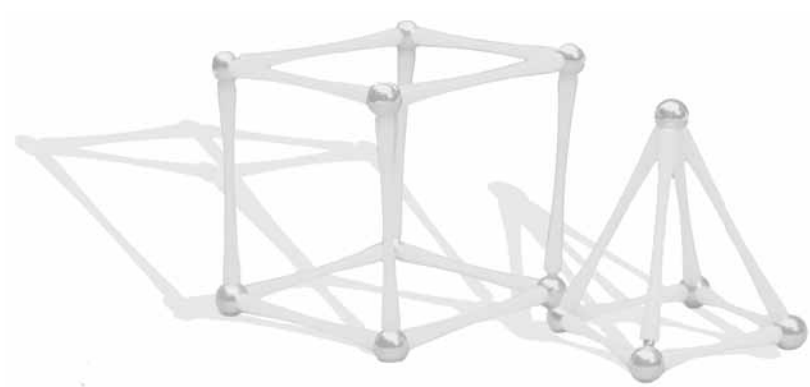
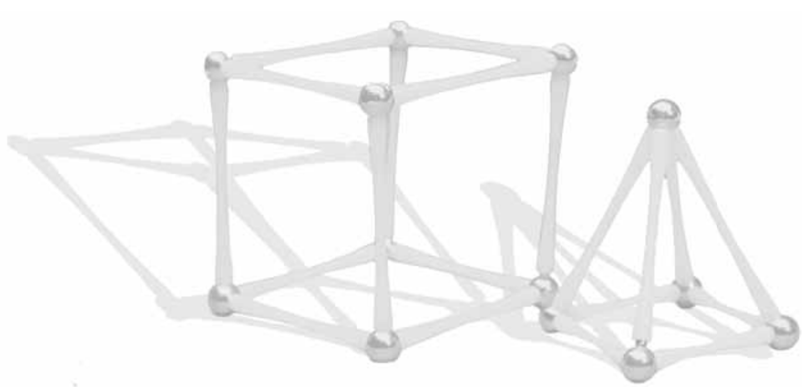
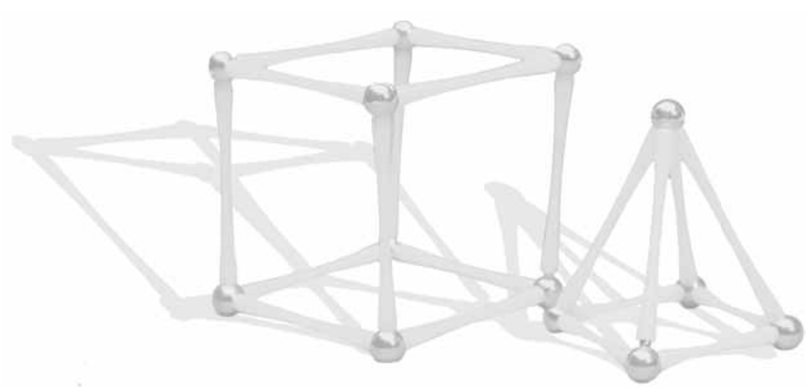


Strategies for Vocabulary Development

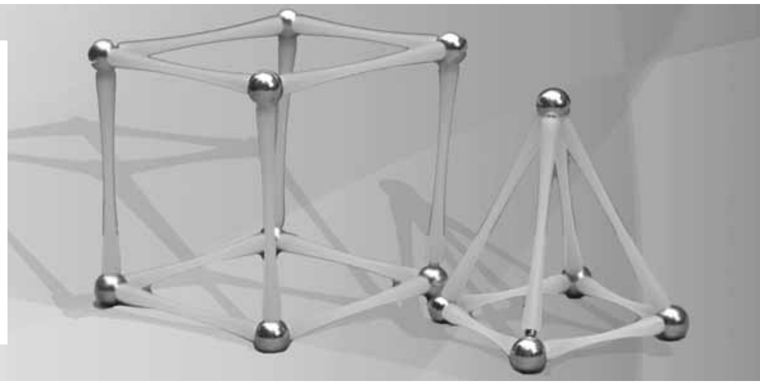




Vocabulary Development Overview

"Vocabulary is truly learned when it can be used naturally in speaking, listening, and writing."

Carol Santa, Lynn Havens, Bonnie Valdes



Common Questions About Vocabulary Development

Why do we need to teach mathematical vocabulary?

In general, words communicate content. If students are unfamiliar with the vocabulary particular to a content area, they will struggle to grasp that content (Santa, Havens, and Valdes 2004).

Mathematics is often seen as a subject that communicates through the manipulation of symbols in orderly ways, not as one that uses words to express ideas. This view is unfortunate and misleading (Burns 1995). In today's classrooms, students must learn content and develop the vocabulary associated with that content to understand, read, write, and speak about what they learn. Therefore, it is essential that teachers deliver the best possible instruction in mathematical vocabulary development.

Mathematical concepts and vocabulary are directly linked, but appropriate mathematical language is rarely used in everyday life. Students need to make connections themselves in order to truly understand the meaning of mathematical vocabulary. Thus, it is imperative that teachers embed learning and vocabulary in contexts that children can understand (Dunlap and Weisman 2006).

How do students best learn mathematical vocabulary?

Teaching mathematics vocabulary is the same as teaching vocabulary in a new language. People from England speak *English*. People from Poland speak *Polish*. In a science class, teachers and students have to speak *Sciencelish*. In a mathematics class, teachers and students must speak *Mathlish*.

By the end of the school year, your students should become bilingual in *Mathlish*. To accomplish this goal, they must *think* in the new language. Therefore, children must think in *Mathlish* to become bilingual in mathematics.

Much like the old strategies for teaching spelling, it is tempting to have students look up the vocabulary words in a dictionary, copy the definition, and write each word multiple times. But, research has shown that we do not learn vocabulary by looking up words in a dictionary and memorizing definitions (Nagy and Scott 2000). Students need to be actively involved to help them retain information. In a series of studies, Bos and Anders (1992) found that students who explored vocabulary words through interactive techniques performed better than students who simply memorized definitions. To help students learn more effectively, teachers should immerse students in hands-on activities, repeatedly expose them to vocabulary, use graphic organizers, and allow them to develop their own strategies.

Vocabulary Development Overview *(cont.)*

Common Questions About Vocabulary Development *(cont.)*

Research also shows that vocabulary knowledge increases when:

- students (not teachers) make connections between vocabulary terms.
- students create their own images and actions to represent word meanings.
- students use new vocabulary terms in multiple ways—writing, talking, organizing, using graphics, etc. (Blachowicz and Fisher, 2000).

Although becoming bilingual in *Mathlish* may seem like a daunting task, the vocabulary-development strategies in this section will help students think about and understand mathematics in ways that are meaningful to them. When students truly understand the meaning of a word, they will be able to use it effectively, and that is powerful knowledge!

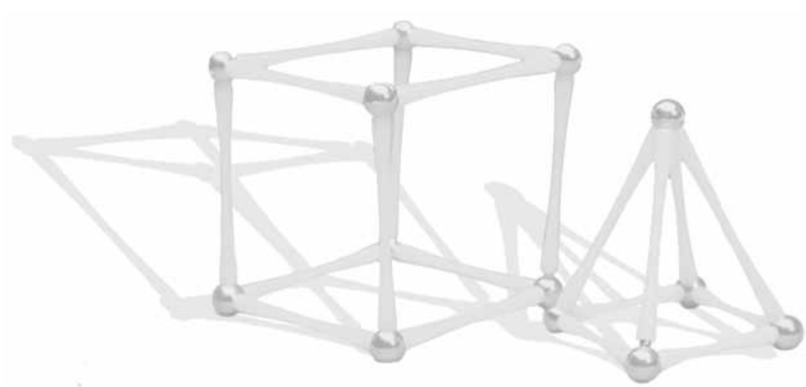
How do I decide which vocabulary words to teach?

Teachers need to ask themselves—*What words and concepts do I want my students to remember in 10 years?* (Santa, Havens, and Valdes 2004). In the real world, it is not necessary for students to have memorized textbook definitions of the vocabulary words taught in each grade. However, students do need to have working understandings of mathematical language and be able to apply that knowledge to other aspects of mathematics and everyday life.

Follow the steps below when choosing vocabulary words to frontload a lesson:

1. Read the standard or objective correlated to the lesson that will be taught (e.g., Students will compare and contrast two-dimensional shapes such as squares, rectangles, triangles, hexagons, rhombi, and trapezoids).
2. Choose specific content words necessary for student understanding to access the mathematics in that standard or objective (e.g., two-dimensional, square, rectangle, triangle, hexagon, rhombus, and trapezoid).
3. Consider other vocabulary words that students will need to grasp in order to have mastery of the standard or objective (e.g., compare, contrast, corner, vertex, side, parallel, perpendicular).
4. Combine the lists to create a master list of vocabulary words for the lesson. As a frontloading activity, have students work with this list of words, using a vocabulary strategy from this section.

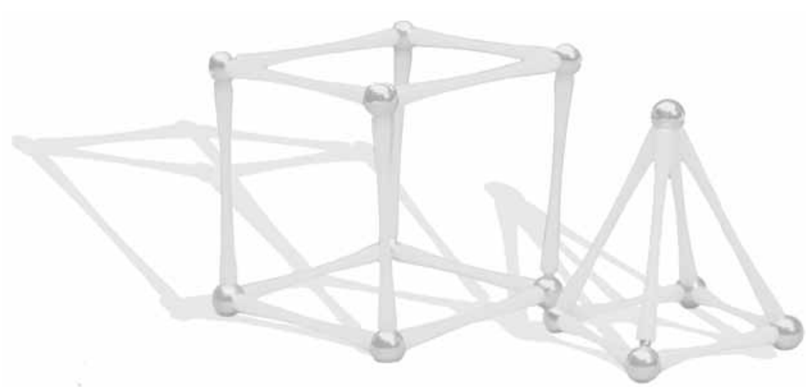
There are also sample vocabulary lists provided in Appendix C (pages 305–310) that have been grouped by grade-level range and mathematical strand. Although they may not list every word that students need to know at each grade level, they give teachers a "jumping off point."



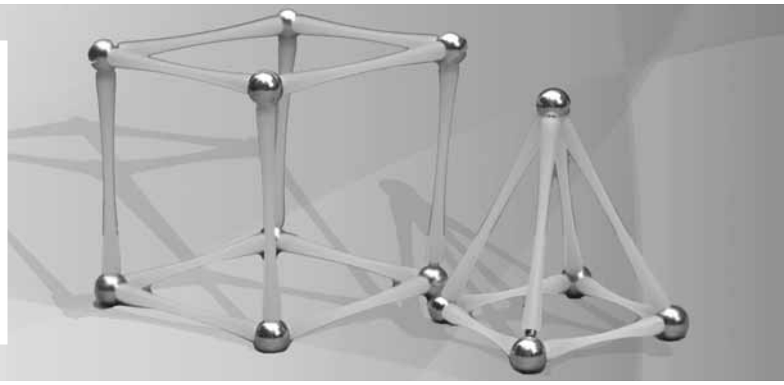
Vocabulary Development Overview *(cont.)*

Tips for Implementing Vocabulary Strategies Successfully

- Introduce the vocabulary words using a vocabulary activity before teaching the lesson. This frontloads the words students will need to access the mathematical content. It is also a tremendous scaffolding opportunity for English language learners.
- Lessons should cover as many of the four domains of language as possible: listening, speaking, reading, and writing. It is important for students to practice using new vocabulary in all four domains in order to take ownership of the words and use them independently. Such repetition will provide students with multiple exposures to a word and give them opportunities to practice the word in a variety of contexts.
- Before students are asked to complete an independent or out-of-seat vocabulary activity, clearly state the purpose for the activity, the behavior expectations, and the consequences for not following those expectations.
- Actively monitor the processes students are using in the lessons. For example, in a speaking activity, ensure that students are correctly practicing mathematical vocabulary, and if not, quickly clarify any vocabulary misconceptions or misunderstandings.
- Vocabulary activities vary in the amount of instructional time necessary for completion. Choose activities based on the allotted classroom time for vocabulary. A vocabulary lesson will take less time as students become familiar with the procedures and expectations.
- Plan extra time when introducing a new vocabulary activity.
- Sometimes an activity will flow better with more vocabulary words. Use this opportunity to add in past vocabulary words. It is always good to review, review, review!
- If students struggle with a vocabulary activity, it is helpful to repeat it the following day.
- It may be beneficial to team with teachers in other content areas and use the same vocabulary-development activities. This will familiarize students with the activities and increase the effectiveness and timeliness of vocabulary activities.
- Give students positive reinforcement as they acquire and master the vocabulary words (certificates, rewards, free time, etc.).



Alike and Different



Standards

- understands level-appropriate vocabulary
- uses level-appropriate vocabulary in speech

Materials

- *Alike and Different* (page 35; page035.pdf)

Background Information

The Alike and Different strategy (Beck, McKeown, and Kucan 2002) gives students the opportunity to examine ways in which selected vocabulary words are both alike and different. It is best utilized after students have had some exposure to the chosen vocabulary words because students are required to analyze the key aspects of the words and make connections that deepen their understanding. This strategy can be either oral or written. Students should be provided time to discuss the words, the connections that exist among the words, and the reasons why students identified those connections.

Procedure

1. Choose a list of vocabulary words that are associated with the content lesson being taught.
2. Pair the words in a way that makes sense. You will be asking students how the pairs of words are both alike and different.
3. Write the word pairs on the board or overhead. Distribute copies of *Alike and Different* (page 35) to students and have them record each word pair on the activity sheet. (Templates for comparing four or six pairs of words are included on the Teacher Resource CD: *alike4.pdf* and *alike6.pdf*.)
4. Read the first pair of words aloud. Have students repeat the words after you.
5. Ask students to tell what they already know about each word in the first pair.
6. Ask students to think about and discuss how the word pairs are alike. You can choose to have students complete this part of the activity independently, in pairs, in small groups, or as a class. Have students record their ideas in the correct place on the chart.
7. Ask students to think about how the words are different. They should record their ideas on the chart as well.
8. Repeat steps 4–7 with the remaining pairs of words. When the activity sheet is completed, review the word pairs as a class and talk about how the words are both alike and different.

Alike and Different *(cont.)*

Example:

Word pair: *square* **and** *cube*

What students might say: A *square* and a *cube* are both shapes. They both have straight sides. A *square* is a two-dimensional shape, and a *cube* is a three-dimensional shape. A *square* has 4 vertices and a *cube* has 8 vertices.

Word pair: *radius* **and** *diameter*

What students might say: The *radius* and the *diameter* are both measurements in a circle. They are both measurements from inside of a circle. The *radius* is half of the distance of the *diameter*. The length of the *diameter* will always be larger than the *radius*.

Differentiation

Above-Level Learners

Give students the list of words and let them make pairs themselves. Then, allow them to complete the activity as directed.

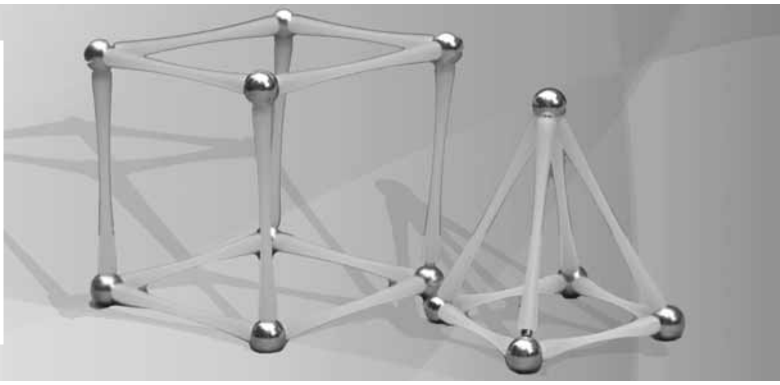
Below-Level Learners

Work with students to create a more detailed list of what they know about each word by brainstorming together. Ask both broad and specific questions to help students generate ideas. Chart their responses in a way that helps students see connections (e.g., a T-chart or a Venn diagram). Then guide students in completing their charts and verbalizing ways in which the words are alike and different.

English Language Learners

Allow students to draw pictures of what they know about each of the words in the pair. Then guide students in completing their charts using both pictures and words. Allow them to use their charts to help them verbalize ways in which the words are alike and different.

Alike and Different *(cont.)*



Standards

- counts whole numbers
- understands basic whole-number relationships

Grades K–2 Sample Lesson

Vocabulary Words

tens	place value	whole number
ones	regroup	hundreds

Procedure

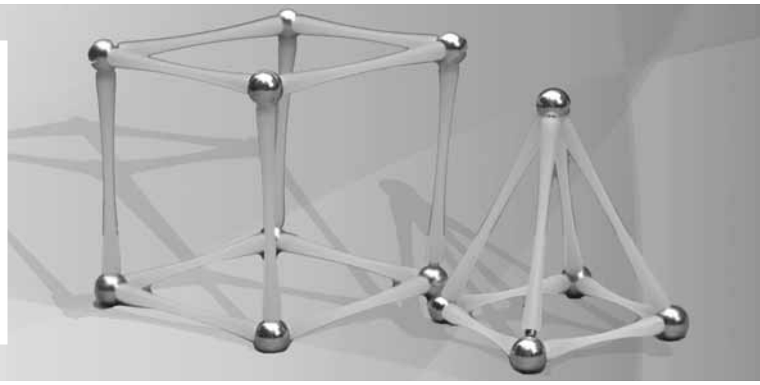
Differentiation suggestions for this strategy are provided on page 31.

1. The vocabulary words above have been paired.
2. Write the word pairs on the board or overhead. Distribute copies of *Alike and Different* (page 35) to students and have them record each word pair on the activity sheet.
3. Read the first pair of words aloud (*tens* and *ones*). Have students repeat the words after you.
4. Ask students to tell what they already know about the words *tens* and *ones*.
5. Ask students to think about and discuss how the words in the pair are alike. You can choose to have students complete this part of the activity independently, in pairs, in small groups, or as a class. Have students record their ideas in the correct place on the chart.
6. Ask students to think about how the words *tens* and *ones* are different. They should record their ideas on the chart.
7. Repeat steps 3–6 with the remaining pairs of words. When the activity sheet is completed, review the word pairs as a class and talk about how the words are both alike and different.

Alike and Different *(cont.)*

Standard

- knows approximate size of basic standard units and relationships between them



Grades 3–5 Sample Lesson

Vocabulary Words

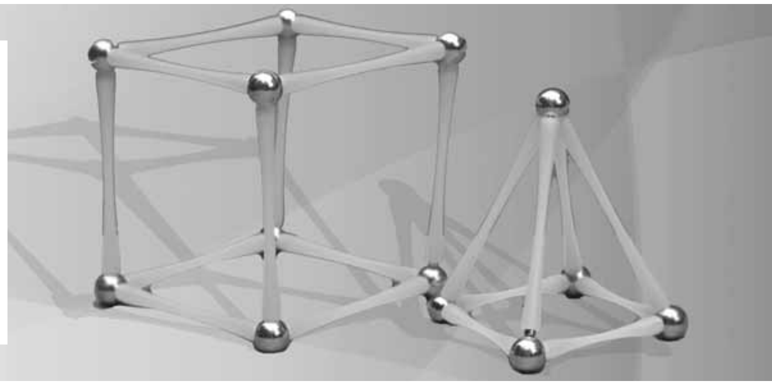
inch	ruler	measure
foot	yard stick	length

Procedure

Differentiation suggestions for this strategy are provided on page 31.

1. The vocabulary words above have been paired.
2. Write the word pairs on the board or overhead. Distribute copies of *Alike and Different* (page 35) to students and have them record each word pair on the activity sheet.
3. Read the first pair of words aloud (*inch* and *foot*). Have students repeat the words after you.
4. Ask students to tell what they already know about the words *inch* and *foot*.
5. Ask students to think about and discuss how the pair of words is alike. You can choose to have students complete this part of the activity independently, in pairs, in small groups, or as a class. Have students record their ideas in the correct place on the chart.
6. Ask students to think about how the words *inch* and *foot* are different. They should record their ideas on the chart.
7. Repeat steps 3–6 with the remaining pairs of words. When the activity sheet is completed, review the word pairs as a class and talk about how the words are both alike and different.

Alike and Different *(cont.)*



Standard

- understands the defining properties of triangles

Secondary Sample Lesson

Vocabulary Words

isosceles	right	angle
scalene	equilateral	triangle

Procedure

Differentiation suggestions for this strategy are provided on page 31.

1. The vocabulary words above have been paired.
2. Write the word pairs on the board or overhead. Distribute copies of *Alike and Different* (page 35) to students and have them record each word pair on the activity sheet.
3. Read the first pair of words aloud (*isosceles* and *scalene*). Have students repeat the words after you.
4. Ask students to tell what they already know about the words *isosceles* and *scalene*.
5. Ask students to think about and discuss how the pair of words is alike. You can choose to have students complete this part of the activity independently, in pairs, in small groups, or as a class. Have students record their ideas in the correct place on the chart.
6. Ask students to think about how the words *isosceles* and *scalene* are different. They should record their ideas on the chart.
7. Repeat steps 3–6 with the remaining pairs of words. When the activity sheet is completed, review the word pairs as a class and talk about how the words are both alike and different.

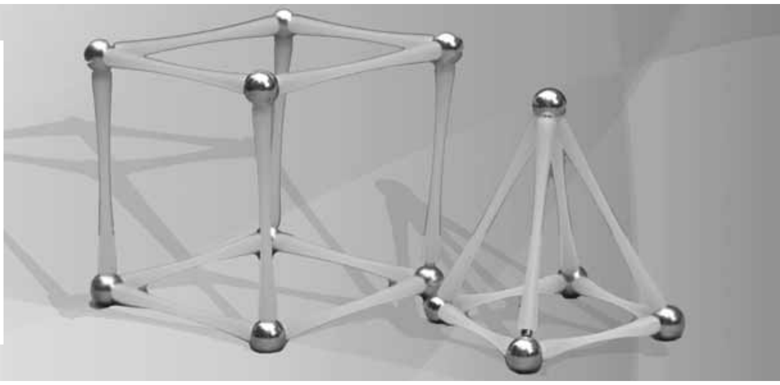
Name ____________________________________

Alike and Different

Directions: Write each word pair in the boxes below. Compare the words by listing the ways they are alike and different. Write your ideas in the columns below each word pair.

Word Pair 1	Word Pair 2	Word Pair 3
Alike: Different:	Alike: Different:	Alike: Different:

Total Physical Response



Standards

- understands level-appropriate vocabulary
- uses level-appropriate vocabulary in speech

Background Information

Total Physical Response (TPR) is a method of vocabulary instruction that allows students to use physical movements as a way to acquire language skills. When utilizing this vocabulary strategy, students apply actions with oral language to concepts and procedures. Teachers can have students perform the action while chorally saying the word related to it.

Procedure

1. Before beginning instruction, choose a set of vocabulary words related to the mathematical concept being covered in class.
2. Write the vocabulary words on the board or overhead.
3. Discuss each word and choose a physical action to represent the word. For example, to represent the word *circle*, students could join fingers above their heads and create the shape of a circle with their arms.
4. Once physical actions of each word are chosen, have students practice the actions either in pairs or in small groups.
5. Instruct all students to stand around the room.
6. Call out each vocabulary word, saying, "Show me ______."
7. Allow students to repeat the word chorally and demonstrate the correct physical action for the called word. (e.g., If the word *circle* was called, the students could form a circle above their heads with their arms.)
8. Demonstrate the correct action for students to self-check. Depending on time and the students' accuracy with the actions, it may be appropriate to call out each word more than once.
9. As an extension, hold up a word card displaying the vocabulary word that is called out. This will give students the opportunity to associate the written word with its verbal equivalent.

Total Physical Response *(cont.)*

Differentiation

Above-Level Learners

Instruct students to show two vocabulary words with one command. For example, "Show me *x-axis* and then show me *y-axis*." Make sure to demonstrate the correct actions for the students so that they also have the opportunity to self-check.

Below-Level Learners

Perform the physical action at the same time as the students instead of waiting and giving them an opportunity to self-check. Once students are comfortable with the words and actions, immediately return to the self-check method of demonstrating the action as discussed on page 36.

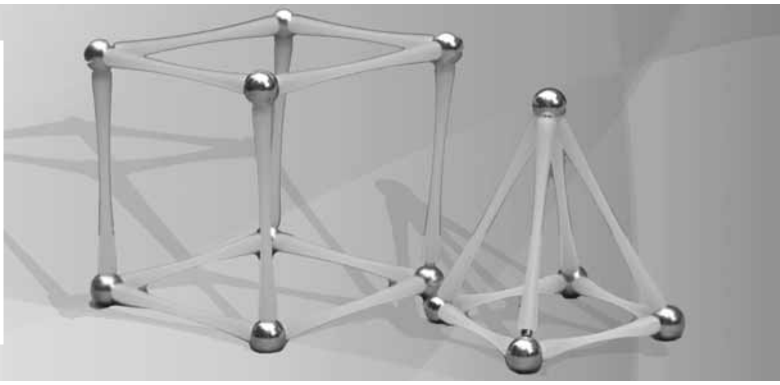
English Language Learners

Allow students to further practice the vocabulary words in pairs. One student plays the role of the teacher, and the other plays the role of the student. Once the student has correctly demonstrated all of the words, the roles are switched.

Total Physical Response *(cont.)*

Standard

- understands basic properties of and similarities and differences between simple geometric shapes



Grades K–2 Sample Lesson

Vocabulary Words

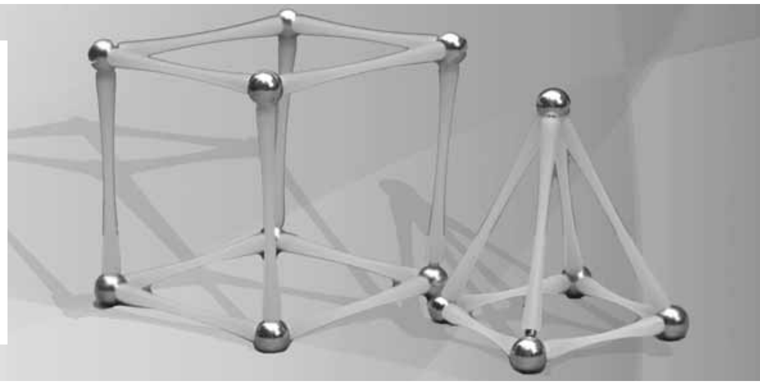
square rectangle triangle symmetrical

Procedure

Differentiation suggestions for this strategy are provided on page 37.

1. Write the vocabulary words above on the board or overhead.
2. Discuss each word and choose a physical action to represent the word.
3. Once physical actions of each word are chosen, have students practice the actions either in pairs or in small groups.
4. Instruct all students to stand around the room.
5. Call out the first vocabulary word saying, "Show me *square*."
6. Allow students to repeat the word chorally and demonstrate the correct physical action for the word *square*.
7. Demonstrate the correct action for students to self-check. Evaluate the time and the students' accuracy with the action. It may be appropriate to call out this word again after a few other words have been practiced.
8. Repeat steps 5–7 for the remaining words on the list.

Total Physical Response *(cont.)*



Standard

- understands the relative magnitude and relationships among whole numbers, fractions, decimals, and mixed numbers

Grades 3–5 Sample Lesson

Vocabulary Words

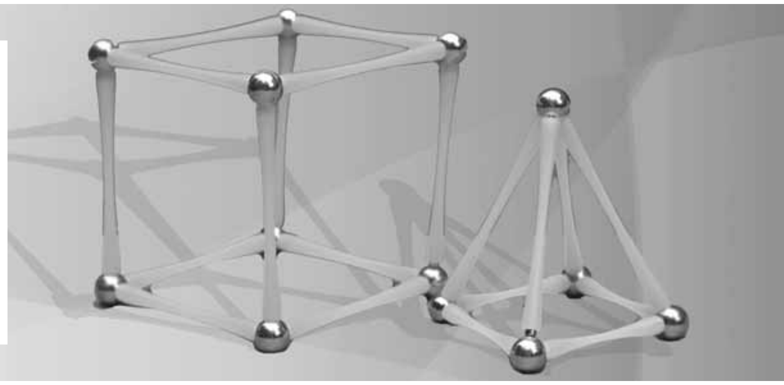
greater than   less than   equal to

Procedure

Differentiation suggestions for this strategy are provided on page 37.

1. Write the vocabulary words above on the board or overhead.
2. Discuss each word and choose a physical action to represent the word.
3. Once physical actions of each word are chosen, have students practice the actions either in pairs or in small groups.
4. Instruct all students to stand around the room.
5. Call out the first vocabulary word saying, "Show me *greater than*."
6. Allow students to repeat the word chorally and demonstrate the correct physical action for the words *greater than*.
7. Demonstrate the correct action for students to self-check. Evaluate the time and the students' accuracy with the action. It may be appropriate to call out this word again after a few other words have been practiced.
8. Repeat steps 5–7 for the remaining words in the list.

Total Physical Response *(cont.)*



Standard

- uses the coordinate system to model and to solve problems

Secondary Sample Lesson

Vocabulary Words

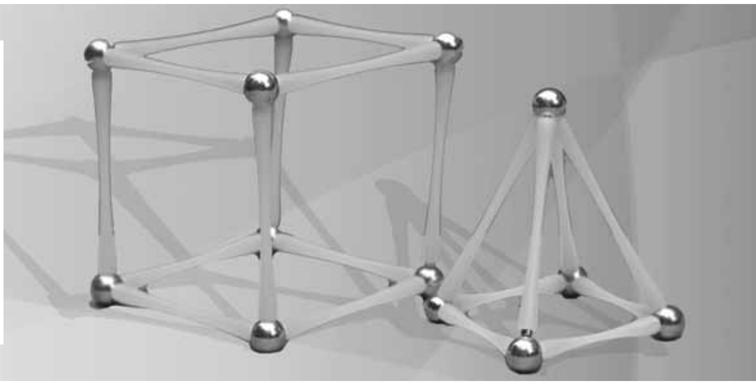
x-axis	y-axis	point
positive slope	negative slope	

Procedure

Differentiation suggestions for this strategy are provided on page 37.

1. Write the vocabulary words above on the board or overhead.
2. Discuss each word and choose a physical action to represent the word.
3. Once physical actions of each word are chosen, have students practice the actions either in pairs or in small groups.
4. Instruct all students to stand around the room.
5. Call out the first vocabulary word saying, "Show me *x-axis*."
6. Allow students to repeat the word chorally and demonstrate the correct physical action for the word *x-axis*.
7. Demonstrate the correct action for students to self-check. Evaluate the time and the students' accuracy with the action. It may be appropriate to call out this word again after a few other words have been practiced.
8. Repeat steps 5–7 for the remaining words in the list.

Math Hunt



Standards

- understands level-appropriate vocabulary
- uses level-appropriate vocabulary in speech

Background Information

Math Hunts are similar to scavenger hunts in that students are gathering items. However, Math Hunts deepen students' understanding of mathematical vocabulary because the objects and pictures they gather are related to specific mathematical concepts. This vocabulary strategy also allows students to have direct and indirect experiences with the words being studied through the process of collecting the objects and pictures.

Procedure

1. Before the Math Hunt begins, create a list of vocabulary words related to the mathematical concept being taught in class. The list should be comprised of both familiar and unfamiliar concepts and should include approximately 10 words.
2. Divide the class into teams (groups of three to four students) and provide each team with a Math Hunt list. (Ideally, this part of the activity should begin one week before the concept is actually taught in class. Then, the pictures and objects that students collect can be shared and discussed during the time allotted in the lesson for vocabulary development.)
3. Explain to students that over the next week they will be searching for things that represent each of the words on the Math Hunt list. Provide them with several examples until students grasp the concept of the activity.
4. Provide students with a point system to use when thinking about the types of objects they would like to collect. For example, 4 points for building a model or collage of the word, 3 points for bringing in a picture of the word, 2 points for finding a book or magazine about the word, and 1 point for drawing a picture of the word or writing a sentence using the word correctly.
5. Give students time to plan their "item-collection strategy" and brainstorm ideas for each word on the list.

Math Hunt *(cont.)*

Procedure *(cont.)*

6. Provide students with time in and out of class to search for pictures and objects related to the vocabulary words on the list. Allow teams to gather as many items as possible and encourage them to use books, the Internet, magazines, environmental print, and any other resources to find representations of the words on the list. Remind groups that they are competing with other teams so they should keep their ideas secret.
7. When the Math Hunt items are due, have teams organize their items by the word they represent.
8. Allow time for each group to share one item for each word on the list. Then, total the points for each team. The winning team gets to create a classroom display featuring the vocabulary words and the items collected.

Differentiation

Above-Level Learners

Before breaking into teams, present students with the list of words. Have them brainstorm any other words they may know that fit into the mathematical concept being studied and create a list. Once the list is completed, have students vote for the word they think best fits with the concept. Add this word to the Math Hunt word list.

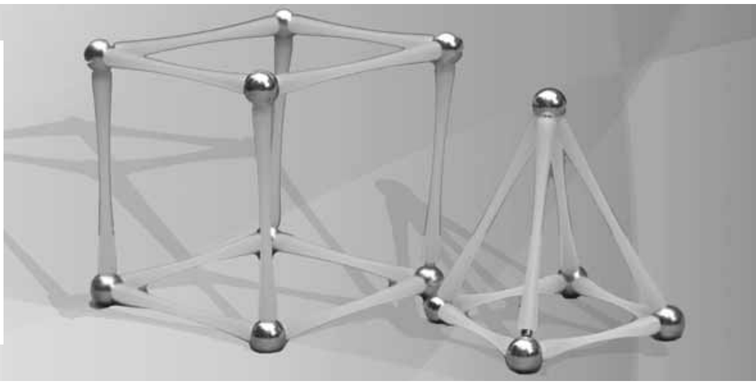
Below-Level Learners

Before breaking into teams, meet with students and discuss the Math Hunt word list as a group. Have students create a personal dictionary defining and graphically representing each word. Allow students to use their dictionaries as they brainstorm ideas for items to collect with their teams.

English Language Learners

Before breaking into teams, meet with students and discuss the Math Hunt word list as a group. Provide students with visual representations of the words and help them create group definitions. These students should be allowed to keep the pictures and definitions they create to assist them while they work with their teams.

Math Hunt *(cont.)*



Standard

- understands basic properties and similarities and differences between simple geometric shapes

Grades K–2 Sample Lesson

Vocabulary Words

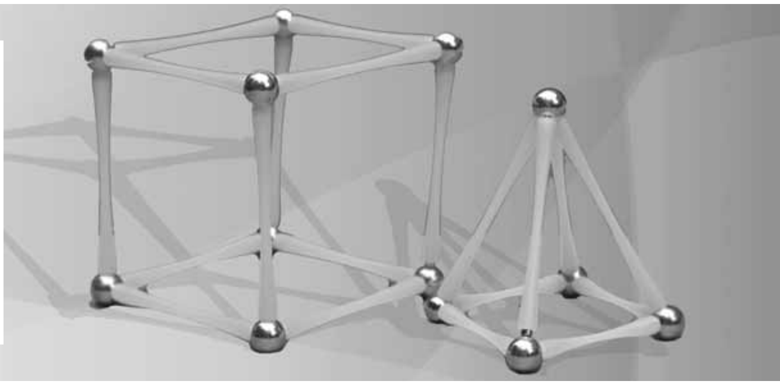
triangle	vertex (corner)	square	side
circle	curved	hexagon	
straight	rhombus	two-dimensional	

Procedure

Differentiation suggestions for this strategy are provided on page 42.

1. Use the words from the list above to create a Math Hunt list.
2. A week before you want to discuss this activity in class, divide the class into teams (three to four students) and provide each team with a list.
3. Explain to students that over the next week they will be searching for things that represent each of the words on the list. Provide them with several examples until students grasp the concept of the activity.
4. Provide students with a point system to use when thinking about the objects they would like to collect. For example, 4 points for building a model or collage of the word, 3 points for bringing in a picture of the word, 2 points for finding a book or magazine about the word, and 1 point for drawing a picture of the word or writing a sentence using the word.
5. Give students 15–20 minutes to plan their "item-collection strategy" and brainstorm ideas for each word on the list.
6. Provide students with time in and out of class to search for pictures and objects related to the vocabulary words on the list. Allow teams to gather as many items as possible and encourage them to use books, the Internet, magazines, environmental print, and any other resources to find representations of the words on the list. Remind groups that they are competing with other teams so they should keep their ideas secret.
7. When the Math Hunt items are due, have teams organize their items by the word they represent.
8. Allow time for each group to share one item for each word on the list. Then, total the points for each team. The winning team gets to create a classroom display featuring the vocabulary words and the items it collected.

Math Hunt *(cont.)*



Standards

- understands that shapes can be congruent or similar
- uses motion geometry to understand geometric relationships

Grades 3–5 Sample Lesson

Vocabulary Words

rotation	*x*-axis	translation	*y*-axis
reflection	symmetry	coordinate plane	
congruent	coordinates	similar	

Procedure

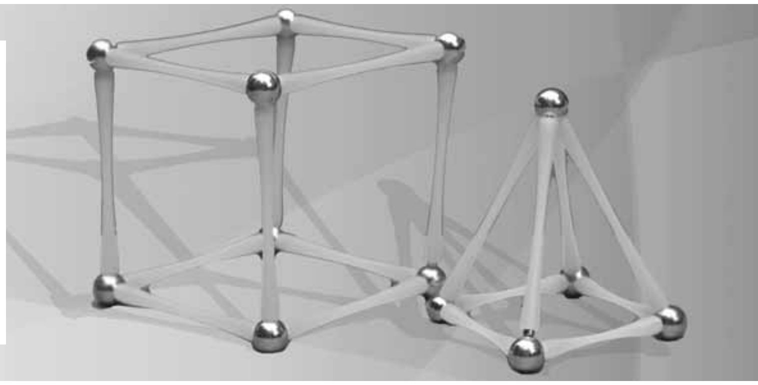
Differentiation suggestions for this strategy are provided on page 42.

1. Use the words from the list above to create a Math Hunt list.
2. A week before you want to discuss this activity in class, divide the class into teams (three to four students) and provide each team with a list.
3. Explain to students that over the next week they will be searching for things that represent each of the words on the list. Provide them with several examples until students grasp the concept of the activity.
4. Provide students with a point system to use when thinking about the objects they would like to collect. For example, 4 points for building a model or collage of the word, 3 points for bringing in a picture of the word, 2 points for finding a book or magazine about the word, and 1 point for drawing a picture of the word or writing a sentence using the word.
5. Give students 15–20 minutes to plan their "item-collection strategy" and brainstorm ideas for each word on the list.
6. Provide students with time in and out of class to search for pictures and objects related to the vocabulary words on the list. Allow teams to gather as many items as possible and encourage them to use books, the Internet, magazines, environmental print, and any other resources to find representations of the words on the list. Remind groups that they are competing with other teams so they should keep their ideas secret.
7. When the Math Hunt items are due, have teams organize their items by the word they represent.
8. Allow time for each group to share one item for each word on the list. Then, total the points for each team. The winning team gets to create a classroom display featuring the vocabulary words and the items it collected.

Math Hunt *(cont.)*

Standards

- understands basic characteristics of measures of central tendency, frequency, and distribution
- reads and interprets data in charts, tables, and plots



Secondary Sample Lesson

Vocabulary Words

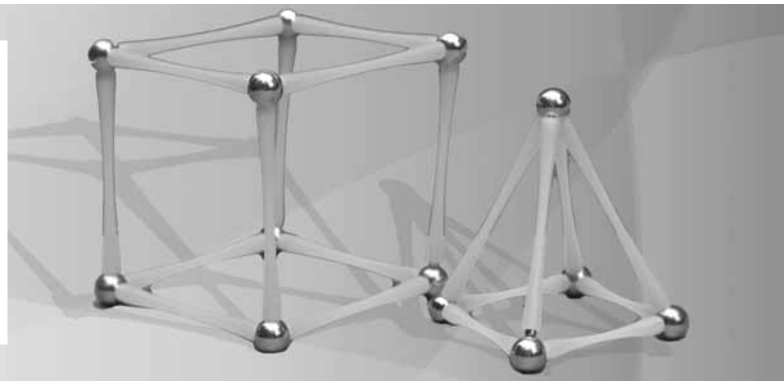
bar graph	mean	line graph	median
data	mode	circle graph	range
pictograph	analyze		

Procedure

Differentiation suggestions for this strategy are provided on page 42.

1. Use the words from the list above to create a Math Hunt list.
2. A week before you want to discuss this activity in class, divide the class into teams (three to four students) and provide each team with a list.
3. Explain to students that over the next week they will be searching for things that represent each of the words on the list. Provide them with several examples until students grasp the concept of the activity.
4. Provide students with a point system to use when thinking about the objects they would like to collect. For example, 4 points for building a model or collage of the word, 3 points for bringing in a picture of the word, 2 points for finding a book or magazine about the word, and 1 point for drawing a picture of the word or writing a sentence using the word.
5. Give students 10–15 minutes to plan their "item-collection strategy" and brainstorm ideas for each word on the list.
6. Provide students with time in and out of class to search for pictures and objects related to the vocabulary words on the list. Allow teams to gather as many items as possible and encourage them to use books, the Internet, magazines, environmental print, and any other resources to find representations of the words on the list. Remind groups that they are competing with other teams so they should keep their ideas secret.
7. When the Math Hunt items are due, have teams organize their items by the word they represent.
8. Allow time for each group to share one item for each word on the list. Then, total the points for each team. The winning team gets to create a classroom display featuring the vocabulary words and the items it collected.

Root Word Tree



Standards

- understands level-appropriate vocabulary
- uses level-appropriate vocabulary in speech

Materials

- *Root Word Tree* (page 50; page050.pdf)

or

- *Root Word Map* (page 52; page052.pdf)

Background Information

The *Root Word Tree* is a graphic organizer that allows students to examine the different parts of a single vocabulary word. When using the *Root Word Tree*, students place a vocabulary word at the base of the tree and break apart the word into recognizable chunks to help them decipher its meaning. Students extend their word knowledge by writing words that are associated with the word parts to help them remember the definition.

Procedure

1. Choose a list of vocabulary words that are associated with the content lesson being taught. Write those words on the board or overhead.
2. Provide students with enough copies of the graphic organizer, *Root Word Tree* (page 50), to use for each of the vocabulary words being studied.
3. Tell students to write one vocabulary word at the base of the tree (e.g., *pictograph*).
4. As a class, break down the word into known word parts and write those parts on the large limbs of the tree (e.g., *pict* and *graph*).
5. Independently, have students write down other words on the branches of the trees with those same word parts (e.g., *picture* and *pictorial*; *autograph* and *photograph*).
6. Have students share their new words with the class, and discuss ideas about the meaning of the vocabulary word based on this new information.
7. As a class, decide on a class definition and have students write that definition beneath the tree.

Root Word Tree *(cont.)*

Differentiation

Above-Level Learners

With appropriate words, have students break the word into three chunks and complete the activity as discussed in the lesson procedure.

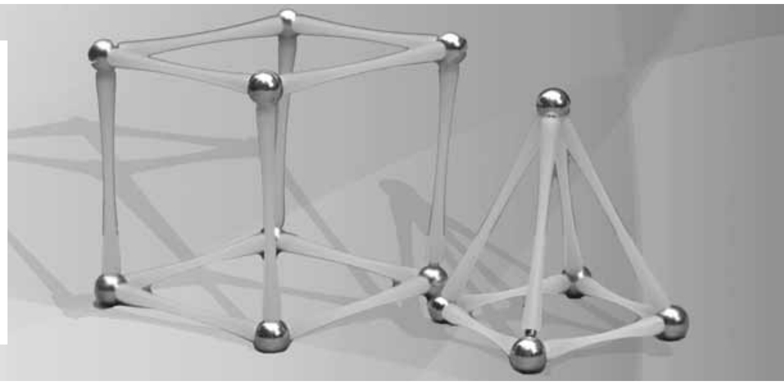
Below-Level Learners

Allow students to complete the *branches* portion of the activity in pairs. They may also draw pictures and/or write words to represent the ideas they want to put in the branches, but encourage them to label the pictures as much as possible.

English Language Learners

Provide English language learners with a picture representing the vocabulary word. Then, allow them to draw pictures and/or write words on the branches of the trees. Students may draw further conclusions about the definition of the vocabulary word by looking at the connected images. This part of the activity may be completed in pairs or small groups.

Root Word Tree *(cont.)*



Standard

- understands basic properties and similarities and differences between simple geometric shapes

Grades K–2 Sample Lesson

Vocabulary Words

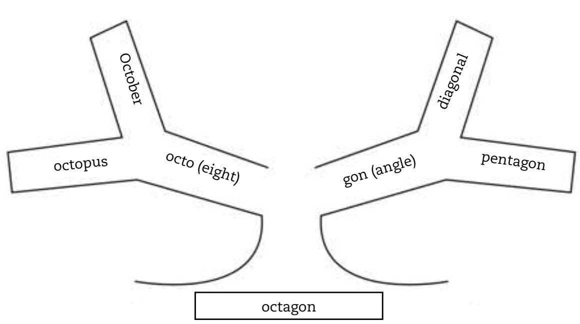
octagon hexagon vertex (vertices) side

Procedure

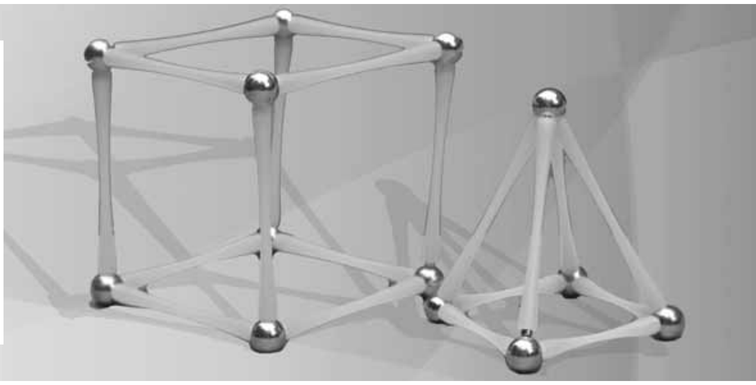
Differentiation suggestions for this strategy are provided on page 47.

1. Write the words from the list above on the board or overhead.
2. Provide each student with four copies of *Root Word Tree* (page 50).
3. Tell students to write the first vocabulary word at the base of one of the trees.
4. As a class, break down the word into known word parts and write those parts on the large limbs of the tree.
5. Divide the students in groups of two to three and have the groups write down other words on the branches of the trees with those same word parts. If appropriate, encourage students to use the Internet or a student dictionary.
6. Have students share their new words with the class and discuss ideas about the meaning of the vocabulary words based on this new information.
7. As a class, decide on a definition and have students write that definition beneath the tree.
8. Repeat steps 3–7 for the remaining vocabulary words on the list.

Sample:



Root Word Tree *(cont.)*



Standard

- knows approximate size of basic standard units and relationships among them

Grades 3–5 Sample Lesson

Vocabulary Words

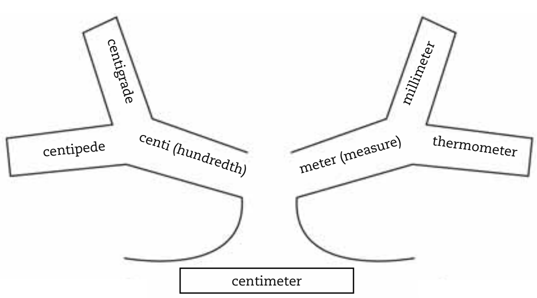
centimeter meter measure ruler

Procedure

Differentiation suggestions for this strategy are provided on page 47.

1. Write the words from the list above on the board or overhead.
2. Provide each student with four copies of *Root Word Tree* (page 50).
3. Tell students to write the first vocabulary word at the base of one of the trees.
4. As a class, break down the word into known word parts and write those parts on the large limbs of the tree.
5. Independently, have students write down other words on the branches of the trees with those same word parts. Encourage students to use the dictionary or Internet to help them with this part of the activity.
6. Have students share their new words with the class and discuss ideas about the meaning of the vocabulary word based on this new information.
7. As a class, decide on a definition and have students write that definition beneath the tree.
8. Repeat steps 3–7 for the remaining vocabulary words on the list.

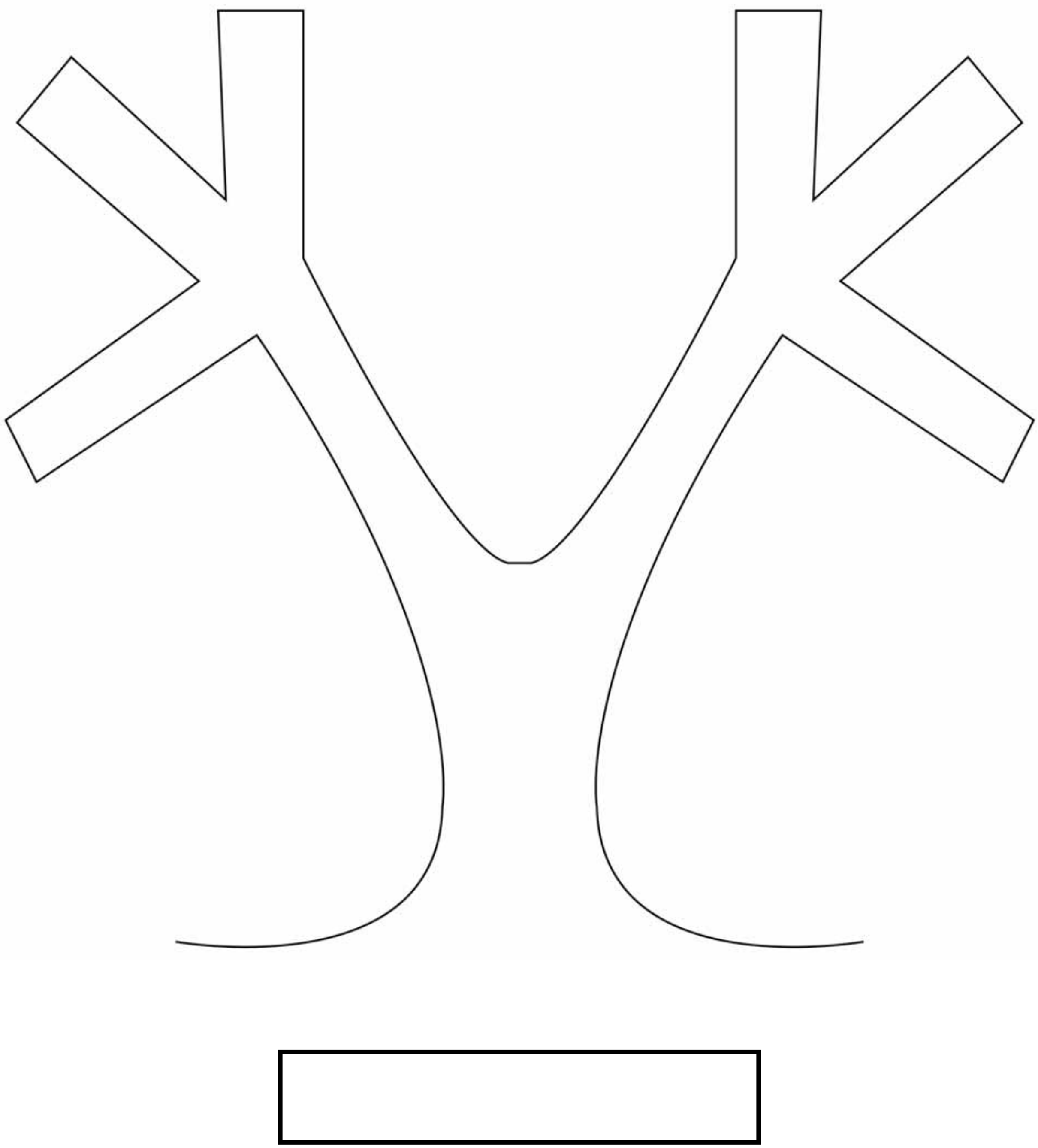
Sample:



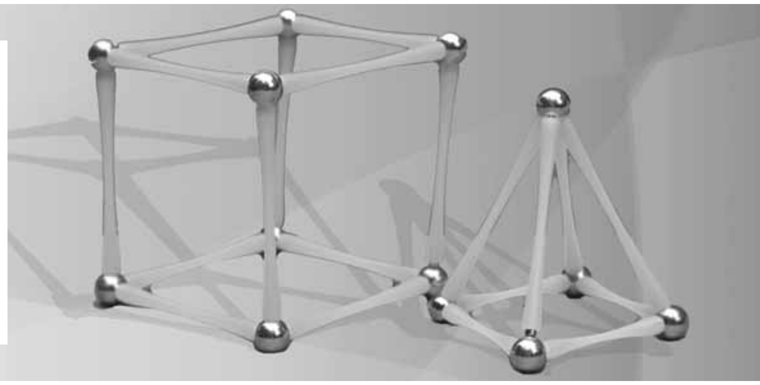
Name ____

Root Word Tree

Directions: Write the unknown word in the box at the base of the tree. Break up the word into parts, and write the parts on the limbs. Think of other words that include the parts, and write those on the branches.



Root Word Tree *(cont.)*



Standard

- understands the relationships between two- and three-dimensional representations of a figure

Secondary Sample Lesson

Vocabulary Words

polygon polyhedron hexagonal prism

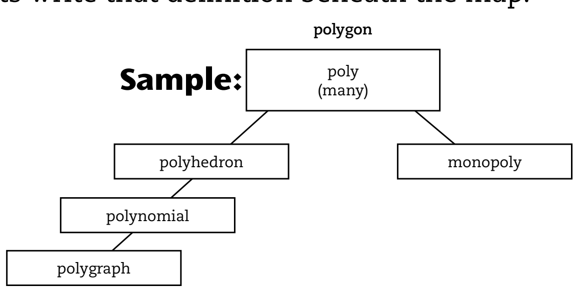
hexagon

Procedure

Differentiation suggestions for this strategy are provided on page 47.

1. Write the words from the list above on the board or overhead.
2. Provide each student with four copies of *Root Word Map* (page 52). This type of graphic organizer is more appropriate for older students and allows them to make deeper connections with the words.
3. Tell students to write the first vocabulary word above the box at the top of the page.
4. As a class, write the root of that word in the box at the top of the page.
5. On a separate sheet of paper, have students independently make a list of other words with that root. The words may have that root at the beginning or the end. Encourage students to use the dictionary or Internet to help them with this part of the activity.
6. Students must then organize their words in a way that makes sense to them, and write them in the spaces provided on the activity sheet.
7. Have students share their new words with the class and discuss ideas about the meaning of the vocabulary word based on this new information.
8. As a class, decide on a definition and have students write that definition beneath the map.
9. Repeat steps 3–8 for the remaining vocabulary words on the list.

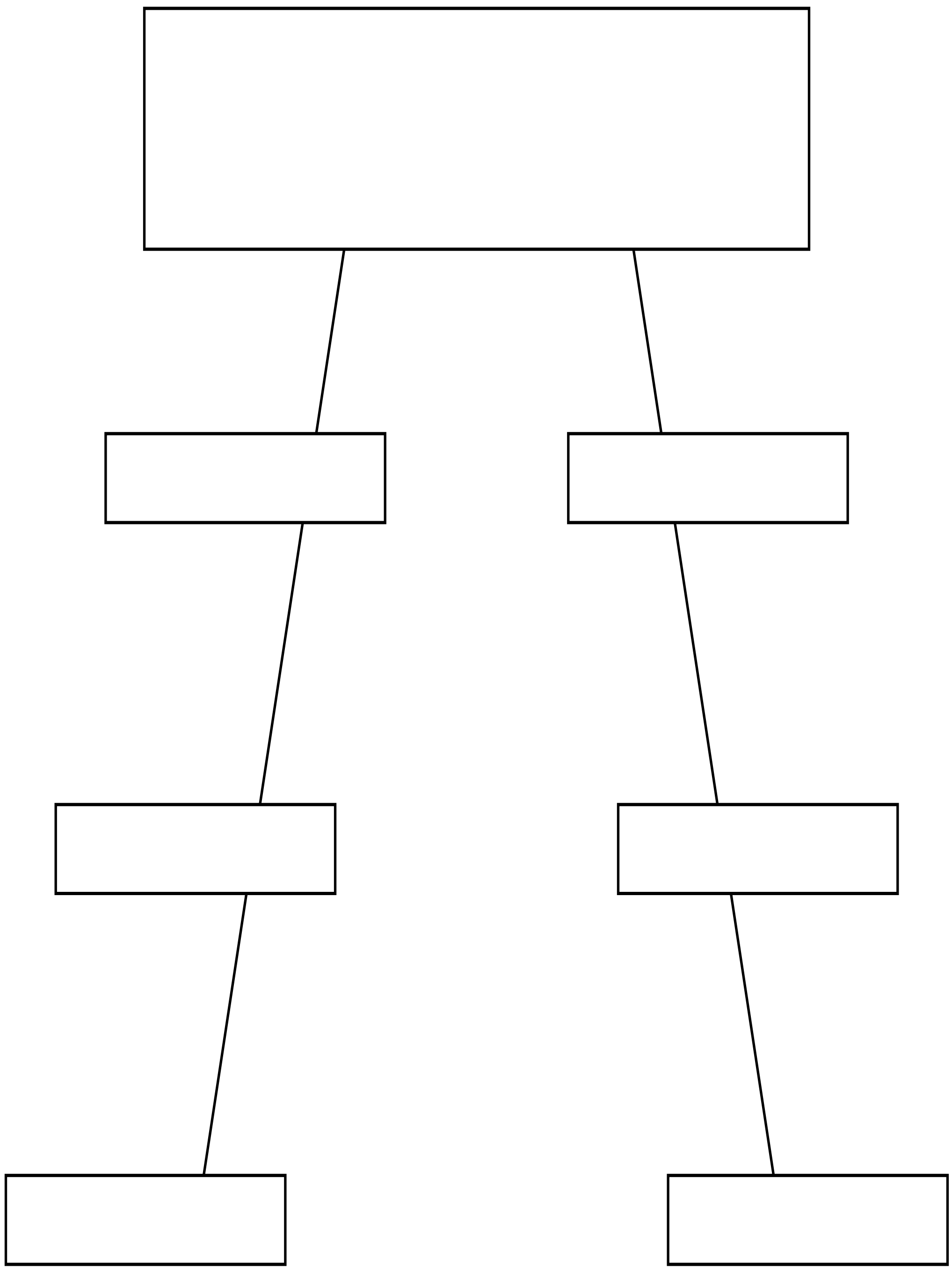
Sample:



Name ____

Root Word Map

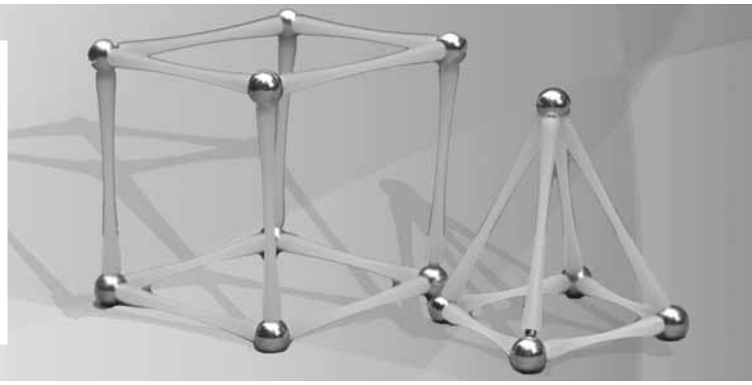
Directions: Write the root of the vocabulary word in the first box. Look in the dictionary or use the Internet to locate words that have the same root. Determine how the words are related to each other, and write them in the boxes below.



Sharing Mathematics

Standards

- understands level-appropriate vocabulary
- uses level-appropriate vocabulary in speech



Materials

- Counters or markers

Background Information

In this vocabulary-development activity, students are asked to share their knowledge of each vocabulary word with other classmates. This activity helps students reinforce proper vocabulary use each time they say the words in sentences. Sharing Mathematics also helps students develop reasoning skills as they listen for correct vocabulary usage in classmates' sentences.

Procedure

1. Write the vocabulary words for the day's lesson on the board or overhead.
2. Discuss the meanings of each of the words and allow several students to share sample sentences using the words correctly. It may be necessary to share examples of strong sentences (A hexagon has six sides and six vertices.) and examples of weak sentences (Hexagons are cool.) before students are allowed to share.
3. Distribute 3 to 5 counters/markers to each student.
4. Place students in groups of three to five.
5. Provide students with the following rules:
   - Every time a student defines a vocabulary word or says a sentence with one of the vocabulary words in it, he or she gives away a marker or counter.
   - Each sentence must be a definition or an example sentence.
   - The students have to use all of their counters/markers.
   - Once a student's counters/markers are "spent," he or she is not allowed to say anything more.
   - Students need to be respectful and listen to the vocabulary sentences from all of their classmates.
6. Allow students time to share sentences within their groups until all students have "spent" their counters.

Sharing Mathematics *(cont.)*

Differentiation

Above-Level Learners

Challenge students to use more than one word in each sentence. If they can do this correctly, allow them to give away two counters during that turn.

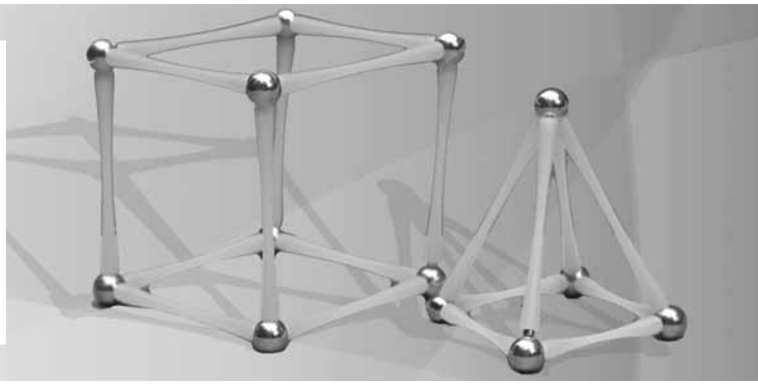
Below-Level Learners

Provide students with a list of the vocabulary words. Allow students to draw their own picture or symbol next to the words that are difficult for them. As the students use the words in sentences, instruct them to cross the words off their lists. Circulate around the room to help struggling students with their sentences.

English Language Learners

Provide each student with a list of the vocabulary words and corresponding pictures. As the students use the words in sentences, instruct them to cross the words off their lists. Circulate around the room to help struggling students with their sentences.

Sharing Mathematics *(cont.)*



Standard

- knows the common language of measurement

Grades K–2 Sample Lesson

Vocabulary Words

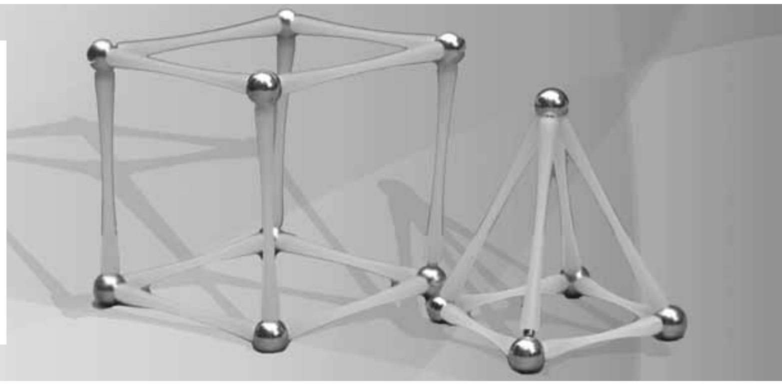
more	less	equal
heavier	lighter	

Procedure

Differentiation suggestions for this strategy are provided on page 54.

1. Write the vocabulary words from the list above on the board or overhead.
2. Discuss the meanings of each of the words and allow several students to share sample sentences using the words correctly. Share examples of strong sentences (I have more apples than my sister.) and examples of weak sentences (I want more.) before students are allowed to share.
3. Distribute 3 to 5 counters/markers to each student.
4. Place students in groups of three to five.
5. Write the following rules on the board or overhead. If necessary, provide simple picture clues to help students remember the appropriate procedures of the activity.
   - Every time a student says a sentence with one of the vocabulary words in it, he or she gives away a counter/marker.
   - Each sentence must be a definition or an example sentence.
   - The students have to use all of their counters/markers.
   - Once a student's counters/markers are "spent," he or she is not allowed to say anything more.
   - Students need to be respectful and listen to the vocabulary sentences from all of their classmates.
6. Allow students time to share sentences within their groups until all students have "spent" their counters.

Sharing Mathematics *(cont.)*



Standards

- understands the basic measures of perimeter, area, volume, capacity, mass, angle, and circumference
- understands relationships among measures

Grades 3–5 Sample Lesson

Vocabulary Words

perimeter	width	area
volume	height	three-dimensional
measure	length	

Procedure

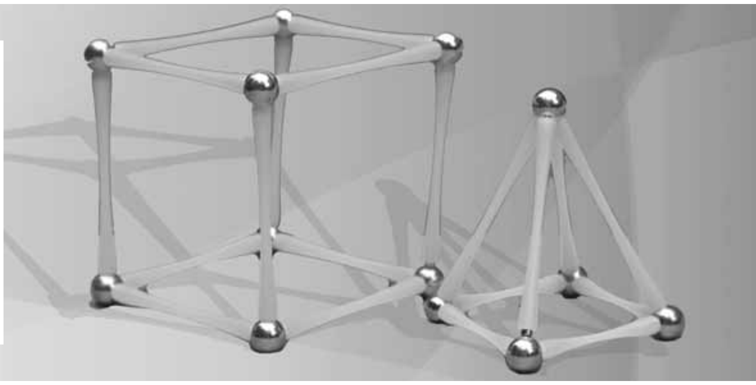
Differentiation suggestions for this strategy are provided on page 54.

1. Write the vocabulary words from the list above on the board or overhead.
2. Discuss the meanings of each of the words and allow several students to share sample sentences using the words correctly. Share examples of strong sentences (Area can be found by multiplying the length and width.) and examples of weak sentences (I like to find the area of objects.) before students are allowed to share.
3. Distribute 5 to 7 counters/markers to each student.
4. Place students in groups of three to five.
5. Write the following rules on the board or overhead:
   - Every time a student says a sentence with one of the vocabulary words in it, he or she gives away a marker or counter.
   - Each sentence must be a definition or an example sentence.
   - The students have to use all of their counters/markers.
   - Once a student's counters/markers are "spent," he or she is not allowed to say anything more.
   - Students need to be respectful and listen to the vocabulary sentences from all of their classmates.
6. Allow students time to share sentences within their groups until all students have "spent" their counters/markers.

Sharing Mathematics *(cont.)*

Standards

- uses geometric methods to complete basic geometric constructions
- understands the defining properties of triangles



Secondary Sample Lesson

Vocabulary Words

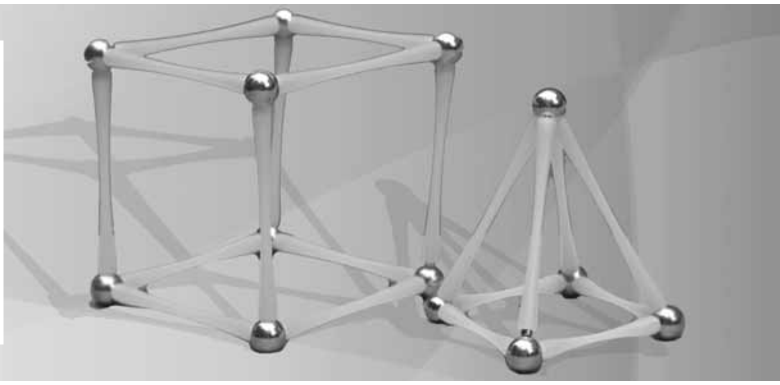
angle	right	acute
protractor	obtuse	degrees
straight	triangle	

Procedure

Differentiation suggestions for this strategy are provided on page 54.

1. Write the vocabulary words from the list above on the board or overhead.
2. Discuss the meanings of each of the words and allow several students to share sample sentences using the words correctly. Share examples of strong sentences (A triangle has three angles that equal 180°.) and examples of weak sentences (Triangles are interesting.) before students are allowed to share.
3. Distribute 7 to 10 counters/markers to each student.
4. Place students in groups of three to five.
5. Write the following rules on the board or overhead:
   - Every time a student says a sentence with one of the vocabulary words in it, he or she gives away a marker or counter.
   - Each sentence must be a definition or an example sentence.
   - The students have to use all of their counters/markers.
   - Once a student's counters/markers are "spent," he or she is not allowed to say anything more.
   - Students need to be respectful and listen to the vocabulary sentences from all of their classmates.
6. Allow students time to share sentences within their groups until all students have "spent" their counters/markers.

Vocabulary Flip Book



Standards

- understands level-appropriate vocabulary
- uses level-appropriate vocabulary in speech

Materials

- *Vocabulary Flip Book* (page 63; page063.pdf)

Background Information

A Vocabulary Flip Book is a foldable method of organizing and defining key vocabulary terms. When using this vocabulary-development strategy, students write a word on the top of each flap. When the flaps are flipped up, students draw pictures, write definitions, or create symbols to remind them of the meanings of the words. Vocabulary Flip Books can be created using the template provided or can be made using regular paper to accommodate the study of more vocabulary words.

Procedure

1. Choose a list of vocabulary words that are associated with the content lesson being taught. Write those words on the board or overhead. This activity works best with eight or fewer vocabulary words.
2. Distribute copies of page 63, titled *Vocabulary Flip Book*. (Teachers can create their own flip-book templates depending on the number of words being studied.)
3. Instruct students to cut out the large rectangle shown by the heavy solid line.
4. Have students fold the paper in half, "hot-dog style," along the thin black line in the middle of the template.
5. Instruct students to make additional cuts on the dotted lines, making sure to stop at the fold.
6. On the top of each flap, instruct students to write each vocabulary word being studied.
7. As a class, discuss the meaning of each word and symbols and pictures associated with it. Depending on the age level of the students, it may be necessary to create student definitions of the words.

Vocabulary Flip Book *(cont.)*

Procedure *(cont.)*

8. Have students lift each flap and draw pictures, symbols, associated words, and definitions in the necessary spaces. Students should choose pictures, symbols, words, and definitions that make sense to them and help them remember the meaning of the word.
9. Students may then review their words with partners using the information they included in their Vocabulary Flip Books. Encourage students to discuss if and why the content under their flaps is different than their partners'.

Differentiation

Above-Level Learners

Instruct students to create a Vocabulary Flip Book using only the words that they feel are the most difficult for them to remember. For some students, this may mean that they only need to work on two or three words.

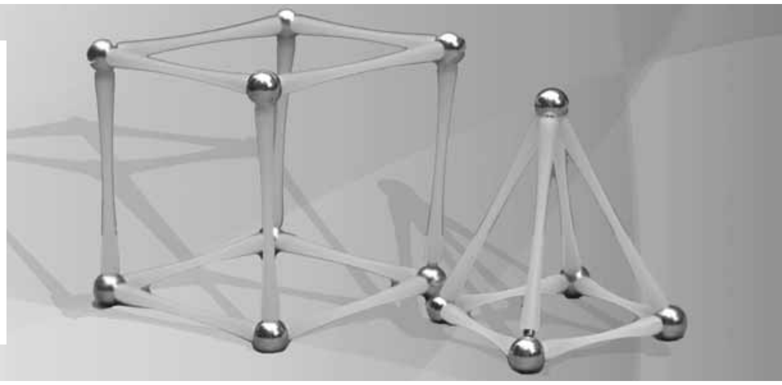
Below-Level Learners

Allow students to complete step 8 in pairs. Encourage them to discuss the words and their choices of drawings, definitions, etc.

English Language Learners

Provide students with precut images or clip art that correspond with the vocabulary words. After the class discussion, allow them to match the images with the corresponding words. This can be done in partners or in small groups. Students should then glue the images under the correct flaps. Then, allow them to add additional information of their choice beneath the flaps.

Vocabulary Flip Book *(cont.)*



Standard

- understands the inverse relationship between addition and subtraction

Grades K–2 Sample Lesson

Vocabulary Words

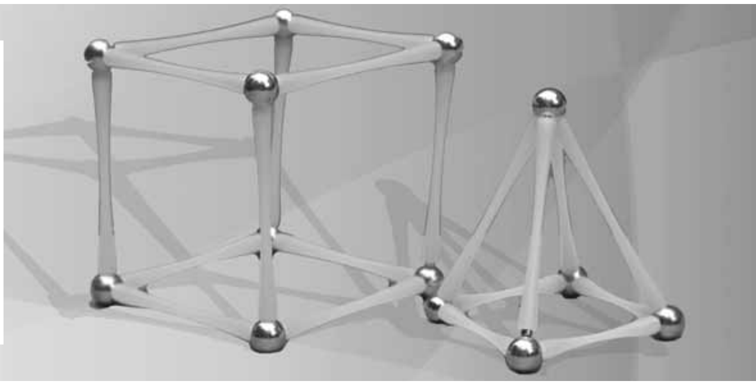
addition subtraction equal

Procedure

Differentiation suggestions for this strategy are provided on page 59.

1. Write the vocabulary words from the list above on the board or overhead.
2. Distribute copies of page 63, titled *Vocabulary Flip Book*.
3. Instruct students to cut out the large rectangle shown by the heavy solid line.
4. Have students fold the paper in half, “hot-dog style,” along the thin black line in the middle of the template.
5. Instruct students to make additional cuts on the dotted lines, making sure to stop at the fold.
6. On the top of each flap, instruct students to write each vocabulary word from the list.
7. As a class, discuss the meaning of each word and symbols and pictures associated with it.
8. Have students lift each flap and draw pictures, symbols, associated words, and definitions in the necessary spaces. Students should choose pictures, symbols, words, and definitions that make sense to them and help them remember the meaning of the word.
9. Students may then review their words with partners, using the information they included in their Vocabulary Flip Books. Encourage students to discuss if and why the content under their flaps is different than their partners’.

Vocabulary Flip Book *(cont.)*



Standard

- understands the relative magnitude and relationships among fractions and mixed numbers

Grades 3–5 Sample Lesson

Vocabulary Words

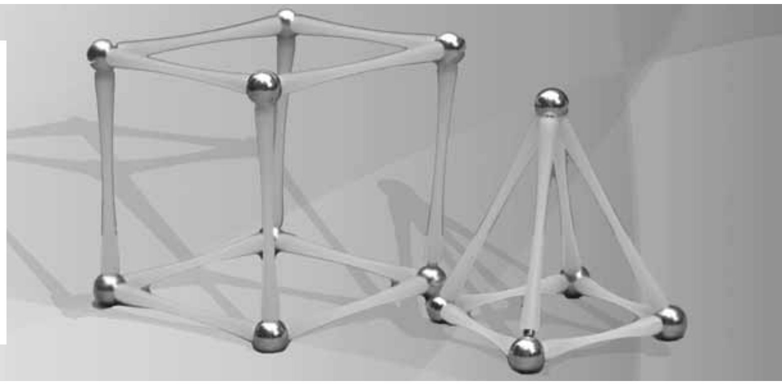
numerator denominator fraction

Procedure

Differentiation suggestions for this strategy are provided on page 59.

1. Write the vocabulary words from the list above on the board or overhead.
2. Distribute copies of page 63, titled *Vocabulary Flip Book*.
3. Instruct students to cut out the large rectangle shown by the heavy solid line.
4. Have students fold the paper in half, "hot-dog style," along the thin black line in the middle of the template.
5. Instruct students to make additional cuts on the dotted lines, making sure to stop at the fold.
6. On the top of each flap, instruct students to write each vocabulary word from the list.
7. As a class, discuss the meaning of each word and the symbols and pictures associated with it. From the discussion, have students create definitions of the words as well.
8. Have students lift each flap and draw pictures, symbols, associated words, and definitions in the necessary spaces. Students should choose pictures, symbols, words, and definitions that make sense to them and help them remember the meaning of the word.
9. Students may then review their words with partners, using the information they included in their Vocabulary Flip Books. Encourage students to discuss if and why the content under their flaps is different than their partners'.

Vocabulary Flip Book *(cont.)*



Standards

- solves linear equations and simple inequalities
- understands that a variable can be used in many ways
- knows that an expression is a mathematical statement using numbers and symbols to represent relationships and real-world situations

Secondary Sample Lesson

Vocabulary Words

linear equation   linear inequality   variable

Procedure

Differentiation suggestions for this strategy are provided on page 59.

1. Write the vocabulary words from the list above on the board or overhead.
2. Distribute copies of page 63, titled *Vocabulary Flip Book*.
3. Instruct students to cut out the large rectangle shown by the heavy solid line.
4. Have students fold the paper in half, "hot-dog style," along the thin black line in the middle of the template.
5. Instruct students to make additional cuts on the dotted lines, making sure to stop at the fold.
6. On the top of each flap, have students write each vocabulary word from the list.
7. As a class, discuss the meaning of each word and the symbols and pictures associated with it. From the discussion, have students create definitions of the words.
8. Have students lift each flap and draw pictures, symbols, associated words, and definitions in the necessary spaces. Students should choose pictures, symbols, words, and definitions that make sense to them and help them remember the meaning of the word.
9. Students may then review their words with partners, using the information they included in their Vocabulary Flip Books. Encourage students to discuss if and why the content under their flaps is different than their partners'.

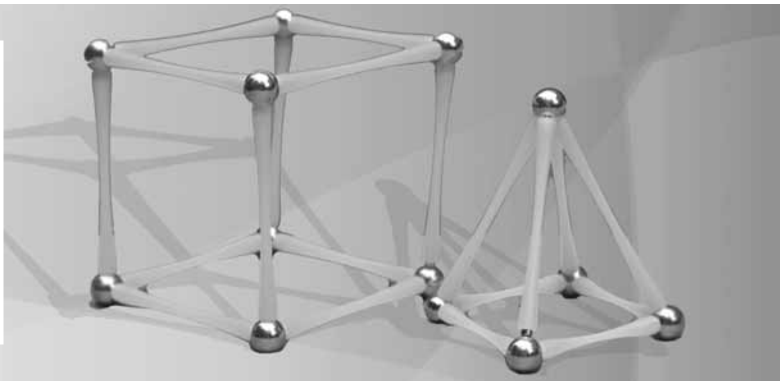
Name ______________________________

Vocabulary Flip Book

Directions:

1. Cut out the Vocabulary Flip Book on the thick black lines.
2. Fold the paper in half on the thin black line in the middle.
3. Cut along the dotted lines. Make sure to stop in the middle!
4. Write your vocabulary words on the top of each flap.
5. On the inside of the Vocabulary Flip Book, write words, definitions, pictures, or symbols to remind you what each word means.

Content Links



Standards

- understands level-appropriate vocabulary
- uses level-appropriate vocabulary in speech

Materials

- note cards (1 per student)

Background Information

Content Links (Yopp, Yopp, and Bishop 2009) is a strategy that helps students see how vocabulary words are connected to each other. It is best utilized after students have had some exposure to the chosen vocabulary words so that they can have meaningful conversations about the vocabulary words and discuss relationships between the words. In this strategy, each student receives a card with a vocabulary word written on it. Then, the class mingles and each student finds someone with whom they can link based on the vocabulary word on his or her card. This strategy is unique because there is not just one correct answer, and students can make their own decisions about why the words are linked. At the end of each round of mingling, students are invited to share their thinking with the class.

Procedure

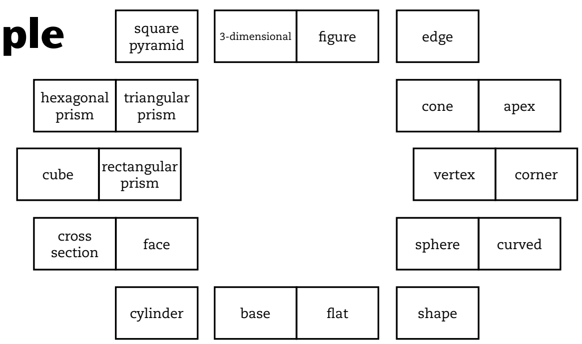
1. Before using this strategy, choose a list of vocabulary words to use. The words should relate to the lesson you are teaching, but students should already be familiar with their meanings.
2. Before beginning the activity, write each of the vocabulary words on note cards or strips of paper. There should be enough words for each student to receive a different word.
3. To begin the lesson, hold up each word card and read the word on the card. Ask students if they need clarification on the words' meanings. Take time to discuss the words as necessary, pointing out any pictures, charts, or other resources around the room to help students create meaning.
4. Explain to students that they each will receive a word card. Then they will be allowed to circulate around the room to find someone who has a word that can be linked with theirs. Once in pairs, they will discuss how the words are related and share their ideas with the class.
5. Distribute word cards, one to each student in the class.
6. Give students time to mingle. Note: There is a possibility that some students will not find a match.

Content Links *(cont.)*

Procedure *(cont.)*

7. Ask students to stand with their partners and form a circle around the room. If there are students without partners, they may stand anywhere they choose in the circle.
8. Have each pair discuss their words, the definitions, and how the words are connected. If some students do not have partners, discuss as a class why those words were difficult to relate to the other words. If there are other words around the circle that could have linked with the unmatched words, point out those words to the students. This will show students how the words could be linked in more than one way.
9. Ask students to share how making connections between vocabulary words helped them better understand those words. If time remains, repeat the activity, reminding students that they may not link with the word they chose in the first round.

Example



Differentiation

Above-Level Learners

Have each student propose six to ten additional vocabulary words that could be added to the list you created. Ask each student to pair his or her words and give a brief explanation of why he or she linked them in that way. Then, have students share and discuss their links with the class.

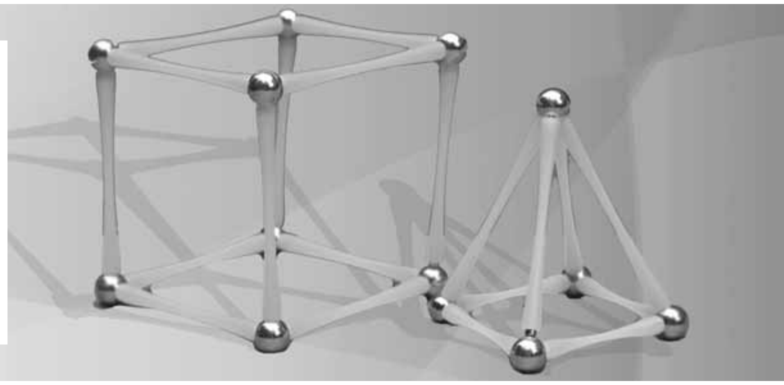
Below-Level Learners

Before beginning the activity with the whole class, present the list of words to these students. In a small group, have students echo-read each word on the cards. Review the definition and explanation of each word using pictures, sketches, or gestures whenever possible.

English Language Learners

Before beginning this activity with the entire class, present the list of words to students. Make sure that they understand what each word means. Use pictures, sketches, or gestures whenever possible. For the whole-class activity, provide students with modified word cards that have a simple visual representation of the meaning of the word included on the back of the card.

Content Links *(cont.)*



Standard

- understands the concept of a unit and its subdivision into equal parts called fractions

Grades K–2 Sample Lesson

Vocabulary Words

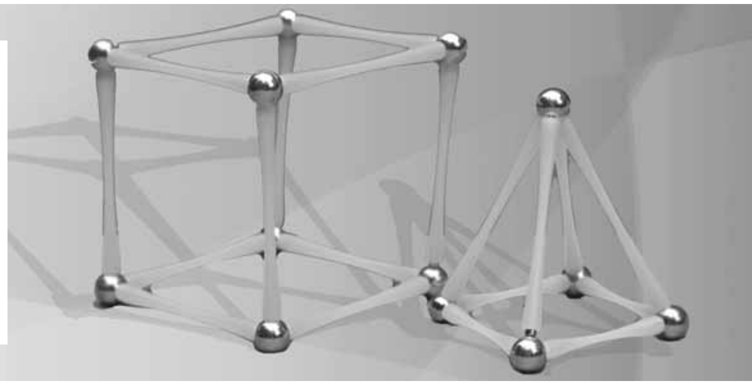
fraction	numerator	denominator	equal
equivalent	whole number	part	whole
piece	less than	greater than	one-half
one-fourth	order	one-eighth	one-third
one-fifth	one	one-sixth	compare

Procedure

Differentiation suggestions for this strategy are provided on page 65.

1. Before beginning the activity, write each of the vocabulary words above on note cards or strips of paper. For this example, it is assumed that there are 20 students in the class.
2. To begin the lesson, hold up each word card and read the word on the card. Have students repeat the word aloud. Ask students if they need clarification on word meanings. Take time to discuss the words as necessary, pointing out any pictures, charts, or other resources around the room to help students create meaning.
3. Explain to students that they will each receive a word card. Then they will be allowed to circulate around the room to find someone who has a word that can be linked with theirs. Once in pairs, they will discuss how the words are related and share their ideas.
4. Distribute word cards, one to each student in the class. Give students time to mingle. Note: There is a possibility that some students will not find a match.
5. Ask students to stand with their partners and form a circle around the room. If there are students without partners, they may stand anywhere in the circle they choose.
6. Have each pair discuss their words, the definitions, and how the words are connected. If some students do not have a partner, discuss as a class why those words were difficult to relate to the other words. If possible, point out other words around the circle that could have linked with the unmatched words.
7. Ask students to share how making connections between vocabulary words helped them better understand those words. If time remains, repeat the activity, reminding students that they may not link with the word they chose in the first round.

Content Links *(cont.)*



Standard

- understands that data can take many forms, represents specific pieces of information about real-world objects or activities, and can be displayed in many ways

Grades 3–5 Sample Lesson

Vocabulary Words

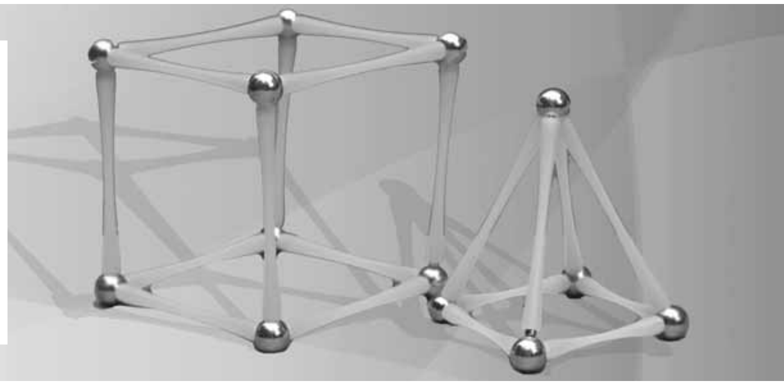
mean	median	mode	range
average	bar graph	line graph	pictograph
circle graph	data	*x*-axis	*y*-axis
key	title	label	compare
analyze	interpret	maximum	minimum

Procedure

Differentiation suggestions for this strategy are provided on page 65.

1. Before beginning the activity, write each of the vocabulary words above on note cards or strips of paper. For this example, it is assumed that there are 20 students in the class.
2. To begin the lesson, hold up each word card and read the word on the card. Have students repeat the word aloud. Ask students if they need clarification on the words' meanings. Take time to discuss the words as necessary, pointing out any pictures, charts, or other resources around the room to help students create meaning.
3. Explain to students that they will each receive a word card. Then they will be allowed to circulate around the room to find someone who has a word that can be linked with theirs. Once in pairs, they will discuss how the words are related and share their ideas.
4. Distribute word cards, one to each student in the class. Give students time to mingle. Note: There is a possibility that some students will not find a match.
5. Ask students to stand with their partners and form a circle around the room. If there are students without partners, they may stand anywhere in the circle they choose.
6. Have each pair discuss their words, the definitions, and how the words are connected. If some students do not have a partner, discuss as a class why those words were difficult to relate to the other words. If possible, point out other words around the circle that could have linked with the unmatched words.
7. Ask students to share how making connections between vocabulary words helped them better understand those words. If time remains, repeat the activity, reminding students that they may not link with the word they chose in the first round.

Content Links *(cont.)*



Standard

- understands the basic concept of a function

Secondary Sample Lesson

Vocabulary Words

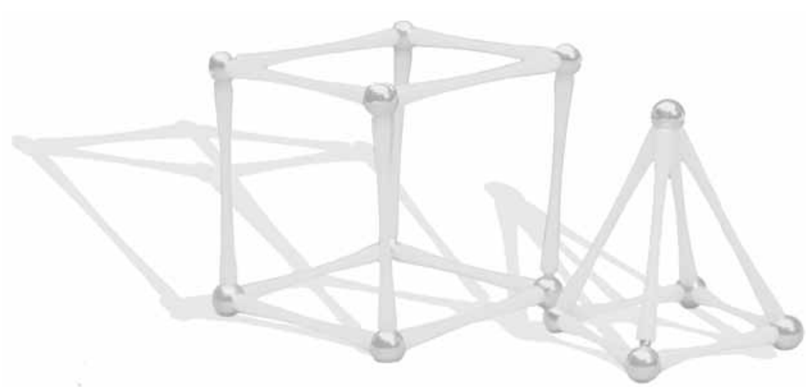
function	range	vertical line test	quadrant
pattern	solution set	*x*-axis	*y*-axis
origin	equation	input	output
relation	ordered pair	coordinate plane	linear
graph	*x*-coordinate	*y*-coordinate	domain

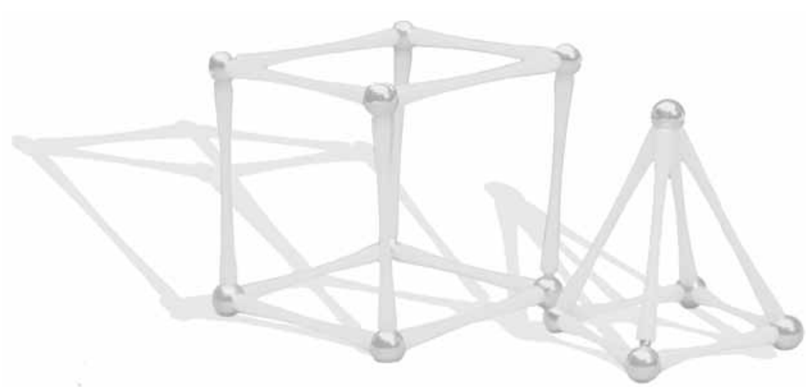
Procedure

Differentiation suggestions for this strategy are provided on page 65.

1. Before beginning the activity, write each of the vocabulary words above on note cards or strips of paper. For this example, it is assumed that there are 20 students in the class.
2. To begin the lesson, hold up each word card and read the word on the card. Have students repeat the word aloud. Ask students if they need clarification on the words' meanings. Take time to discuss the words as necessary, pointing out any pictures, charts, or other resources around the room to help students create meaning.
3. Explain to students that they will each receive a word card. Then they will be allowed to circulate around the room to find someone who has a word that can be linked with theirs. Once in pairs, they will discuss how the words are related and share their ideas.
4. Distribute word cards, one to each student in the class. Give students time to mingle. Note: There is a possibility that some students will find no match.
5. Ask students to stand with their partners and form a circle around the room. If there are students without partners, they may stand anywhere in the circle they choose.
6. Have each pair discuss their words, the definitions, and how the words are connected. If some students do not have a partner, discuss as a class why those words were difficult to relate to the other words. If possible, point out other words around the circle that could have linked with the unmatched words.
7. Ask students to share how making connections between vocabulary words helped them better understand those words. If time remains, repeat the activity, reminding students that they may not link with the word they chose in the first round.

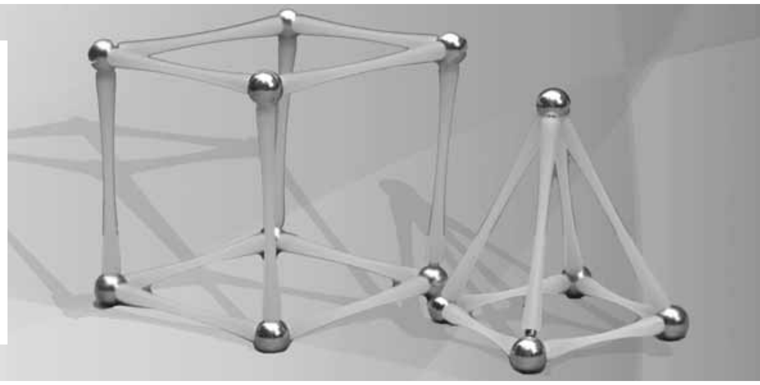
Strategies for Using Manipulatives





Manipulatives Overview

The value of the manipulative is not in the cost, but in its use.



Common Questions About Manipulatives

What are manipulatives?

Manipulative materials are colorful, intriguing materials constructed to illustrate and model mathematical ideas and relationships and are designed to be used by students in all grades (Burns and Sibley 2000). Manipulatives are sometimes called objects to think with (Kennedy, Tipps, and Johnson 2008).

Manipulatives can take many forms in the mathematics classroom. They can be as simple as a piece of paper that is folded and cut to show congruency or as elaborate as a full-class set of base ten blocks to show place value. Fraction bars, rulers, counters, pattern blocks, algebra tiles, and tongue depressors are all examples of manipulatives. Today, there are commercial, teacher-made, student-made, and virtual manipulative materials.

The value of the manipulative is not in the cost, but in its use. Matching the manipulative to the mathematical concept is the most important step that any teacher makes during lesson preparation.

Why should I use manipulatives?

Research indicates that lessons using manipulatives are more likely to help children achieve mathematically than lessons without manipulatives (Sowell 1989). This is because students often struggle to relate to concepts and make sense of abstract ideas without some type of first-hand experiences with them. Manipulative use gives students hands-on experiences and support learning by creating physical models that become mental models for concepts and processes (Kennedy, Tipps, and Johnson 2008).

Also, long-term use of concrete materials is positively related to increases in student mathematics achievement and improved attitudes toward mathematics (Grouws and Cebulla 2000). When students can directly relate to a concept through the use of manipulatives and feel successful, they do not fear failure and have greater success seeing relationships and connections among the five designated areas of mathematics. Using manipulatives also “helps students understand mathematical concepts and processes, increases thinking flexibility, provides tools for problem-solving, and can reduce math anxiety for some students” (The Education Alliance 2006).

Manipulatives Overview *(cont.)*

How often should I use manipulatives?

Manipulatives should be used on a daily basis throughout most of the year in the mathematics classroom. The students should also have access to them when they are assigned an independent activity. Once the transition begins to the pictorial, or representational, phase, the students may choose which method is easier for them. Manipulatives should be used as often as needed to fit the objective of the lesson. Ideally, the materials should be available for students to use at any time to help them think, reason, and solve problems (Burns and Sibley 2000).

Does every student need his or her own set of manipulatives?

Pride of possession is important to students. Often, it is most efficient for each student to have his or her own set of manipulatives. If there are not enough materials available in your classroom, borrow from other teachers or pool necessary materials. Manipulative blackline masters are frequently available and can be duplicated, laminated, cut, and placed in individual bags. However, a particular activity may specifically call for cooperative learning, in which case assigning two to three students per set of manipulatives is necessary.

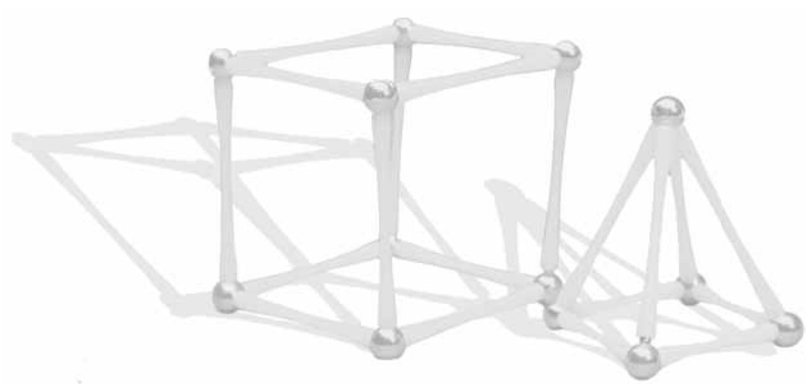
Students need the kinesthetic experience of working with manipulatives in order to help them remember mathematical concepts. Handling the manipulatives appears to help students construct mathematical ideas and retain them better than seeing someone else demonstrate a concept. Although they may not need a whole set to themselves, enough pieces are required for them to successfully complete the task using the manipulative.

Why is free exploration necessary?

Free-exploration time is a necessity when working with manipulatives. Imagine what would happen if you gave a child a new toy and did not allow him or her to play with it. Chaos would erupt. The same happens when students are first exposed to manipulatives. Students need time to explore and openly discover in order to satisfy their curiosities. If they are not given time, they will become distracted by the potential for fun, and not pay attention to the learning involved in the lesson.

It is sometimes difficult to gauge how much time is needed for free exploration. Ultimately, the time limit depends on the manipulative being introduced and the students. Sometimes, a task can be assigned to the students after five minutes of free exploration, and sometimes it may take longer before students are comfortable enough with the manipulative that they can focus on the instruction being given by the teacher.

Do not forget to give free-exploration time to older students. They need the same opportunities for discovery as younger students.



Manipulatives Overview *(cont.)*

Using Manipulatives Effectively

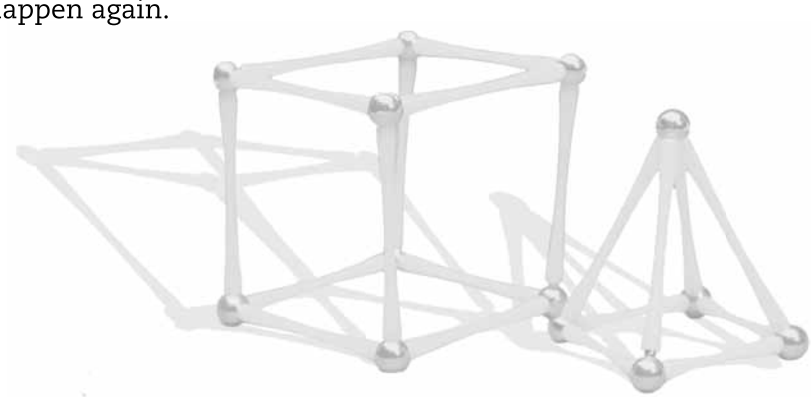
Solely using manipulatives does not guarantee success. The teacher is still the key to successful instruction and implementation of manipulatives in a lesson. The teacher should be certain that:

- the manipulatives have been chosen to support the objective(s) of the lesson.
- the lesson moves from concrete to abstract when appropriate.
- significant plans have been made to prepare students in the use of manipulatives.
- each lesson includes time for reflection and assessment.
- each student actively participates.

Effective teachers use manipulatives, when appropriate, to explore problems and provide experiences that help children make sense of mathematics and build their mathematical thinking (Reys et al. 2007). In some cases, teachers may be resistant to manipulatives. They may feel that manipulatives are for younger students, or they may see manipulatives as "toys" that inhibit rather than enhance learning. A middle school teacher once remarked to a parent, "My students don't play with those silly toys." What this teacher did not understand was that some of his students were unable to grasp certain mathematical concepts, and by using manipulatives many students may have experienced greater success in understanding mathematical concepts. Manipulatives are appropriate for students of all ages and ability levels.

Using manipulatives effectively requires planning and organization on the part of the teacher (Frei 2008). Here are some tips for effective manipulative use in the classroom:

- Be clear with students about expectations while using manipulatives. Discuss the similarities and differences between using manipulatives and playing with toys or games at home. One difference might be that at home, the students make up their own rules when playing with toys; but at school, the teacher will decide what tasks he or she wants the students to complete during the course of the class period.
- Set the boundaries for using manipulatives on the first day of use. Students must understand the consequences for misusing them. Tell them that the manipulatives will be taken away if they are misused (thrown, stolen from others, found on the floor, etc.).
- Do not punish the entire class for the misbehavior of a few students. If it is necessary to take the materials away from a student, take the materials away for only a portion of the assigned task. For example, if a student throws a block, take his or her blocks away for that task only. When the next task is assigned, give the blocks back. Continue this until the student no longer misuses the materials. Usually, once the student sees that he or she is missing out on a fun activity, misuse will not happen again.



Manipulatives Overview *(cont.)*

Strategies for Acquiring Manipulatives

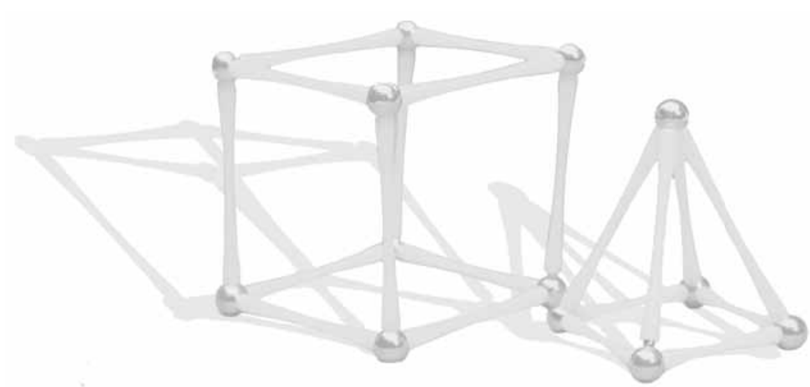
It is important to keep in mind that acquiring a large variety of manipulatives takes time. In some districts, the school administrator may budget money for classroom supplies. Sometimes a grade-level materials list is sent out to each family in the school. Even with this support, the teacher may decide to supply his or her own consumable supplies. This can become very expensive.

The key to acquiring supplies is to be creative. Not all manipulatives need to be purchased from a teacher supply store in order for effective learning to occur in the classroom. Here are some helpful strategies for acquiring manipulatives:

- Send home a classroom wish list on the first day of school. When the students bring in supplies, they can be collected and put in a central supply area. Even if everyone does not bring in all of his or her supplies, at least there are some supplies for immediate classroom use.
- Utilize the supplies found at the grocery store. Kids love food even if they are not allowed to eat it. Food such as beans, pasta, cereal, candy, and crackers all have multiple uses in the mathematics classroom.
- Purchase inexpensive materials from local dollar stores that can be used to concretely teach mathematics topics.
- Share supplies with other teachers on your grade-level/content-area team.
- Gather household supplies such as buttons, water bottle caps, or used stamps from friends, family, and neighbors.
- Set a goal to purchase 1–3 items each year. After a few years, you will have many types of items to use in the classroom.

Strategies for Organizing Manipulatives

It is essential for the teacher to organize all the materials that will be used in the mathematics classroom. Often, there are many pieces in a set of manipulatives, and they are small to begin with. Depending on a teacher's management style and the type of activity being conducted, there are many ways to organize manipulatives.



Manipulatives Overview *(cont.)*

Strategies for Organizing Manipulatives *(cont.)*

When organizing manipulatives, keep in mind the following questions:

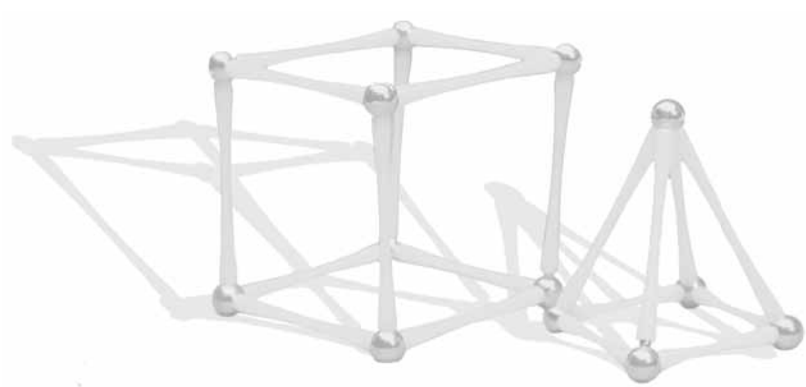
- What outcome are the students supposed to walk away with at the end of this activity?
- What size and how many of the manipulatives does each student/group of students need?
- How can the manipulatives be arranged to be most easily used in the activity?
- What is the size of the students' workspace?
- What can be done to make sure that students do not take the manipulatives away from the activity or classroom?
- Does the organization method being utilized enable more positive learning opportunities to occur?

Using and Organizing Individual Bags

One of the most efficient ways to organize manipulative materials is by using heavy-duty, zip-top plastic bags. The bags should be heavy duty or freezer bags to be sturdy enough to last the entire school year. Flimsier bags will break, causing manipulatives to get mixed up or lost. It is also very effective to organize the materials so that each student has an individual bag of each manipulative. Then, assign every student in the classroom a number that should not change throughout the school year. If students have numbers that are used in the grading system, use them for the manipulative bags as well. That makes one less number that they and you have to remember! Using a permanent marker, numbers should be written on the heavy-duty, zip-top bags so there is one bag for each student in the class. (For example, student 1 in the grade book will use bag 1 for each activity that utilizes manipulatives.)

If the school is departmentalized, and teachers teach two or more sections of mathematics, this works especially well. Each section will have a 1, 2, etc., student and a 1, 2, etc., bag of materials. For more expensive manipulatives, fill a gallon-sized, zip-top bag with enough materials for two to three students. For example, students 1–3 will use the bag marked 1, and students 4–6 will use the bag marked 2. Students quickly learn which groups of manipulatives they should use.

It is important for teachers to place the same number and kinds of manipulatives in each bag. This ensures that every student has the same set of materials. At the end of the class period, the students should clean up and reorganize their own materials so that the materials will be ready for the next time they are used. This gives students a sense of ownership of these materials, and they will usually make sure all of the materials are placed back into the bag.



Manipulatives Overview *(cont.)*

Strategies for Organizing Manipulatives *(cont.)*

Using and Organizing Individual Bags *(cont.)*

In some cases, it may be necessary to number each individual block with a permanent marker. If this is the case, students will line up their manipulatives in numerical order and place them back in the bag. After the manipulatives are placed back in the bags, the bags are placed in numerical order in large plastic bins with an identifying picture and/or label of the manipulative on the outside of each bin. If flat bins are used, the small bags should be placed in numerical order. This organization may seem to take a long time and be unnecessary. However, the students learn to do this very rapidly and it is time well spent. The materials will always be organized and ready for the next lesson without you having to spend extra time after the school day is over.

Using and Organizing Boxes and Containers

Organizing boxes and containers can be difficult if there is limited storage space. The most important thing to consider when storing manipulatives in this way is whether students will be expected to access them at any time throughout the school year. If students are allowed to get and return the manipulatives, then the boxes or containers need to be within their reach.

Labeling the boxes or containers is also important. Depending on the age of the students and their access level to the manipulatives, it may also be necessary to include picture labels.

It is also important to consider buying or ordering identical types of containers or boxes. This is helpful for stacking purposes. When multiple types of containers are used to store manipulatives all in one place, it can be difficult to stack them efficiently and safely because they are not all the same shape or size.

All of the same numbering and labeling systems can be used for organizing individual boxes and containers as described in the section titled, *Using and Organizing Individual Bags*.

Using and Organizing Supply Caddies

Each group of students can also have a container for materials and supplies, such as markers, scissors, or glue. Plastic laundry caddies work well because each caddy has individual compartments for the different materials. Number the scissors and marker sets using a permanent marker. If the scissors belong in caddy 1, all of the scissors in caddy 1 should be labeled with a 1–1, 1–2, 1–3, etc. In this way, students can help keep track of the materials.

Using and Organizing Calculators

Calculators can be organized using the same strategy as discussed above. Using a permanent marker or sticker, number each calculator and the calculator cover. Store them in numbered calculator pockets. These pockets can be purchased commercially. Or, a hanging shoe bag may be used.

Manipulatives Overview *(cont.)*

Strategies for Organizing Manipulatives *(cont.)*

Using and Organizing Calculators *(cont.)*

You can also store the calculators in boxes or containers. To do this most efficiently, label the containers 1–10, 11–20, 21–30, etc. The numbered calculators are stored in their respective numbered containers. That way, when students are coming to the boxes to retrieve their calculators, they are not all going to one container. It might also be helpful to place the containers at separate corners of the room so that students are going to different places to retrieve the devices.

Strategies for Individual Use of Manipulatives

When students are working individually, it is important for them to have defined workspaces. Felt squares or placemats can be used to provide an individual workspace for each student and to muffle the noise of plastic or wooden manipulatives. (This strategy can also be used when students are working in groups but are each required to work with the manipulatives.) The students must complete their work with the manipulatives on the felt squares. This also enables unfinished work to be easily moved and saved for later use. The felt square belongs to each student and can be kept in his or her desk or in the supply caddies.

When students are transitioning from using manipulatives to the pictorial representation of the mathematical concept, it is important to remind them that manipulatives are still available. However, make sure that students do not see manipulative use as a crutch. Using the phrase "Manipulatives are available for use" instead of "You can get manipulatives if you need them" will help students understand that manipulatives are meant to increase their understanding, not to only be used when they are stuck on a concept.

Strategies for Free Exploration of Manipulatives

Free exploration is a necessary part of the successful use of manipulatives in the mathematics classroom. Play comes naturally to students because they are intrinsically curious. It is important to allow students to explore the manipulatives and be free to use them however they wish. This will allow them to explore the manipulative, see how it works, and gain mathematical understanding before any direct instruction begins.

Use the following strategies during free exploration:

- Set minimal guidelines for free-exploration time. These guidelines should be related to student safety, not to set boundaries for the ways in which students can explore the manipulatives.
- Make sure students understand that free exploration time belongs to them. You will only take the manipulatives away if the safety guidelines are not followed. Then, mathematics instruction time belongs to you. Students must understand that manipulatives will be taken away if they engage in free-exploration type activities during instructional time.

Manipulatives Overview *(cont.)*

Strategies for Free Exploration of Manipulatives *(cont.)*

- Allow the students to explore the manipulatives however they see fit, according to the classroom boundaries set in place.
- Provide students with a time frame in which they can conduct free exploration. Use a kitchen timer or bell to signal when time is through.
- Once free exploration time is over, allow students to share what they noticed or learned about the manipulative based on their explorations. Praise all responses equally, even if they are not mathematically based. For example, a student could notice that the manipulative is hard. Although that is not a mathematical observation, it is still an observation and is valid in the discussion.

Strategies for Modeling While Using Manipulatives

It is important to understand that students will not "automatically draw the conclusions their teachers want simply by interacting with particular manipulatives" (Ball 1992). Students need to understand how they are to use the manipulatives, what mathematical concept is to be shown, and what conclusions they should be able to draw by using the manipulatives.

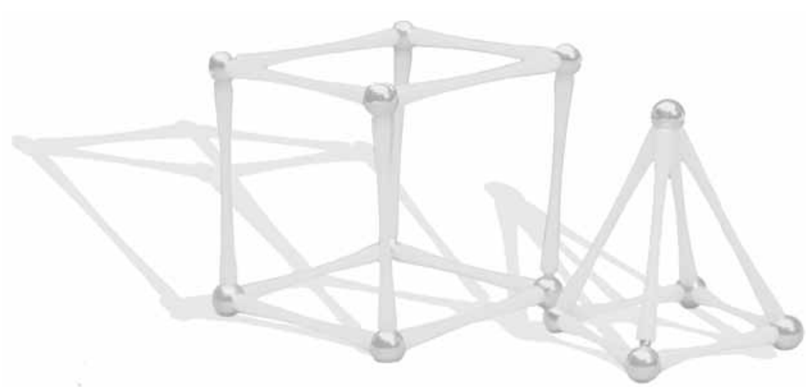
Because mathematical understanding is not implicit in the manipulatives themselves, it is crucial for teachers to develop the context for which the manipulatives are appropriate (Ball 1992). Modeling is one way to develop the mathematical context for the students.

Modeling with manipulatives means that you show students how to use the materials in order to solve a problem, illustrate a mathematical concept, or display a mathematical relationship. Depending on students' understanding of the concept and their familiarity with the manipulative, it can be appropriate to have the students watch as you model, participate at certain parts while you model, or follow along directly with their own manipulatives as you model.

It is important to plan the modeling before "performing" it in front of the students. This will enable you to think critically about the relevance of the manipulative in the concept you are teaching. It will also help you plan the vocabulary and the other language that is best used during the model, which is one of the most important aspects. Modeling is your opportunity to reinforce the language of mathematics and show students how to use that language appropriately and in context.

The strategies below and on the next page can also be used when modeling manipulative use.

- Do not assume that your students have an understanding of the procedure, the manipulatives, or the concept you are modeling. Make sure to model, discuss, and ask students questions about everything.



Manipulatives Overview *(cont.)*

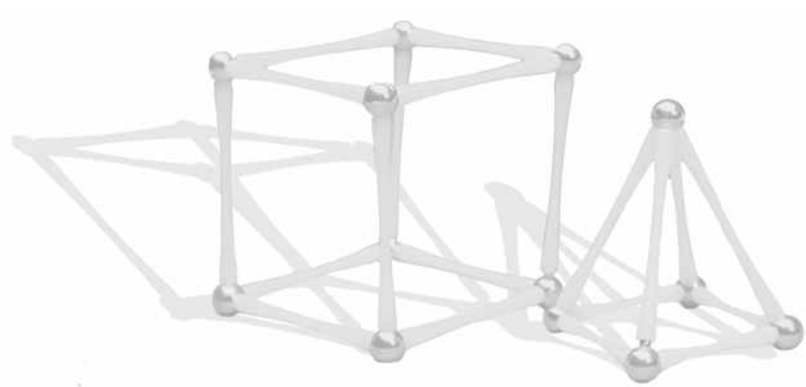
Strategies for Modeling While Using Manipulatives *(cont.)*

- Model the mathematical concept/procedure by thinking aloud. The students will benefit from hearing all of your thoughts as you are working. In turn, they learn to approach the concept/procedure in the same way that you did.
- Use an overhead projector or an electronic projection system to complete the modeling. This will allow you to easily display what you are doing in an engaging way for the students. If you do not have overhead sets of the manipulatives with which you are working, create your own, using permanent markers and blank transparency sheets. Then, cut out the manipulatives and use them accordingly.
- Create larger versions of the manipulatives, mats, graphic organizers, and other materials the students are to use. This is appropriate if you are modeling for students while they sit on the floor or if you are using the board or an easel as your space for modeling. Using larger versions of the materials will enable them to see what is going on as you model.
- Have class discussions about the concept/procedures being modeled. Ask students questions and allow them to discuss with partners in order for them to verbalize their mathematical understandings.
- When appropriate, have students work with you as you model.
- At times, make an error during your modeling. Then, go back and check your work to show students how to correct the error. When modeling an error, be sure to use the think-aloud format so that students hear and "see" your thinking process as you correct the error.

Strategies for Grouping Students While Using Manipulatives

Effective teachers actively engage students with meaningful tasks using a variety of instructional approaches. When teaching while using manipulatives, it is often appropriate to group students. It is important for students to realize that these groups are flexible. That means that students will often be grouped with different students depending on the skill being taught. Below are two different types of grouping strategies:

- **Homogeneous groups:** Students are grouped according to similar content-specific ability levels. For example, group students who have a weak understanding of addition together and put the students who have a strong understanding of addition together.
- **Heterogeneous groups:** Students are grouped according to various readiness or ability levels. For example, in one group, each student could have a different understanding of measuring angles.



Manipulatives Overview *(cont.)*

Strategies for Grouping Students While Using Manipulatives *(cont.)*

Regardless of the strategy used when grouping, students need to realize the following about groups:

- cooperation is valued above competition—students should help and encourage one another
- participation is integral to group success—all ideas are important
- working collaboratively is important and can foster learning that might not have taken place by working alone
- group time is not free time—active engagement is key
- time management is important

Strategies for Assigning Roles Within Groups

When working with manipulatives, it is often helpful to assign students roles within their groups. This will help each student feel important and give him or her responsibility for the success of the activity. It is important to rotate roles so that every student has a chance to have each role at least once during the year.

The chart below shows examples of various roles that could be given during instruction with manipulatives:

Role Name	Role Description
Timekeeper	• keeps track of the time elapsing during the activity • motivates group members to stay on task and use time wisely • notifies team members when it is time to clean up the activity
Materials Officer	• is in charge of getting the materials for one week • makes sure each student has everything needed for the assigned task
Materials Organizer	• counts materials to make sure none were lost during the activity • is in charge of making sure that all of the supplies are back in the correct containers or bags at the end of the activity
Group Leader	• encourages people to participate in discussion • is the only student who may approach the teacher with a question from the group

Manipulatives Overview *(cont.)*

The Manipulative Survival Guide

Teachers often ask for a list of concepts and the manipulatives that match mathematical concepts. The National Council of Supervisors of Mathematics developed such a list in 1994. An adaptation of this list is found below.

Mathematical Concept	Manipulative
Angles	protractors, compasses, geoboards, tangrams, pattern blocks
Area	geoboards, color tiles, pattern blocks, cubes, rulers
Classifying And Sorting	attribute blocks, cubes, pattern blocks, tangrams, color tiles, counters, dominoes, geometric solids, money, number cards, dice, candy, cereal, plastic animals or insects, crackers
Coordinate Geometry	geoboards
Counting	linking cubes, two-color counters, counters, color tiles, hundred boards, dominoes, number cubes, dice, pasta, cereal, candy, beans
Decimals	base ten blocks, money, calculators, dice, spinners
Equations/Equivalence	algebra tiles, balance scales, calculators, color tiles, dominoes, money, two-color counters, counters, Cuisenaire® rods
Estimation	color tiles, balance scales, Cuisenaire® rods, calculators, rulers, beans, cereal, candy, linking cubes, pasta
Facts	two-colored counters, hundreds chart, dominoes, spinners, dice, money, calculators
Fractions	fraction models, tangrams, pattern, Cuisenaire® rods, blocks, color tiles, number cubes, spinners
Integers	two-color counters, thermometers, color tiles
Logical Reasoning	attribute blocks, dominoes, pattern blocks, tangrams, dice, spinners
Measurement	balance scales, rulers, capacity containers, thermometers, clocks, geometric solids, base ten blocks, color tiles, pattern blocks
Mental Math	hundreds chart, dice, spinners
Money	coins, bills
Odd, Even, Prime, or Composite Numbers	color tiles, two-colored counters
Patterns	pattern blocks, attribute blocks, tangrams, calculators, color tiles, Cuisenaire® rods, dominoes, pasta, beans, cereal, candy
Percent	base ten blocks, color tiles
Perimeter and Circumference	geoboards, color tiles, tangrams, pattern blocks, rulers
Place Value	base ten blocks, Cuisenaire® rods
Probability	spinners, dice, money, color tiles, two-color counters, cereal, pasta, candy, beans
Ratio and Proportion	color tiles, tangrams, pattern blocks, two-color counters, cereal, candy, crackers, beans, pasta
Similarity and Congruence	attribute blocks, pattern blocks, tangrams, geoboards
Size, Shape, Color	attribute blocks, color tiles, geoboards, geometric solids, pattern blocks, tangrams, pasta, beans, cereal, candy, crackers
Square and Cubic Numbers	color tiles, cubes, base ten blocks, geoboards
Surface Area	color tiles, cubes
Tessellations	pattern blocks, attribute blocks
Transformations	geoboards, pattern blocks, tangrams
Volume	capacity containers, cubes, geometric solids
Whole Numbers	base ten blocks, dice, spinners, color tiles, money, dominoes, rulers, calculators, clocks, two-color counters, pasta, cereal, candy, beans

Adapted from National Council of Supervisors of Mathematics (1994)

Manipulatives Overview *(cont.)*

Preventing Manipulative Dependency

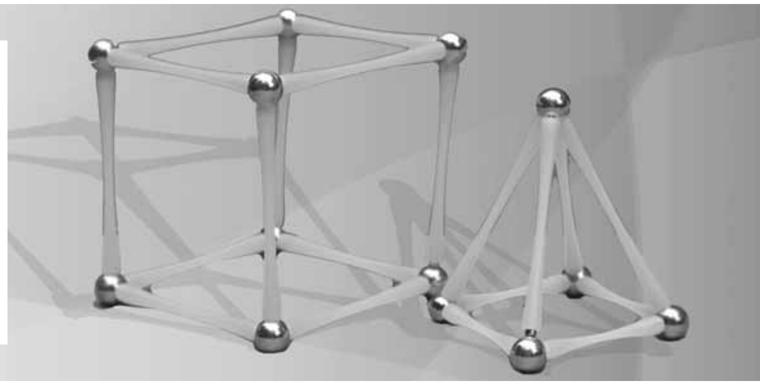
Students do not automatically connect symbolic representations of concepts to the activities and concepts they learn using manipulatives. They often think that finding solutions using manipulatives is completely separate from those using a paper and pencil. Effective teachers help their students make the connections between manipulatives and written problems. Helping students connect the concrete objects to standard math symbols and operations is essential (Allen 2003).

Students need many experiences with concrete materials before they are ready to move on to abstract representations of those concepts. Initially, working with and talking about problems with concrete objects will help students develop deeper conceptual knowledge. When the conceptual knowledge has been well developed, effective teachers then demonstrate how pictures can represent the concrete objects. Then, students need to be taught how to create the pictorial representations for the concrete models. Once students are proficient with the pictorial stage of a concept, they can be shown how symbols are used to record what has been expressed with the pictures. Discussions should also be included in all of these stages of teaching so that students see how the three stages of representations all show the same concept.

If lessons are repeatedly taught in the sequence discussed above, students will not become manipulative dependent. The length of time spent in each phase of the sequence depends on the students and the content being taught. The steps are as follows:

1. **Free Exploration Time:** After an introduction using overhead manipulatives, students should be given free exploration time. On the first day using the manipulatives, an entire class period may be needed.
2. **Teaching with Concrete Representations:** At the end of the free exploration period, the students are given a task to perform using the manipulatives.
3. **Practicing Concrete Representations:** The students will use only concrete models or manipulatives until the majority of students understand the concept.
4. **Modeling Pictorial Representations:** While the students are using the concrete materials, the teacher will begin modeling the pictorial representations.
5. **Teaching Pictorial Representations:** The teacher teaches the students how to draw pictorial representations.
6. **Practicing Pictorial Representations:** The students will practice drawing the pictorial models.
7. **Modeling Symbolic (Abstract) Representations:** As the students continue to work with the concrete and pictorial, the teacher will begin using the symbols.
8. **Teaching Symbolic (Abstract) Representations:** The students will be taught how to write the symbols.
9. **Practicing Symbolic (Abstract) Representations:** The students begin using the symbols instead of the concrete and pictorial models (Mink and Fackler 1996).

Using Counters for Skip Counting by 2s and 5s



Standard

- extends simple patterns of numbers

Grades K–2 Sample Lesson

Materials

- two-color counters
- *Counting by 2s and 5s* (page 85; page085.pdf)
- sheets of paper
- clear bowl

Procedure

1. Begin by giving each student 5 counters. Have students hold up both hands. Ask them, "How many hands are you holding up?" *(two)*
2. Have students put a counter in each hand. Ask them, "How many counters are you holding?" *(two)*
3. Have a student come up to the front of the room and place his or her counters in a clear bowl. As a class, count "one, two" as the student places the counters into the bowl. Write the number 2 on the board or overhead and draw two circles above the number to represent the counters.
4. Have another student drop his or her 2 counters in the bowl. As a class, count "three, four" while the student drops the counters. Write the number 4 on the board or overhead and draw two circles above the number to represent the additional counters that are in the bowl. Repeat this with another student to total 6 counters in the bowl.
5. Distribute a sheet of paper to each student. Have them write the sequence from the board on their papers. Instruct them also to draw the counters. Ask students which two numbers would be next. Provide students with more counters to use to figure this out and have them write their answers on their papers.
6. Repeat steps 3 and 4 with two other students so that the class can check their work. Explain to students they are counting by 2s because each student is dropping 2 counters into the bowl.
7. Pair students according to ability level and give each pair 50 counters.

Using Counters for Skip Counting by 2*s* and 5*s* *(cont.)*

Procedure *(cont.)*

8. Distribute the activity sheet, *Counting by 2s and 5s* (page 85). Complete the first problem as a class. Students should place the appropriate counters in the boxes on their papers before drawing them. If students have difficulty starting at 6, allow them to place counters to the side of the problem representing the numbers 2 and 4 in the sequence.
9. Instruct the pairs to place 5 counters in front of them in a pile. Write the number 5 on the board or overhead and draw 5 circles above the number.
10. Instruct pairs to put another pile of 5 counters in front of them. Ask students how many counters they have altogether. *(10)* Write the number 10 on the board or overhead and draw 5 more circles above it. Repeat these steps up to the number 30.
11. Complete question 4 on the activity sheet as a class.
12. With their partners, have students complete the activity sheet according to the suggestions in the differentiation section below.

Differentiation

Above-Level Learners

After completing questions 3, 5, and 7 on the activity sheet, have pairs use counters to investigate other numbers to use for skip counting. Pairs should record and draw one sequence on the backs of their papers. For example, pairs could count by 10s, 3s, or 4s.

Below-Level Learners

If starting in the middle of a sequence is difficult, allow students to create the sequences beginning at 2 or 5 in order to find the missing numbers in the sequence.

English Language Learners

Work with these students in a small group. Provide them with number and picture cards to use to match with the quantities used in the sequences.

Name ______________________________

Counting by 2*s* and 5*s*

Directions: Use counters to count by 2.

1. 6, 8, ________, 12, 14

2. 12, 14, ________, ________, 20

3. 10, 12, ________, ________, ________

Directions: Use counters to count by 5.

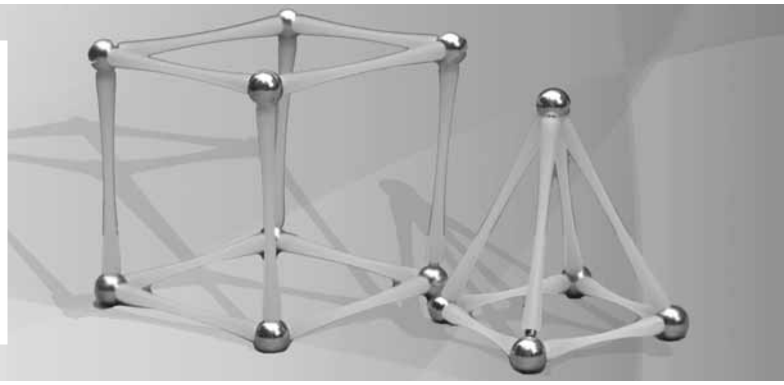
4. 10, 15, ________, ________, ________

5. 15, 20, ________, ________, 35

6. 5, 10, ________, ________, ________

7. 20, 25, ________, ________, ________

Using Counters for Multiplication with Arrays



Standard

- multiplies whole numbers

Grades 3–5 Sample Lesson

Materials

- bags of 30 counters per student
- *Using Arrays* (page 88; page088.pdf)
- sheets of paper

Procedure

1. Begin by placing students in groups so that they can make human arrays. For example, a group of 6 students can stand and make a 2 × 3, 3 × 2, 1 × 6, or 6 × 1 array.
2. In their groups, have students decide on how many arrays they can make. Instruct them to draw their human arrays on their sheets of paper.
3. Distribute the bags of counters to the students. Invite one group to come up to the front of the class to create its different arrays. When the group creates its first array, have the rest of the class create it using counters. Draw the array on the board or overhead and write the number of students in the row and column. For example, the group of 6 students can create the array of 2 students in a row by 3 in a column. Draw the array on the board or overhead and label the width 2 and the length 3. Then write 2 × 3 and say "2 by 3." Ask students, "How many total students created the array?" (*six*)
4. Repeat step 3 for all of the arrays that the group creates. Do not erase any of the drawings and labels so that students can use them to record. Students should also keep each array that they create in front of them. The arrays will be revisited later in the lesson.
5. After the group demonstrates all of its human arrays, discuss the drawings created by its work. For example, discuss how 2 students in each row multiplied by 3 students in each column equals 6 total students. Write 2 × 3 = 6 on the board or overhead and read the mathematical equation to the students.
6. Ask for student volunteers to help you write a multiplication problem for each array on the board or overhead. Show students that they can check their work by counting the total number of circles/counters used in the array.

Using Counters for Multiplication with Arrays *(cont.)*

Procedure *(cont.)*

7. Place students in pairs.
8. Using their counters, have each pair create as many arrays as possible with a total of 12 counters. Students should also write the multiplication problem for each array.
9. Review with students all the possible arrays that can be made with 12 counters. Explain to students that the number in the row and column of the rectangle are called factors and the total number of counters in the array is called the product.
10. Distribute the activity sheet *Using Arrays* (page 88) to the students.
11. Have students complete the activity sheet according to the differentiation suggestions listed below.

Differentiation

Above-Level Learners

Instruct students to complete problems 2 and 3 independently. With remaining time, instruct students to write at least three sentences about the patterns they see in the multiplication sentences produced by each number.

Below-Level Learners

Allow students to complete the activity sheet in pairs. Students should create all of the arrays with counters before drawing them on their activity sheets. This will help them visualize all of the combinations and check their work before recording it.

English Language Learners

Complete the first problem as a group. Discuss the terms *array*, *product*, *multiplication sentence*, *length*, and *width*. Choose one array and label it using the discussed terms. Allow students to complete the rest of the activity in pairs or small groups.

Name ____

Using Arrays

Directions: Use counters to create as many arrays as possible that show the given number. Draw the arrays you create and write the equation that is shown by each array.

1.

18

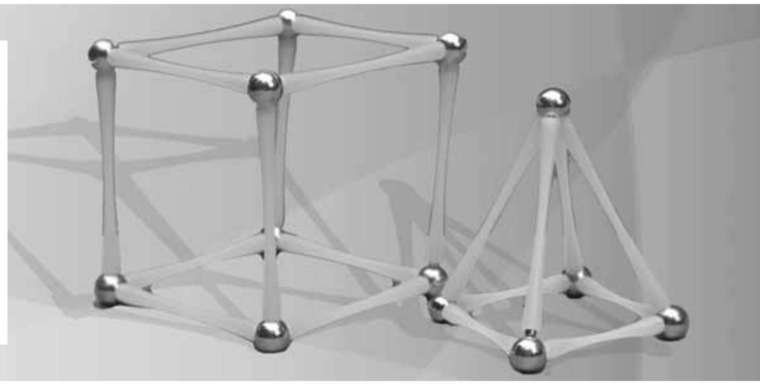
2.

24

3.

20

Using Counters for Solving Linear Equations



Standard

- solves linear equations using concrete, informal, and formal methods

Secondary Sample Lesson

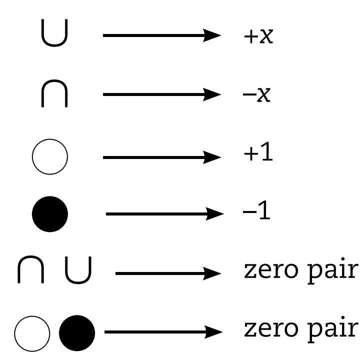
Materials

- small cups (medicine sized)
- two-color counters (chips)
- pencils
- overhead cups and chips (Cut the cups in half heighth-wise so that they lie flat on the overhead projector.)
- *Cups and Chips* (page 91; page091.pdf)

Procedure

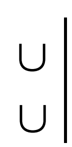
1. Before the lesson, create student sets of cups and chips. Each set should contain 30 counters (chips), 10 cups, and a pencil.
2. To begin the lesson, present the value of the cups and chips to the students. An "up cup" equals positive x, a "down cup" equals $-x$, the yellow face of the chip equals positive 1, and the red side of the chip equals –1. Also show students that a zero pair can be made by combining 1 up cup and 1 down cup or by combining 1 yellow chip and 1 red chip. Draw the symbols shown to the right on the board and discuss what they represent and how they correlate to each piece of their student sets.

Cups and Chips



3. Write the equation $2x = 10$ on the board or overhead. Using the overhead set of cups and chips, set up the equation as shown on the right, explaining that the line down the middle of the problem (the pencil in their student sets) acts like the equal sign in the equation. As you lay out the cups, explain to students that the number of cups is designated by the equation. In this equation there are two xs, so there are two cups.

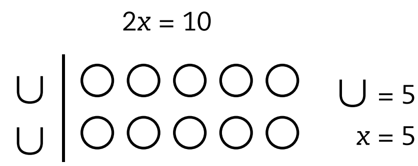
$2x = 10$



Using Counters for Solving Linear Equations *(cont.)*

Procedure *(cont.)*

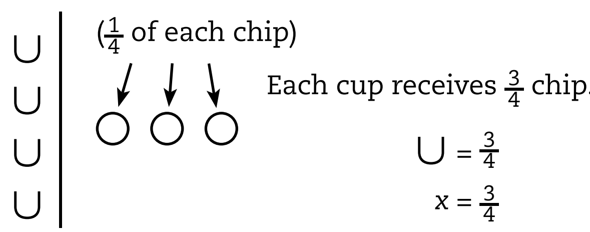
4. Explain next that there are 10 positive integers, so there needs to be 10 yellow chips used. These chips should be divided equally into the cups. Each cup gets one whole chip at a time until there are no more chips to distribute. Because 2 can divide evenly into 10, there are no chips left over.



5. Distribute the student sets. Repeat this problem, having the students follow along using their sets of cups and chips. When the students have finished, have them move their manipulatives to the sides of their desks so that they can watch as you present the next example.

6. Now display the equation $4x = 3$. Ask students how many cups you should display on the left side of the equation and why. *(4; There are 4 xs on the left side of the equation.)* Then ask students how many chips are needed to solve the equation and why. *(3; There are 3 integers on the right side of the equation.)*

7. Ask students if they can evenly divide the 3 chips into the 4 cups. *(No)* Then explain that they can place a fraction of each chip into each cup, and demonstrate this for the students. It may be necessary to create paper chips for students who need to actually cut the chip into pieces in order to visualize the fraction of each chip that belongs in each cup.



8. Repeat this problem, having the students follow along using their sets of cups and chips.

9. As a class, complete the following problems:
   - $6x = -9$
   - $5x = -7$
   - $4x = 11$
   - $2x = 13$
   - $2x + x = 15$
   - $3x = -3 + 15$

10. Distribute *Cups and Chips* (page 91) to the students. Have them complete the activity sheet according to the appropriate differentiation suggestions below.

Differentiation

Above-Level Learners

Have students work in pairs to complete problems 6–9. Then present pairs with the following problem: $4x - 2 = 3x + 1$. Challenge pairs to use the cups and chips to figure out a method for solving that problem.

Below-Level Learners

Have students work in pairs to complete problems 1–6. With remaining time, allow pairs to choose one other problem to work as a challenge.

English Language Learners

Use paper chips to solve the problems and allow students to physically cut the chips when fractions are involved. Allow students to complete the activity sheet in pairs or small groups.

Name ______________________________

Cups and Chips

Directions: Solve the problems below using cups and chips. Show your work.

1. $8x = 26$

2. $3x = 17$

3. $4x = -9$

4. $2x = -18$

5. $3x = -21$

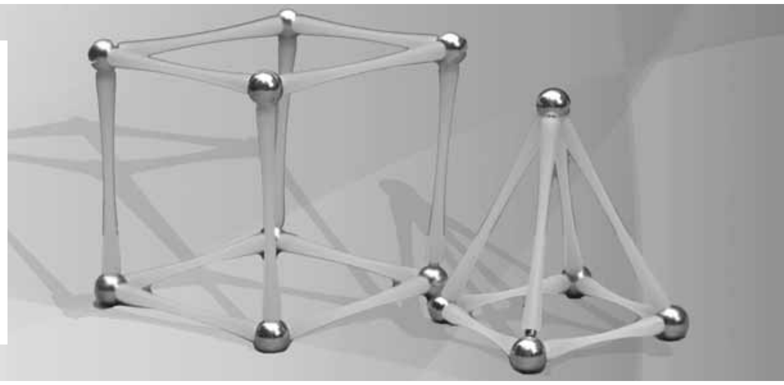
6. $4x + 2x = 24$

7. $5x = 23 - 4$

8. $2x = 5 + 16$

9. $3x + 4x = -22$

Using Linking Cubes for Data Analysis



Standard

- collects and represents information about objects or events in simple graphs

Grades K–2 Sample Lesson

Materials

- linking cubes
- *Our Pocket Graph* (page 94; page094.pdf)
- small pieces of paper

Procedure

1. Begin by having students stand behind their desks and examine their clothes. Ask volunteers to tell what types of clothes they are wearing, such as a skirt, pants, or jacket.
2. Tell students that they will be collecting data about their pockets today. Ask them for suggestions as to how they could find out how many pockets each person has on his or her clothes. List their suggestions on the board or overhead.
3. Have students look around the room and make a prediction about which person in the class they think is wearing the most pockets and which is wearing the least pockets. Distribute small pieces of paper to the students and have them record the names of those two students.
4. Ask for volunteers to share their predictions and tell why they chose those particular students.
5. Explain to students that they will be using linking cubes to help them count the number of pockets they are wearing.
6. Distribute a handful of linking cubes to each student in the class.
7. Have them put one cube in each of their pockets. Once they have placed a cube in each of their pockets, collect the leftover cubes.
8. Instruct each student to take all of the cubes out of his or her pockets and link them together by stacking one on top of another.
9. When everyone has finished, instruct students to hold their sticks of cubes above their heads and view the other sticks. Tell students that they have now created data.
10. Instruct students to sit down at their desks. Call two students to the front of the room. Have them place their sticks side by side and display them for the rest of the class to see. Tell the class that those sticks represent two pieces of data.

Using Linking Cubes for Data Analysis *(cont.)*

Procedure *(cont.)*

11. Ask each student to tell the class how many pockets his or her stick represents. Then, ask the class a question that compares the two sticks, such as, "Whose stick is larger?" or "How many more pockets does student 1 have than student 2?"
12. Repeat steps 10–11 with other pairs of students. If comparing two sets of data is easy for your students, consider inviting three or four students to the front of the room.
13. Have the whole class stand once again. Give the students a directive such as, "Walk slowly around the room and find someone who has a different number of pockets than you. Stand with that person. Figure out who has more pockets." Tell students that a group of three can form, if necessary.
14. Once students have all found partners, ask each group one at a time to hold up their sticks and tell the class who has more pockets and how they know that to be true.
15. Repeat steps 13–14 with other directives such as, "Find someone who has a different number of pockets than you and determine who has fewer pockets," "Find someone who has an equal number of pockets as you," or "Find someone who has at least two more pockets than you."
16. Divide the students into groups of 4–5 and allow them to sit at tables or desks around the room. On the tables, instruct groups to lay their sticks side by side. Ask them what type of graph this looks similar to. *(bar graph)*
17. Distribute a copy of *Our Pocket Graph* (page 94) to each student. Have them complete the graph with their groups. Remind them to include all of the parts of the graph: title, data, and labels.
18. Use the differentiation suggestions below to help students complete the rest of the activity sheet.

Differentiation

Above-Level Learners

Instruct students to independently write at least three questions that can be answered using the information on their graphs. When they have completed writing their questions, they should switch papers with partners and answer the questions they receive.

Below-Level Learners

Instruct students to work in pairs to write at least two questions that can be answered using one of their graphs. When finished, each pair should switch papers with another pair and answer the questions it receives.

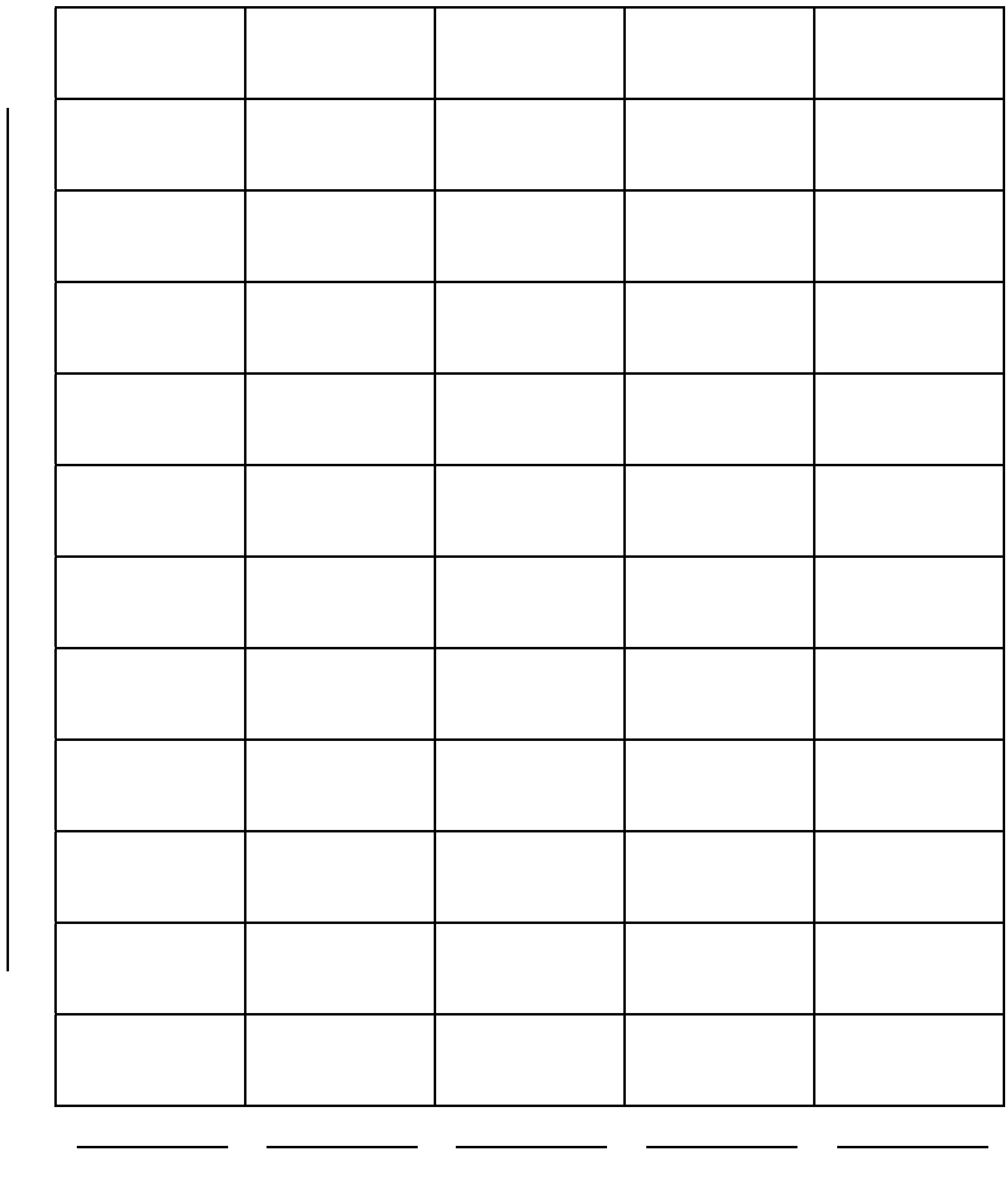
English Language Learners

Work with these students in a small group. Discuss the components of the graphs, having the students point to each component on their graphs as it is discussed. Then, choose 2–3 graphs to discuss together. Ask students questions that can be answered using the information in the graphs.

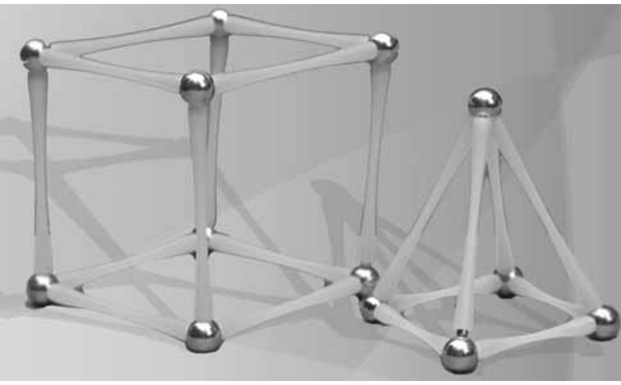
Name ______________________________

Our Pocket Graph

Directions: Create a bar graph using the grid below. The graph should show the number of pockets you and the members of your group have. Make sure to label all of the parts of the graph.



Using Linking Cubes for Understanding Area and Perimeter



Standard

- understands the basic measures of perimeter and area

Grades 3–5 Sample Lesson

Materials

- bag of 50 linking cubes for each pair of students
- *Areas and Perimeters* (page 97; page097.pdf)

Procedure

1. Write the following on the board or overhead:

   Would you need to find the area or the perimeter for the following?

   a. border around a classroom bulletin board *(Answer: perimeter)*

   b. tiles for a floor *(Answer: area)*

   c. wallpaper for a wall *(Answer: area)*

   d. fence around a yard *(Answer: perimeter)*

2. Place students in pairs. Have them determine if perimeter or area is needed for each scenario. Review the answers with the students.
3. Give each pair 50 linking cubes.
4. Using scenario **a**, have the pairs show the perimeter of 20 feet. Tell students that each side of the cube is a foot. Have the pairs share their answers. Students could use the linking cubes to create an array or an outline of the rectangle using the given dimensions. They may also think of other ways to use the cubes to show the perimeter. Allow several groups to share their methods with the class using the overhead or another electronic display device.
5. When each pair shares its answer, write the measurement of the width and length of each side. Show students how the perimeter was calculated. For example, one pair may have made a rectangle with a width of 4 feet and a length of 6 feet. Write on the board or overhead the following:

   w = 4 feet l = 6 feet P = 4 feet + 4 feet + 6 feet + 6 feet = 20 feet

Using Linking Cubes for Understanding Area and Perimeter *(cont.)*

Procedure *(cont.)*

6. Using scenario **b**, have the pairs show the area of 24 $ft.^2$ Tell students that each side of the cube is a foot. Have the pairs share their answers. Allow several groups to share their methods with the class using the overhead projector or other electronic display device.
7. When each pair shares its answer, write the measurements of the width and length of each side. Show students how the area was calculated. For example, one pair may have made a rectangle with a width of 3 feet and a length of 8 feet. Write on the board or overhead the following:

   w = 3 feet l = 8 feet A = 3 feet • 8 feet = 24 $ft.^2$
8. Have the pairs create a rectangle with a perimeter of 38 feet and a width of 9 feet. Tell students that each side of the cube is a foot in length. Give pairs time to solve the problem. Then, allow several groups to share how they solved for the length of the rectangle with the class, using the overhead or another electronic display device.
9. Have the pairs create a rectangle with an area of 48 $ft.^2$ and a length of 12 feet. Tell students that each side of the cube is a foot in length. Give pairs time to solve the problem. Then, allow several groups to share how they solved for the width of the rectangle with the class, using the overhead or another electronic display device.
10. Write the problem below on the board or overhead.

    Kiara wants to cover her journal with paper. Her journal has a length of 8 inches and a width of 6 inches. What is the area of the journal that she needs to cover? *(48 $in.^2$)*
11. Review the problems with the students and demonstrate how the linking cubes can be used to solve them. Then, distribute copies of *Areas and Perimeters* (page 97) to the students and have them complete it according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 5–7 independently and then check their work in pairs. If they answer the questions correctly, they may write word problems of their own. If they do not answer the questions correctly, they must complete the rest of the activity sheet for additional practice.

Below-Level Learners

Have students work in groups of 3–4 to complete the activity sheet. Assign students the roles of Timekeeper, Materials Officer, Materials Organizer, and Group Leader (see page 80 for more information). When groups need assistance, they may confer with another group. If they still have a question after conferring with the other group, then they may come to you for help.

English Language Learners

Work with these students in a group. Model solving each problem with the linking cubes and have students follow along as you work. Make sure there is open discussion about the definition of the terms *length*, *width*, *area*, and *perimeter*. It may be helpful to create a group chart that visually displays the definitions of each of the terms.

Name ____

Areas and Perimeters

Directions: Use the linking cubes to find the area and perimeter. Draw a diagram in the space below to show how you used the linking cubes to solve the problems.

1. l = 8 ft.
w = 4 ft.
A = ______________
P = ______________

2. l = 12 ft.
w = 2 ft.
A = ______________
P = ______________

3. l = 9 ft.
w = 5 ft.
A = ______________
P = ______________

4. l = 11 ft.
w = 4 ft.
A = ______________
P = ______________

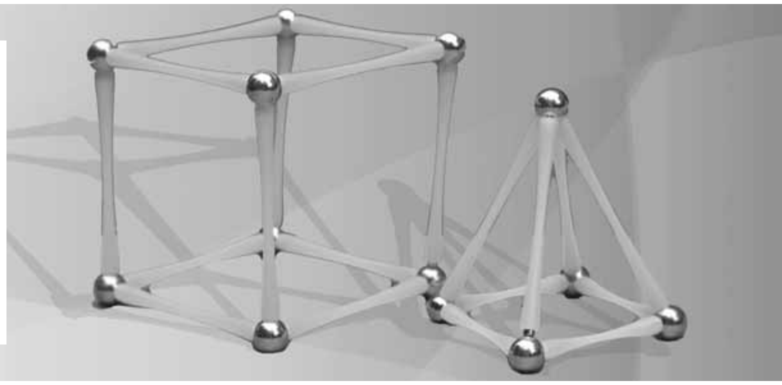
Directions: Use the linking cubes to find the length of each rectangle. Draw a diagram in the spaces below to show how you used the linking cubes to solve the problems.

5. Salena is planting a garden. She has 46 feet of garden border. The length of the garden is 13 feet. What is the width of the garden?

6. Hans is covering a poster board that has an area of 20 $in.^2$ The width of the poster board is 4 inches. What is the length of the board?

7. Omar has made a toy box as a present for his younger sister. The toy box is 3 feet wide and 6 feet long. He has decided to glue a piece of ribbon around the perimeter of the toy box as a decoration. How long does the ribbon need to be to fit around the perimeter of the toy box?

Using Linking Cubes for Understanding Probability



Standards

- determines probability using simulations or experiments
- understands the relationship between the numerical expression of a probability (e.g., fraction) and the events that produce these numbers

Secondary Sample Lesson

Materials

- bags with 20 linking cubes: 6 red, 3 yellow, 2 orange, 5 brown, 3 green, and 1 blue (1 bag per pair of students)
- *Cube Probability* (page 100; page100.pdf)

Procedure

1. Organize students into pairs according to ability level.
2. Give each pair of students a bag of linking cubes and a copy of the *Cube Probability* activity sheet (page 100). Explain to the students that they will be conducting an experiment by randomly drawing cubes out of their bags.
3. Instruct students to sort the cubes by color and then record the theoretical probability of drawing each color in the first chart on their sheets. For example, since 5 out of 20 of the cubes are brown, the theoretical probability of drawing a brown is 25%.
4. Have each pair put the cubes back in the bag.
5. To complete the activity, the students will work with their partners to determine if the experimental probability corresponds to the theoretical probability and if the accuracy of experimental probability increases as more trials are completed.
6. To complete the next section of the activity sheet, have one partner shake the bag and pull out one cube. The other partner should record the color of the cube on the sheet.
7. After putting the cube back into the bag, the partners should continue drawing cubes and recording their colors until there have been 50 draws. Pairs should then record the totals in the table provided.
8. As a class, discuss whether the draws matched what they expected. Also discuss why there is a possibility that each pair has different results at this time.

Using Linking Cubes for Understanding Probability *(cont.)*

Procedure *(cont.)*

9. Instruct pairs to complete 50 more draws and record the results. They should then combine the 100 draws and record the final totals in the last chart on the activity sheet.
10. Allow students to answer the final question according to the differentiation suggestions below. Then, lead a class discussion about students' observations. They should come to the conclusion that experimental probability more closely matches theoretical probability as the number of trials increases.

Differentiation

Above-Level Learners

Have students complete the last question as written. Then have students make a prediction about experimental probability if 500 trials were recorded.

Below-Level Learners

Allow students to create bulleted lists of their observations as they pertain to the last question on the activity sheet. It may be necessary for pairs to work in groups of four in order to see enough data to make their generalizations.

English Language Learners

Allow these students to dictate their observations and generalizations to you or other English-proficient helpers in the classroom. Record their thoughts on the backs of their papers. If students feel comfortable responding to parts of the last question on their own, allow them to do so before beginning to take dictation.

Name ________________________________

Cube Probability

Directions: Fill in the sheet as instructed by your teacher.

1. Sort the cubes from your bag by color. What is the theoretical probability that each color will be drawn from the bag?

Yellow	Red	Blue	Orange	Green	Brown

Draw a cube from the bag and, using the code, write the corresponding letter in the boxes below. Return the cube to the bag after each draw.

Yellow = **Y** Red = **R** Blue = **B** Orange = **O** Green = **G** Brown = **BR**

2. Record 50 trials.

3. What was the experimental probability that each color was drawn from the bag?

Yellow	Red	Blue	Orange	Green	Brown

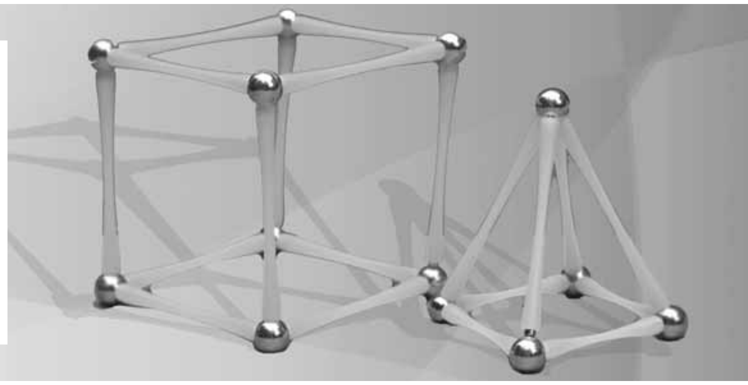
4. Record 50 more trials.

5. Combine the two sets of trials. What was the experimental probability that each color was drawn from the bag?

Yellow	Red	Blue	Orange	Green	Brown

6. Compare the theoretical probability to the experimental probability after 50 trials and to the experimental probability after 100 trials. On a separate sheet of paper, write a paragraph about what you can conclude about theoretical and experimental probability when there is a greater number of trials.

Using Base Ten Blocks for Addition



Standards

- adds and subtracts whole numbers
- understands the basic meaning of place value

Grades K–2 Sample Lesson

Materials

- base ten blocks
- bags of base ten blocks (1 flat, 14 rods, and 25 units—1 bag per student)
- place-value mats
- *Addition Fun* (page 103; page103.pdf)

Procedure

1. Distribute bags of base ten blocks and place-value mats to the students. If place-value mats are unavailable, the students can fold pieces of paper in thirds and label the top of each section with one of the following: *hundreds*, *tens*, *ones*.
2. Have student volunteers remind the class of the value of each of the items in their bags. *(unit = 1; rod = 10; flat = 100)*
3. Have students place 4 units in the ones place on their mats. Underneath the 4 units, have them place 3 units. Ask students how many units are now on the mats. *(7)* Invite students to share the addition sentence that goes with what the students just did on their mats. *(4 + 3 = 7)* Record this on the overhead.
4. Have the students place 15 units in the ones place on their mats. Underneath the 15 units, have them place 10 more units. Ask students to add or combine the two sets and invite a student to share the result. *(25)* Ask students if it is appropriate to have 25 units in the ones place. *(no)* Complete these same steps on the overhead, using base ten blocks as a demonstration tool.
5. Use the overhead and base ten blocks to show them how to exchange 10 units for 1 rod. To make sure students see the correlation, have them line up the 10 units to make the exact same size as 1 rod. Then put the rod in the tens place because it stands for 10 units.

Using Base Ten Blocks for Addition *(cont.)*

Procedure *(cont.)*

6. Ask students if any more rods can be made from the remaining units in the ones place. *(yes, 1)* As a class, exchange another group of 10 units for 1 rod and place the rod in the tens place. Ask how many units are left after regrouping *(5)* and how many rods they have now. *(2)* Complete these same steps on the overhead using base ten blocks as a demonstration tool.
7. Discuss the steps that the students just followed to create the addition sentence 15 + 10.
8. Allow students to look at their place-value mats to figure out the sum of the two numbers. *(25)*
9. Repeat the procedures from steps 4–8 to answer the following problems: 23 + 20; 17 + 30; 42 + 14. When beginning with numbers greater than 20, have students create them using rods and units to begin with, not only units.
10. Have the students complete the *Addition Fun* activity sheet (page 103) according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 6–9 on the activity sheet. Then instruct students to create an addition problem of their own where the answer is a three-digit number. Have them write at least two sentences to tell how they could find the answer using base ten blocks.

Below-Level Learners

Have students complete questions 1–6 of the activity sheet in pairs. With any remaining time, have pairs choose one of the problems they completed and write at least two sentences to tell how they used base ten blocks to solve it.

English Language Learners

Before beginning the activity sheet, review the names of the base ten blocks with the students. If necessary, draw visual representations of the blocks on a chart and write the words associated with each block. Then complete questions 1–6 in a small group. Students who understand the concept quickly should be allowed to complete the activity sheet in pairs away from the rest of the group.

Name ______________________________

Addition Fun

Directions: Solve the problems below using base ten blocks and a place-value mat.

1. 18 + 26 = ____

2. 42 + 36 = ____

3. 38 + 7 = ____

4. 71 + 20 = ____

5. 52 + 37 = ____

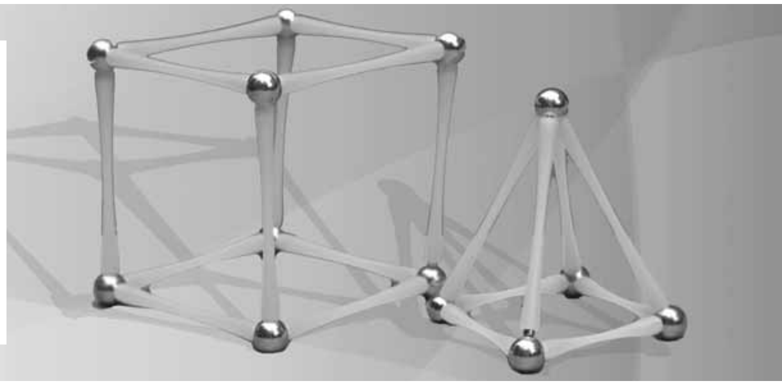
6. 49 + 29 = ____

7. 58 + 17 = ____

8. 65 + 21 = ____

9. 43 + 38 = ____

Using Base Ten Blocks for Division



Standards

- multiplies and divides whole numbers
- understands the basic meaning of place value

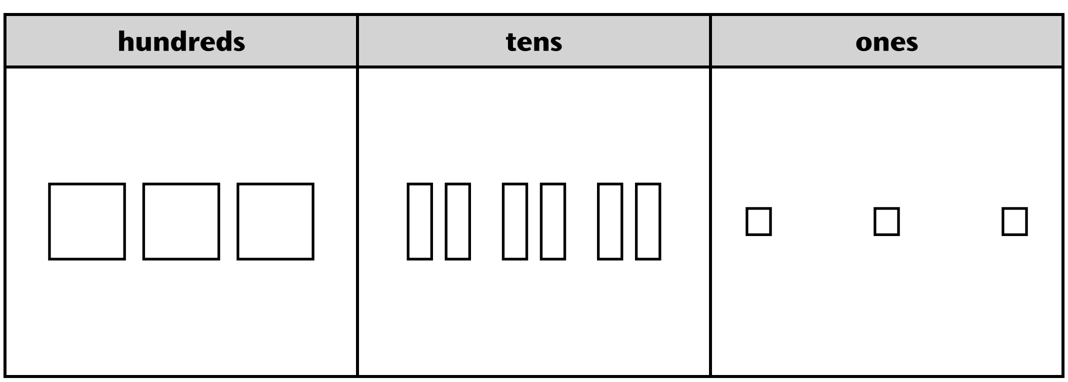
Grades 3–5 Sample Lesson

Materials

- base ten blocks
- bags of base ten blocks (4 flats, 20 rods, and 25 units—1 bag per student)
- place-value mats
- *Daring Division* (page 106; page106.pdf)

Procedure

1. Distribute the bags of base ten blocks and the place-value mats to the students. If place-value mats are unavailable, the students can fold pieces of paper in thirds and label the top of each section with one of the following: *hundreds*, *tens*, *ones*.
2. Have students place blocks representing the number 363 on their mats. *(3 flats, 6 rods, and 3 units)* While modeling on the overhead, have students divide the blocks into 3 equal groups. This represents the division number sentence 363 ÷ 3. To do this, begin by dividing the flats evenly, then the rods, and finally the units. (See the visual below.) Then, ask students how many blocks are in each group. *(121—1 flat, 2 rods, and 1 unit)* Therefore, 363 ÷ 3 = 121.



Using Base Ten Blocks for Division *(cont.)*

Procedure *(cont.)*

3. Have students place 345 on their mats. While modeling on the overhead, have the students divide the blocks into 3 equal groups. This represents the division number sentence 345 ÷ 3. To do this, divide the flats evenly. There is 1 in each group. Divide the rods evenly. There is 1 in each group, and there is 1 rod left over. Have students use the rules of regrouping to trade the rod for 10 units. There are now a total of 15 units. Divide the 15 units evenly into the three groups. There are 5 units in each group. Ask students what the total number of blocks is in each group. (115—1 flat, 1 rod, and 5 units) Therefore, 345 ÷ 3 = 115.
4. Divide the class into groups of three or four. Assign each group one of the problems below. Allow the groups to work their assigned problems and discuss how they will present their problems to the rest of the class. Then, allow each group to present its problem to the class. The class should follow along with the group that is presenting so that each student gets practice in working the problems.
   - 284 ÷ 2
   - 348 ÷ 3
   - 426 ÷ 2
   - **185 ÷ 5**
   - **236 ÷ 4**
   - **381 ÷ 3**
5. Have students complete *Daring Division* (page 106) according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 7–12. Then, allow them to write their own problem and illustrate the solution in the place-value mat at the bottom of the page. Have students write the problem and the answer below the place-value mat.

Below-Level Learners

Have students complete questions 1–7 in pairs and choose one problem's solution to illustrate on the place-value mat at the bottom of the page. With remaining time, allow pairs to choose one of the additional problems on the page to solve. Pairs should choose a problem that they think would be a challenge for them.

English Language Learners

Review the procedure for solving each problem and the vocabulary associated with the manipulatives. Allow students to choose 7 problems from the activity sheet to solve either in pairs or small groups. Challenge students to pick some problems that they think may be a little difficult for them. Students should also choose one problem's solution to illustrate on the place-value mat at the bottom of the page.

Name ______________________________

Daring Division

Directions: Use base ten blocks and a place-value mat to solve the problems below. Choose one problem that you solved to illustrate on the place-value mat at the bottom of the page.

1. 252 ÷ 2 =

2. 168 ÷ 4 =

3. 318 ÷ 3 =

4. 428 ÷ 2 =

5. 393 ÷ 3 =

6. 184 ÷ 2 =

7. 165 ÷ 5 =

8. 248 ÷ 4 =

9. 453 ÷ 3 =

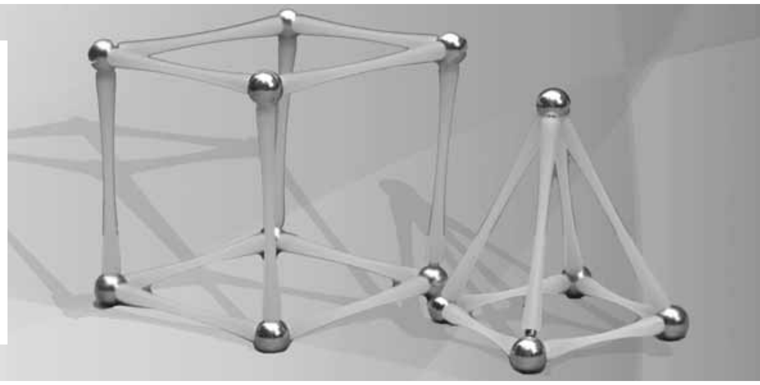
10. 205 ÷ 5 =

11. 310 ÷ 2 =

12. 486 ÷ 3 =

hundreds	tens	ones

Using Algebra Tiles for Collecting Like Terms



Standard

- Understands basic operations (e.g., combining like terms) on algebraic expressions

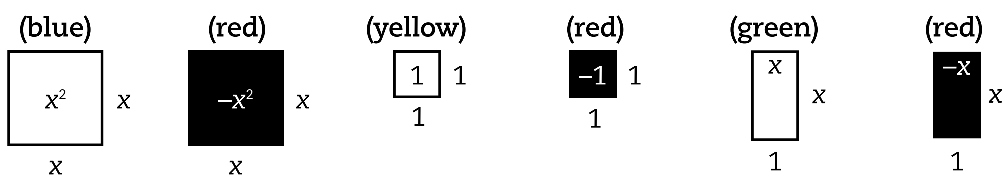
Secondary Sample Lesson

Materials

- overhead algebra tiles
- bag of algebra tiles for each student
- paper and pencils
- 1 red, yellow, green, and blue marker per pair of students
- *Combining Like Terms* (page 110; page110.pdf)

Procedure

1. Divide the class into pairs. Distribute the paper and markers to each pair.
2. Draw the diagram below on the board or overhead and have students create it on their papers.



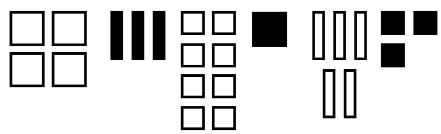
3. Distribute the bags of algebra tiles to the pairs. Have students find a tile to match each part of the diagram on their papers. As a class, discuss the tiles and their values.
4. Have a "pop quiz" by holding up each tile and asking, "What does this stand for?" Complete this several times until students are comfortable with the values of the manipulatives.
5. Explain how the tiles can be combined to make zero pairs. For example, a green rod (x) and a red rod ($-x$) combine to make a zero pair.
6. Model measuring the length and width of the blue square. They are the same. Tell the students that instead of using the actual length, they will let x represent the measurement. Tell students that the blue square is named for its area: $x \bullet x = x^2$. This area concept is the same for the other tiles.

Using Algebra Tiles for Collecting Like Terms *(cont.)*

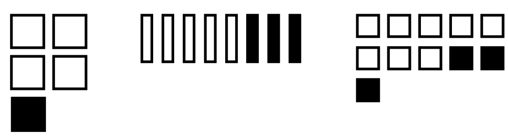
Procedure *(cont.)*

7. Write the following expression on the overhead: $4x^2 - 3x + 8 - x^2 + 5x - 3$. Have students record the same expression on their papers.
8. Model how to place the appropriate tiles under the first three terms of the expression and have students place the same tiles on their papers.
9. Have student volunteers come to the overhead projector and place algebra tiles under the remaining three terms.

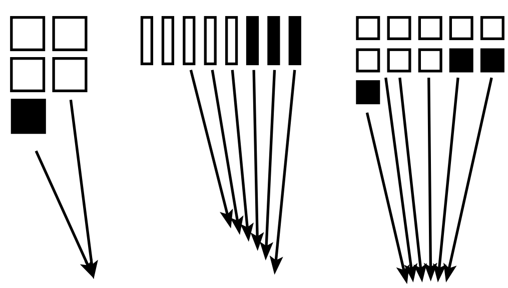
$$4x^2 - 3x + 8 - x^2 + 5x - 3$$



10. Show students how to arrange the tiles in descending order, and have them complete the same step after you model.

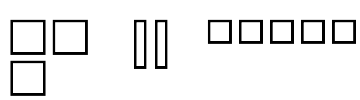


11. Then pull zero pairs to the side and have students do the same.



12. Look at the values of the remaining algebra tiles to determine what expression remains. ($3x^2 + 2x + 5$)

$3x^2$ $2x$ 5



Using Algebra Tiles for Collecting Like Terms *(cont.)*

Procedure *(cont.)*

13. Complete the problems below as a class. First, write the expression and allow pairs to copy it from the overhead. Then, turn the overhead off and instruct them to set up the expression using the algebra tiles. Place the correct algebra tiles on the overhead while students are working. Turn the overhead back on for pairs to check their work. Turn the overhead off and have pairs combine like terms while you do the same. Turn the overhead back on for pairs to check their work. As a class, write the expression for the remaining terms.
    - $2x^2 + x - 6 - 3x^2 - 4x + 8$
    - $4x - 5x^2 - 2x + 5$
    - $-5x - 3x^2 + 4 + x - 2$
    - $x^2 + 6 + 5x + 3x^2 - 6$
14. Allow students to complete *Combining Like Terms* (page 110) according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Instruct students to choose the four problems that they think are most challenging and complete those. Then allow each student to create an expression and find a partner with whom to solve each other's expressions.

Below-Level Learners

Allow students to work in groups of 3 or 4 to complete the activity sheet. Instruct them to review the chart created at the beginning of the lesson before solving any problems. Give each person in the group a role. (See page 80 for more information on assigning roles.)

English Language Learners

Review the chart created at the beginning of the lesson. In homogeneously grouped pairs, have students choose six problems that they think are right for them. Instruct students to discuss the steps they are following as they combine terms in each expression.

Name ____

Combining Like Terms

Directions: Use algebra tiles to solve each problem below.

1. $4x^2 + 7x - 3 + x^2 - 3x + 2$

2. $-6x^2 + 3x - 8 + 5x^2 + 2x + 12$

3. $7x - 5x^2 - 6 + 2x^2 - 8x + 4$

4. $x^2 - 2x + 1 + 8x^2 - 4x - 5$

5. $8x^2 + 2x - 9 + 2x^2 - 4x + 5$

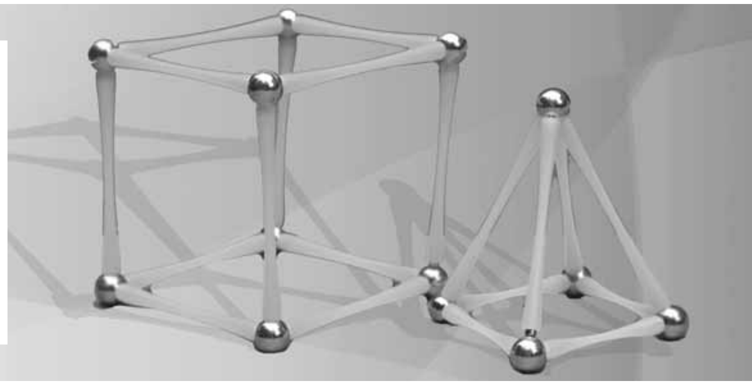
6. $9x + 3x - 3x^2 - 5 + 2$

7. $8 + 9x - 3 + 6x^2 - 3x$

8. $x^2 - 4x - 8 + 5x^2 + 7$

9. $4x + 7x^2 + 4 - 3x^2 - 3x + 6$

Using Pattern Blocks for Spatial Visualization



Standard

- knows that geometric shapes can be put together or taken apart to form other shapes

Grades K–2 Sample Lesson

Materials

- overhead pattern blocks
- bags of pattern blocks (1 bag per student; bags need 6 of each pattern-block shape)
- paper and pencils
- felt mats *(optional)*
- *Pattern Block Combinations* (page 113; page113.pdf)

Procedure

1. Distribute one felt mat and one bag of pattern blocks to each student. Give students several minutes for free exploration using their shapes.
2. With the person next to him or her, have each student review the names of each of the shapes in their bags.
3. Call out the name of a shape and have students choose that shape from their bags and hold it above their heads. Repeat this for each pattern-block shape.
4. Have students place a trapezoid on their mats in front of them. Tell students that they are going to try to make a trapezoid using pattern-block shapes that are not trapezoids. Give students the option of recreating the shape next to the original trapezoid or on top of the original trapezoid.
5. As students are working, circulate around the room and ask questions of students who are having difficulty. It is best for students who are having difficulty to practice building the new trapezoid on top of the original.
6. Ask the following questions:
   - "Which shapes have you used already? Did any of those seem to fit?"
   - "What do you notice when you place a triangle on the corner of the trapezoid?"
   - "Are you going to have to use more than one shape to make the new trapezoid? How do you know?"

Using Pattern Blocks for Spatial Visualization *(cont.)*

Procedure *(cont.)*

7. Once students have had time to work, ask for a volunteer to come up and build his or her new trapezoid using the overhead pattern blocks. Place a trapezoid on the overhead projector and have the student volunteer build his or her trapezoid below the original. Have the student explain the steps he or she took to figure out how to make the new trapezoid and tell the number of and type of shapes he or she used.
8. Ask the students to raise their hands if they built the new trapezoid in the same way as the student volunteer did.
9. Ask the students if anyone found a different way to build the trapezoid. Invite one of those students to the overhead projector to build his or her new trapezoid under the other two trapezoids, for the class.
10. Ask the students to raise their hands if they built the new trapezoid in the same way as the student volunteer.
11. Lead a discussion about the similarities and differences between the three trapezoids.
12. Distribute copies of *Pattern Block Combinations* (page 113) to the students. Tell them that they are going to be doing the same type of activity they just completed as a class, but using a hexagon instead of a trapezoid. Challenge the students to find as many combinations as possible without repeating a combination, and to record the combinations on the activity sheet.
13. Allow students to complete the activity sheet according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete the activity independently. With remaining time, students should create their own shape, using three pattern blocks. Instruct each student to trace the outline of his or her new shape and try to fill it, using various other shapes.

Below-Level Learners

Allow students to complete the activity sheet in pairs. If tracing the shapes to complete the second column of the chart is difficult, consider providing students with paper cutouts of the pattern blocks.

English Language Learners

Explain the assignment so that students clearly understand it, and allow them to ask clarification questions. As a group, create a reference sheet that lists the shapes and their names. Have students use this sheet to complete the activity. Students should complete the activity in pairs.

Name ______________________________

Pattern Block Combinations

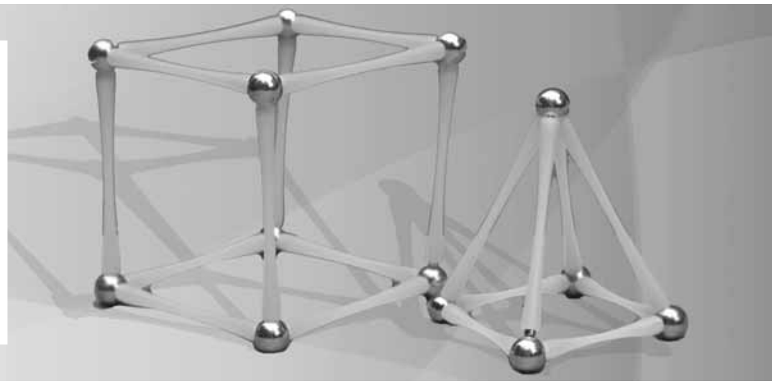
Directions: Find as many combinations as possible without repeating a combination, and record the combinations in the chart below.

Pattern Block and Number	Drawing
1.	
2.	
3.	
4.	
5.	
6.	
7.	

Using Pattern Blocks for Transformations in Quadrant 1

Standard

- uses motion geometry (e.g., turns, flips, slides) to understand geometric relationships



Grades 3–5 Sample Lesson

Materials

- pattern blocks
- graph paper
- *Coordinate Planes* (page 117; page117.pdf)
- *Transformations* (page 118; page118.pdf)

Procedure

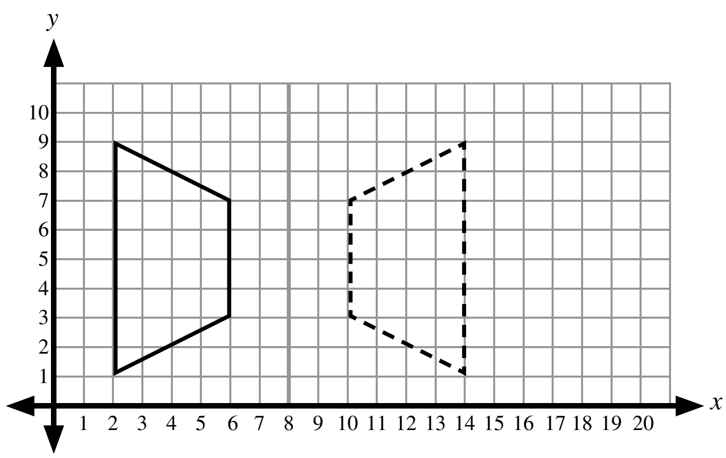
Note: This lesson uses the terms *slide*, *turn*, and *flip*. Depending on the level of your students, you may choose to use the terms *translation*, *rotation*, and *reflection*.

1. Distribute a handful of pattern blocks to the students. Allow them several minutes for free exploration.
2. Distribute a copy of *Coordinate Planes* (page 117) to each student.
3. Have students place a green triangle along the *x*-axis where one of the vertices touches the origin of the plane (0, 0). Instruct students to trace the triangle. Review sliding with students. Have them slide it three units to the right and trace the triangle again.
4. Have students remove the triangle from their coordinate plane and discuss what they did with a partner. Prompt students to discuss the direction of the slide, what *slide* means, and the similarities and differences between the original triangle and the slide. Ask for student volunteers to share their observations.
5. Write the following on the board or overhead and have students work the following with their partners:
   - slide the triangle two units up
   - slide the triangle four units to the right and three units up
   - slide the triangle six units to the right and five units up
6. Tell students to use the triangle and start by placing it on the *x*-axis with one of the vertices touching the origin (0, 0). Each slide should be completed in a different coordinate plane.
7. Choose student volunteers to demonstrate the slides using the board or overhead so that the class can check their work.

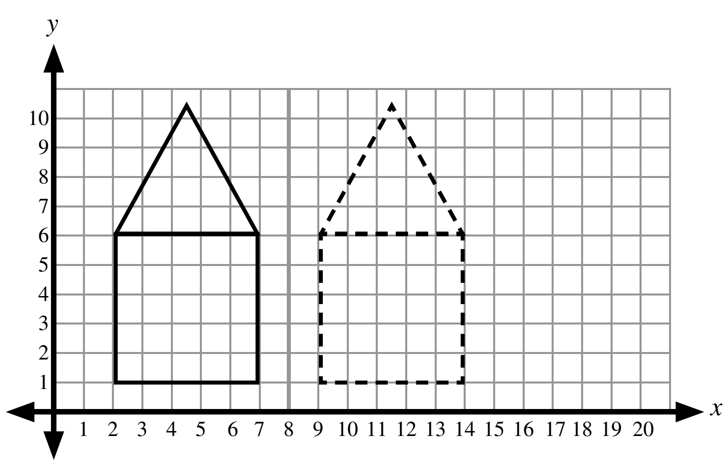
Using Pattern Blocks for Transformations in Quadrant 1 *(cont.)*

Procedure *(cont.)*

8. Distribute a second copy of *Coordinate Planes* (page 117) to students. In each coordinate plane on their sheets, have students draw a vertical line through x = 8 as shown below.
9. Explain that flipping a figure is like a mirror image. Have students place a trapezoid on the first coordinate plane like the one below. Instruct them to trace the trapezoid.
10. Ask students to flip the trapezoid along the vertical line. Then, have them trace the new position of the trapezoid.



11. Now have students place the square and triangle on the grid like the one below. Instruct them to trace the figure.
12. Ask students to flip the figure along the vertical line. Then, have students trace the flip.



13. Allow students time to discuss with partners the flips they just completed. Prompt students to discuss what *flip* means and the similarities and differences between the original images and the flipped images. Ask for student volunteers to share their observations.

Using Pattern Blocks for Transformations in Quadrant 1 *(cont.)*

Procedure *(cont.)*

14. Have students stand up. Tell them that they are going to practice rotating. Instruct students to turn to the right 90°. Complete the turn with the students. Ask them to describe the motion that they just made.
15. Have students return to their original positions. Repeat step 14, having students turn 180° and 270°. Allow the students to practice turning several times so that they can associate a degree of turning with the correct position.
16. Instruct students to sit down and then place a square anywhere on the coordinate plane. Have them place a dot on the coordinate plane at the bottom right corner of the square. Show students how to rotate the square 90°, 180°, and 270°. Trace the square at each position and discuss the images once all of the rotations have been completed.
17. Divide students into pairs. Have one of the partners choose a pattern block. In the last coordinate plane on the activity sheet, the other partner must rotate the block 90°, 180°, and 270°. Then the partners switch roles. Invite partners to share their work when the class is finished.
18. Have students complete *Transformations* (page 118) according to the differentiation suggestions listed below.

Differentiation

Above-Level Learners

Instruct students to complete questions 1, 3, and 4 independently. With remaining time, allow students to use three or more pattern blocks to create a figure. Have each student find a partner and let each student slide, flip, and rotate the partner's figure.

Below-Level Learners

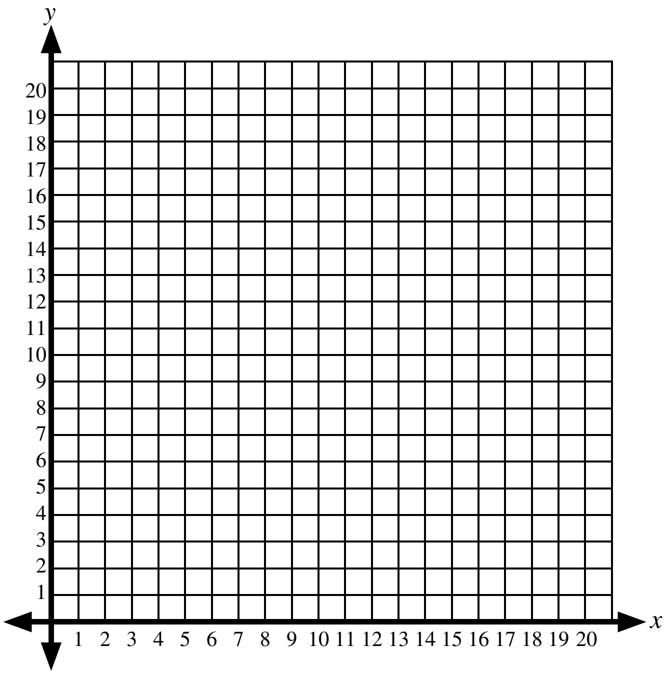
Instruct students to complete questions 1–4 in pairs. Have students practice the transformations and let their partners check their work before tracing. With remaining time, allow students to use two pattern blocks to create a figure. Have each student find a partner and let each student slide, flip, and rotate the partner's figure.

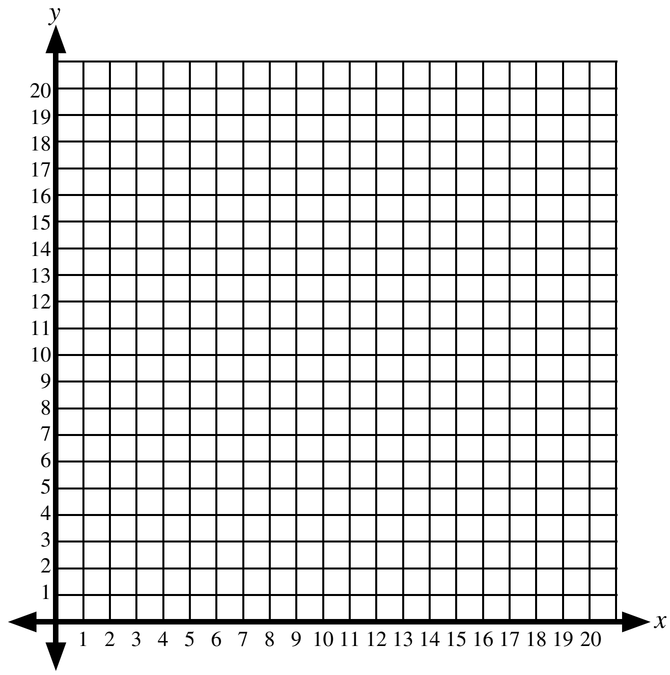
English Language Learners

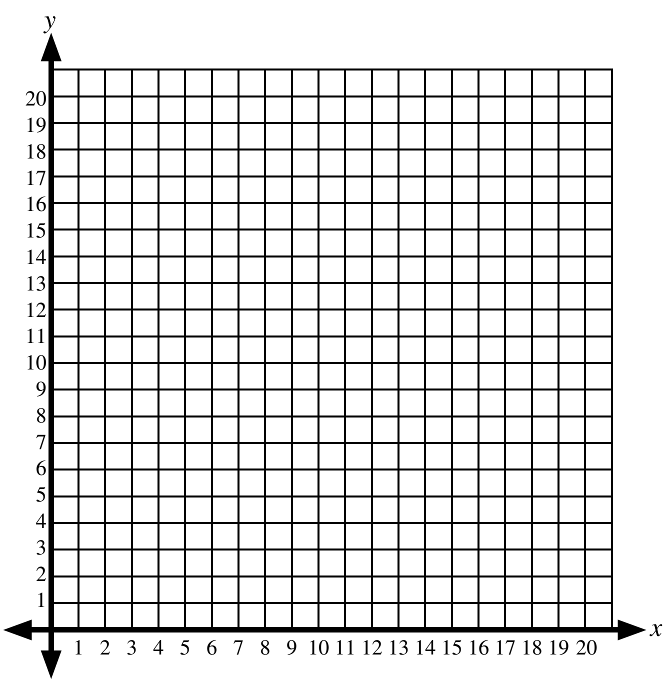
Work with these students in a small group. Discuss sliding, flipping, and rotating figures. Create a picture dictionary of the terms *slide*, *flip*, and *rotate* using only one pattern block. Allow students to use their dictionaries when completing the activity sheet in pairs.

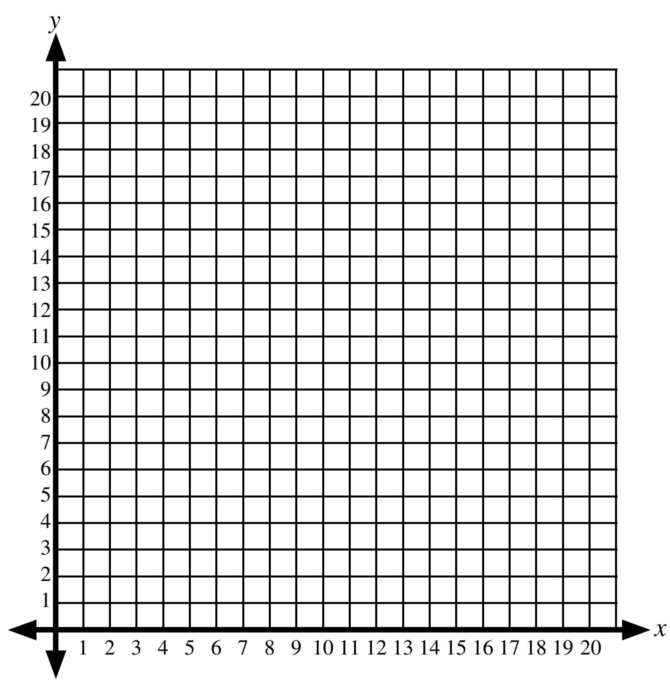
Name ________________________________

Coordinate Planes









Name ____

Transformations

Directions: Use pattern blocks to solve each problem. Create a coordinate plane for each problem using graph paper. Label the axes of each coordinate plane from 1–15. Draw the answer on the grid.

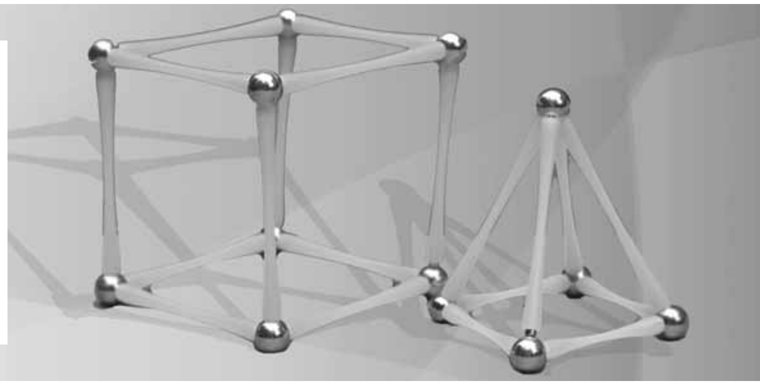
1. Place a hexagon on the *x*-axis of the coordinate plane and trace its location. Slide the hexagon 4 units to the right and 5 units up. Trace its new location.

2. Place a square on the *x*-axis of the coordinate plane and trace its location. Slide the square 3 units to the right and 2 units up. Trace its new location.

3. Draw a vertical line at x=10. Create a figure to the left of x=10 and trace it. Flip the figure across the vertical line and trace its new location.

4. Place a rhombus anywhere on the coordinate plane and trace it. Rotate the figure and trace its new location. This figure was rotated ______°.

Using Pattern Blocks for Rotational Symmetry



Standard

- understands geometric transformations of figures (e.g., rotational symmetry)

Secondary Sample Lesson

Materials

- sheets of construction paper
- paper pattern blocks
- protractors and scissors
- *Rotational Symmetry* (page 121; page121.pdf)

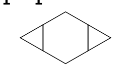
Procedure

1. Tell students that in this activity they will investigate rotational symmetry. Distribute the necessary supplies to each student.
2. Instruct each student to place a triangle pattern block in the left-hand corner of his or her *Rotational Symmetry* activity sheet (page 121). Also display the triangle on the overhead. Then have students trace around the perimeter of the triangle using a pencil.
3. Instruct students to make a dot on one corner of the equilateral triangle pattern block and make another dot in the corresponding corner on the shape on their papers. Then have them rotate the triangle until it fits exactly into the drawn shape. The center of the triangle is the center of rotation. Ask students whether the corner is in its original place. *(no)* Repeat the rotating and questioning a second time.
4. Explain to students that when a figure is rotated around a center point by fewer than 360° and the figure appears unchanged, the figure has rotational symmetry. Ask students if the triangle demonstrates rotational symmetry based on the experiment they just performed. *(yes)*
5. Allow students to conduct the same experiment with the remaining pattern blocks and record their results.
6. Once students have completed the experiment, discuss the results as a class. *(The triangle, hexagon, square, and both rhombuses have rotational symmetry. The trapezoid does not have rotational symmetry.)*

Using Pattern Blocks for Rotational Symmetry *(cont.)*

Procedure *(cont.)*

7. Show students the design below. Trace around it on the overhead and on construction paper. Cut out the design on the construction paper to use for testing rotational symmetry.



8. Mark one of the outside vertices (on the construction paper) with a dot and make an "A" on the outline in the same place. Rotate the design until it fits perfectly into the outline without going all the way around. Mark the new position of the dot with a "B" on the outline.
9. Remind students that the center of the design is the center of rotation.
10. Demonstrate the angle that is formed from the center of rotation to the beginning position marked "A," and from the center of rotation to the ending position marked "B." Identify the number of degrees of this angle by measuring it with a protractor. *(180°)* Tell students that this shape has 180° of rotational symmetry.
11. Divide the students into pairs. Tell the pairs that they must create at least two different pattern-block designs using two or more pattern blocks. One design must show 90° of rotational symmetry and the other shape must show 120° of rotational symmetry.
12. Write the following steps on the board or overhead:
    1. Create a design using pattern blocks. Trace the design on your activity sheet and on a piece of construction paper. Cut the design out on the construction paper only.
    2. Mark a dot on one of the vertices of the construction paper shape and an "A" on the corresponding vertex on the traced design.
    3. Test for rotational symmetry. If it exists, use a protractor to measure the degrees.
13. Have students complete the activity sheet according to the differentiation suggestions below. Once students have completed the experiment, allow them to share their findings with the class or in small groups.

Differentiation

Above-Level Learners

Challenge students to design a shape that has both 90° and 120° of rotational symmetry. Students should be instructed to only create one design if they choose to meet both rotational requirements.

Below-Level Learners

Have pairs complete only one design. The pairs should choose whether they want to create a design that demonstrates 90° of rotational symmetry or 120° of rotational symmetry.

English Language Learners

Write the steps of the activity on a large sheet of display paper. As a group, work with students to draw pictures that represent the necessary steps to take to complete the activity. If additional support is needed to understand the vocabulary, have students also add those pictures and words to the bottom of the display paper.

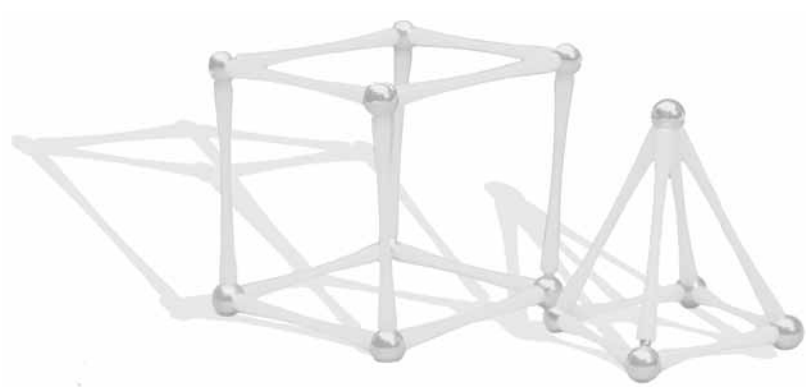
Name ____

Rotational Symmetry

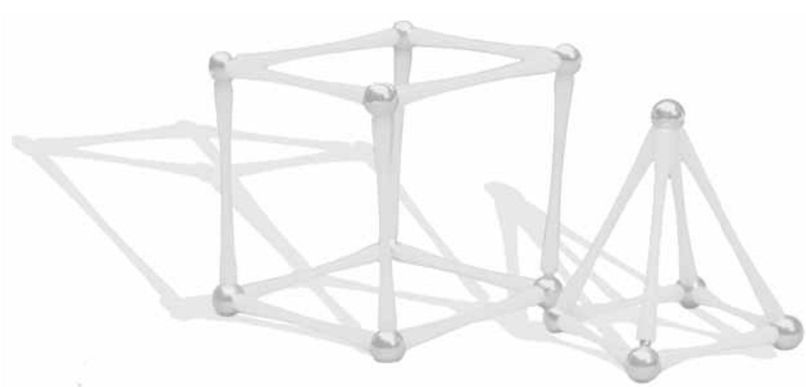
Directions: Test pattern blocks for rotational symmetry using the directions from your teacher.

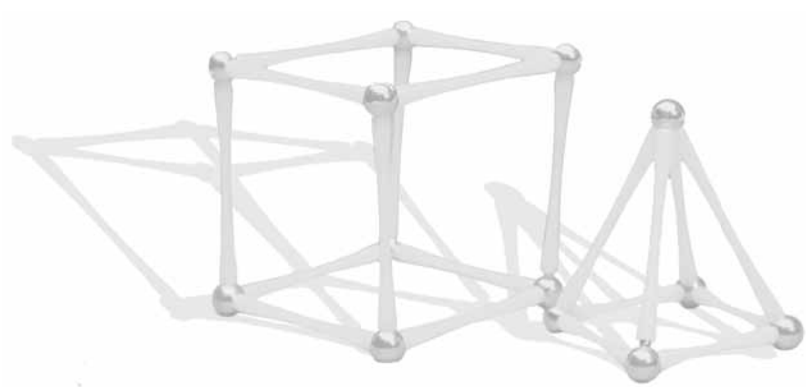
Directions: Use the space below to create at least two different pattern-block designs using two or more pattern blocks. One design must show 90° of rotational symmetry and the other shape must show 120° of rotational symmetry. If you need more space, use the back of this page.

Notes



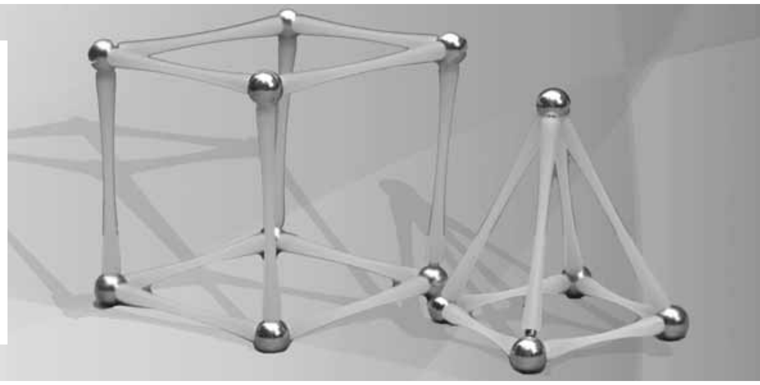
Strategies for Teaching Procedures





Teaching Procedures Overview

In order for students to make meaning in mathematics, they must understand both the how and why.



When adults think back to their experiences in mathematics instruction, they often remember the procedures being drilled into their heads. They can remember the steps in how to complete a long division problem or find the value of *x* in a given equation, but they cannot explain why they are completing the problem in that particular way. There is a clear disconnect between *completing* procedures and *understanding* procedures. But in order for students to make meaning in mathematics, they must understand both the how and why.

To adequately meet the needs of your students, it is important to build on their mathematical understanding of each topic covered throughout the year. At its core, mathematics instruction implies learning skills that must build on one another (Dean and Florian 2001). Teachers must help students build on their prior knowledge and apply it to new concepts to deepen students' mathematical understanding. This process can be accomplished by following the steps in the outline below. As this chapter progresses, the steps will be elaborated upon, and specific strategies will be provided for steps 3 and 4.

1. Use mathematical content standards, not your textbook, as a guide for instruction.
2. Find out your students' prior knowledge.
3. Build understanding for each new procedure being taught.
4. Practice each new procedure taught.
5. Assess students to gauge understanding of each new procedure.

Teaching Procedures Overview *(cont.)*

The information that follows explains how to identify concepts taught at your grade level and suggests beneficial strategies to use when teaching procedures.

1. **Use mathematics content standards, not your textbook, as a guide for instruction.**
    - The next grade level's teachers will assume that students know what is explained in the content standards. You must teach those standards to students so that next year's teachers can do the same.
    - Textbooks may not cover all of the necessary concepts for your state, or they may cover too many concepts. Using your state's content standards is the best guide for what to teach in mathematics.
    - Ask for clarity from the curriculum specialist at your school or district if a standard seems ambiguous or difficult to understand.
2. **Find out your students' prior knowledge.**
    - Consult the previous grade level's standards to see which concepts/procedures should have been covered. However, do not assume that students mastered those concepts.
    - Ask students to diagram or explain what they know about a particular concept. They could use graphic organizers such as KWL charts, lists, anticipation guides, and concept maps to generate ideas about the concept.
3. **Build understanding for each new procedure being taught.**
    - Explicitly teach and model the procedure being taught. Provide students with many examples that link to their prior knowledge.
    - Allow students time to verbalize the procedure with each other.
    - Ask questions and provide scaffolds to help students understand the procedure.
    - Link the procedure to real-life contexts.
    - Provide students with sufficient time for guided practice.
4. **Practice each new procedure taught.**
    - Independent practice should mirror guided practice so that students achieve success.
    - Students should practice in different ways to match their learning styles.
    - Allow students adequate, focused time to practice independently, and monitor their progress.
    - Harder problems/scenarios should only be introduced when students are near mastery of a procedure.
5. **Assess students to gauge understanding of each new procedure.**
    - Review the procedure before moving on to the next aspect of the mathematical concept.
    - If students do not show adequate understanding, consider reteaching.
    - For more information about assessing mathematical thinking, see pages 263–293.

Teaching Procedures Overview *(cont.)*

Strategies for Building Understanding

Strategies for scaffolding

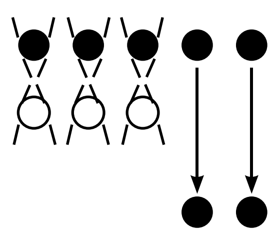
Modeling—In a mathematics classroom, modeling is the process that teachers engage in when they explicitly show a procedure or concept to the students. Often, modeling involves a visual or concrete representation of the procedure or concept and helps students directly relate to it. When modeling, it is essential to tell students exactly what you are doing and thinking in order for them to fully understand why each step is important.

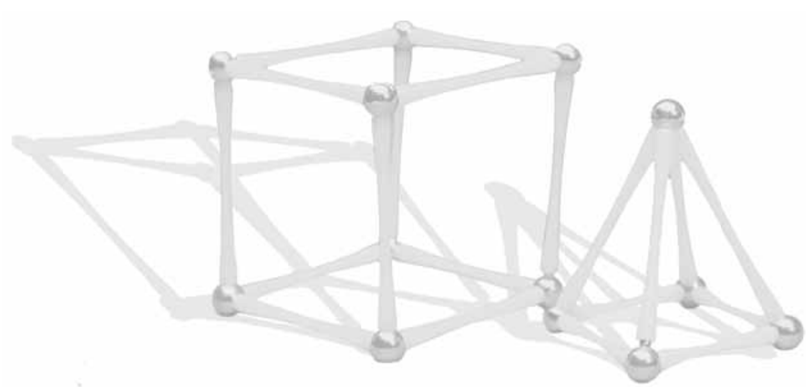
For example, when modeling to students how to sort shapes by number of sides into a Venn diagram, you would hold up a concrete representation of each shape, say the name of the shape, count the number of sides out loud, tell the students where you want to put the shape in the Venn diagram, and tell them why the shape belongs in that particular location.

Graphic Organizers—Graphic organizers are great tools to use for scaffolding procedures. If using this strategy, choose a graphic organizer that fits the type of procedure you are teaching. If a graphic organizer does not exist that adequately organizes the procedure you are teaching, create one yourself.

For example, after modeling the procedure of adding positive and negative integers using two-color chips, you can use a T-chart graphic organizer to link the concrete representation of the procedure to the written representation of the procedure. The figure below demonstrates how to use a T-chart for this scenario.

Adding Integers

Concrete Procedure Adding Integers	Written Procedure
–5 + 3 	• write the problem • put out the negative chips • put out the positive chips • find zero pairs and cross them out • count what is left • –5 + 3 = –2

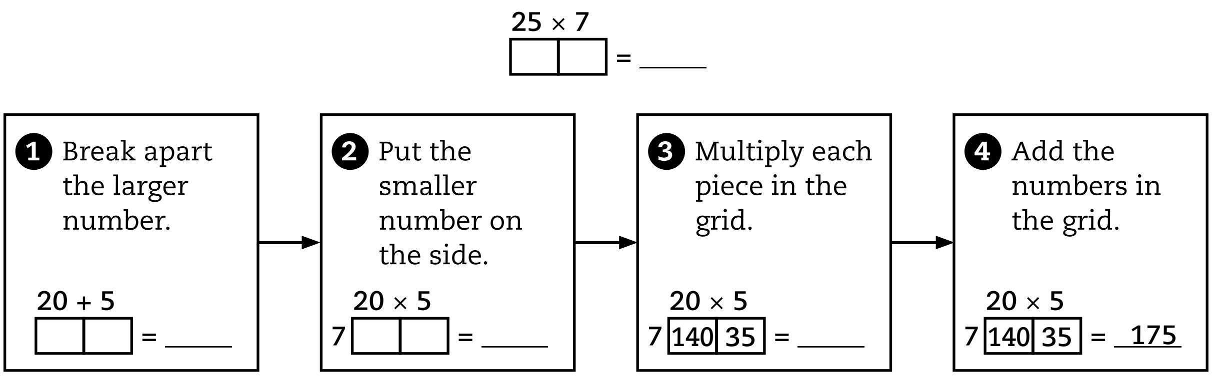


Teaching Procedures Overview *(cont.)*

Strategies for Building Understanding *(cont.)*

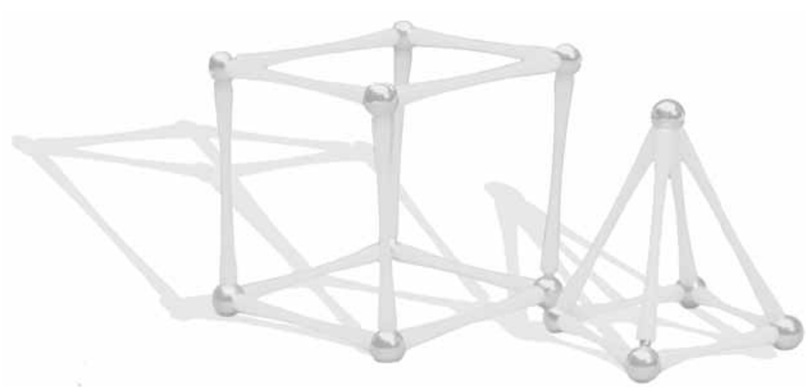
Strategies for scaffolding *(cont.)*

Graphic organizers such as flow charts can also be used to show procedures. For example, when teaching students the procedure for the grid method of multiplication, a flow chart can give information about what to do in each part of the procedure. Students can add words and images to help them remember what to do at each step. The figure below demonstrates how to use a flow chart for this scenario.



Visual/Concrete Materials—Visual or concrete materials can be used as a scaffold for student learning. When teaching a new procedure, students need to have a representation of the concept to which they can relate. The concept needs to be real for them. Using visual or concrete materials will help students understand the concept in the most basic way. Then, when students are ready to move to the pictorial stage of the concept, they have a concrete experience to relate the pictorial model to.

For example, when teaching about beginning number sense and place value, it is important to start with base ten blocks without connecting them to the numerical representation. The procedure for building two-digit numbers becomes finding the correct number of tens rods first, and finding the correct number of units second. Or, after teaching students the procedure for addition by counting on, provide students with a number line. Show them how to find the larger of the two numbers on the number line, and then count on to find the solution to the addition problem. This visual clearly demonstrates the procedure.



Teaching Procedures Overview *(cont.)*

Strategies for Building Understanding *(cont.)*

Strategies for scaffolding *(cont.)*

Acronyms—Acronyms are words that are formed by the first letters of each of the parts of a compound term or phrase. These mnemonic devices can help students remember the steps or components of the procedures being taught. There are classic acronymns, such as **P**arentheses, **E**xponents, **M**ultiplication and **D**ivision, **A**ddition, and **S**ubtraction (**PEMDAS**), which are taught in many classrooms around the world. However, acronyms can be made up by you and your students to help them remember specific procedures for the concepts taught in your class. Acronyms can even be created by individual students to make personalized mnemonics that are meaningful to themselves.

For example, to help students remember the procedure for the standard algorithm of long division, ask the question: "**D**oes **M**om **S**erve **C**heese**B**urgers?" This will help them remember the steps of **d**ividing the divisor by the dividend, **m**ultiplying the quotient with the divisor, **s**ubtracting, **c**hecking their work for computation errors, and **b**ringing down the next number in the dividend.

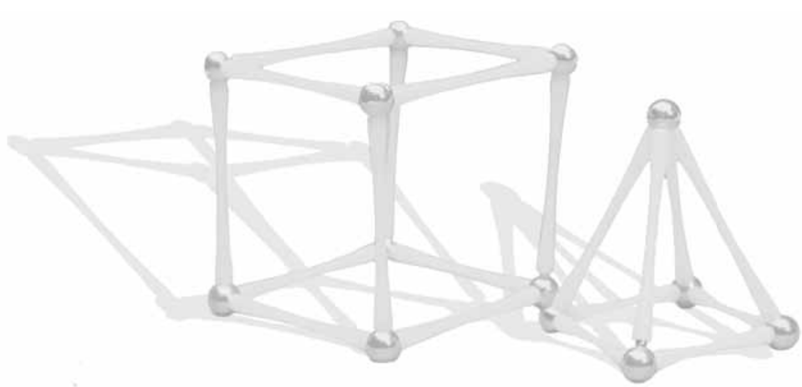
The acronym **FATT A SAT** can help students remember the procedure for adding three single-digit numbers: **F**irst **A**dd **T**wo **T**hen **A**dd **S**um **A**nd **T**hird.

Chunking—Chunking mathematics is a strategy to use when the procedure is lengthy and difficult. With this strategy, you teach students one piece (chunk) of the procedure at a time. Students must practice and understand the first chunk before moving on to the next.

For example, in order to factor an equation, like terms must be combined. The first part of the procedure is to combine like terms. However, this is a difficult skill for many students, and it must be mastered before the procedure of factoring can be taught.

Think Alouds—This is a very effective strategy when teaching procedures. A think aloud is just what it says: thinking out loud. Using this strategy provides students with an opportunity to look inside the window of your mind and actually "hear" your thoughts as you go through the procedure.

When doing a think aloud for your students, it is important to tell them why you are doing each step of the procedure and how you mentally check yourself and your work as you continue through the procedure.



Teaching Procedures Overview *(cont.)*

Strategies for Building Understanding *(cont.)*

Strategies for scaffolding *(cont.)*

Multiple Exposures—When presenting a procedure to students, it is important to remember that they may not understand it the first time. By exposing the students to the procedure multiple times, you are reinforcing the concepts being taught. This can also be an opportunity to present the information in different styles, thereby targeting students' various learning modalities.

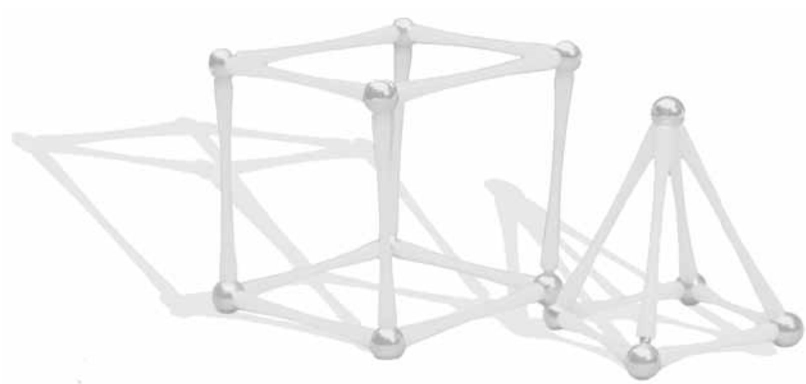
For example, when presenting the idea of combining like terms in algebra, the students' first exposure to the procedure can be through modeling, the second exposure can use concrete materials, and the third exposure can be verbal discussions in small groups.

Note Taking—Note taking is a great strategy to use for scaffolding the procedure. When used properly, students will be able to study their notes and use them as a bridge for learning the procedures. It is important to teach your students how to effectively take notes.

The first step in effective note taking is recording. It is important to write the notes that you expect the students to copy on the board or overhead as you discuss or model the procedure. This will help them understand the most important concepts.

The second step in effective note taking is organizing. Once the notes are complete, instruct students to review their notes and add extra comments in the margins or underline key words and phrases that are important to them. This will help make their notes meaningful and will help them see parts of the procedure in which they still have confusion.

The final step in effective note taking is studying. Students need to be taught how to study their notes in order to best learn the procedure being described. Remind them that the comments and labels they created in the margins will provide them with an overview of their notes; specific information about the procedure is found within the notes.



Teaching Procedures Overview *(cont.)*

Strategies for Building Understanding *(cont.)*

Strategies for linking the procedure to real-life contexts

Visual Aids—Using visual aids such as posters, photos, and diagrams can help students link the mathematical procedures and their importance to real life. When students understand why mathematics is important and how it affects them on a daily basis, their motivation to learn mathematics increases.

For example, bring in a wallpaper border sample from a local wallpaper store. Show students how the pattern repeats and discuss how the designer had to follow the same procedures the students do when creating the repeating pattern.

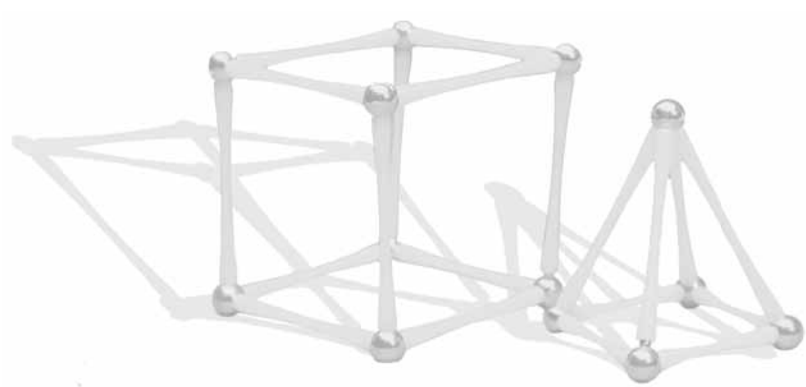
When studying the procedure for determining central tendencies, show students a graph from the newspaper. They could discuss the content of the graph and how knowing how to calculate central tendencies for the data in the graph deepens their understanding of the information.

Children's Literature—Creating the connection between mathematics and real life for students is integral to their motivation. Often, fiction and nonfiction children's literature present many contexts in which mathematics is used in real life. However, the explicit connection does not have to be made within the story line of the book in order for you to use it as a springboard for teaching the procedure.

For example, a book about traveling can be used to discuss the procedure of using the distance formula as it relates to the methods of transportation presented in the book. A book about food can be used to discuss the procedure of multiplying fractions in a recipe using the food items presented in the book. A book about the zoo can be used to discuss the procedure of addition or subtraction in the context of animals coming to or leaving the various exhibits discussed in the book.

Open-Ended Questions—Once students understand the basic steps to the procedure being taught, it is effective to further their thinking by asking open-ended questions that link the procedure to real life.

For example, after teaching the procedure for calculating area, ask students how they think area might be important for farmers. After teaching the procedure for extending visual patterns, ask students why a clothing designer needs to know about patterns. After teaching a procedure for subtracting decimals, ask students how they could give change from a $100 bill for an item that cost $56.73.



Teaching Procedures Overview *(cont.)*

Strategies for Practicing Procedures

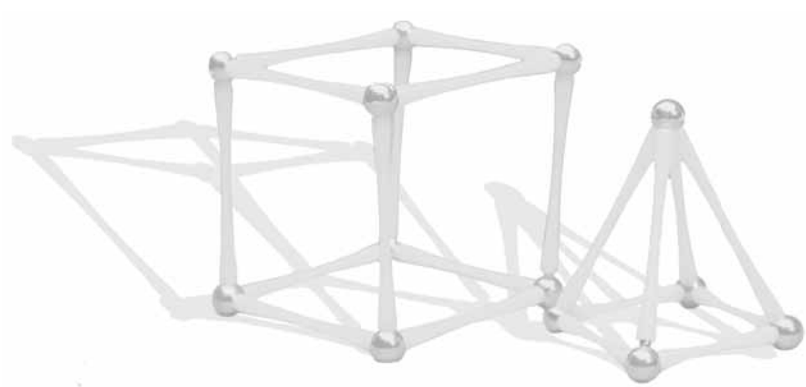
Strategies for implementing focused practice of the procedure

Grouping Students—Grouping students can be a very powerful strategy to use when practicing procedures. There are many ways to group students. It is important to consider the desired outcome of the grouping experience before choosing the type of grouping to use with the students. Depending on the type of activity being completed within the group, it can also be helpful to assign specific roles such as recorder, moderator, encourager, or timekeeper to each student so that the activity can be kept on track. However, it is also important to remind students that even though they have specific roles within the group, they are not free from contributing to the work involved in the activities.

Often, students are grouped in pairs or small groups of 3–4 students. These types of grouping strategies work well when intensely practicing procedures. However, the ability readiness levels of the students within the groups should also be considered. Depending on the activity, students can be grouped homogeneously, where all students in the group have the same ability readiness level for the procedure being taught, or heterogeneously, where each student in the group has a different ability readiness level for the procedure being taught.

When grouping students, it is important to remember that different strategies should be used as often as possible so that students see how flexible and positive grouping can be within the classroom. Group activities lose their positive effect when students are constantly put into the same groups for every practiced procedure. Students already know with which concepts they struggle, and continuously grouping them in ways that point out their struggles does not increase their motivation to work harder. It is also important to note that grouping is not a method for the higher-level students to "tutor" and help the lower-level students 100 percent of the time. This is frustrating for both students and further enhances their frustrations with learning.

Tiering Assignments—This strategy can be used to specifically target each student's level of understanding of the procedure being taught. Tiered assignments are parallel tasks that have varied levels of complexity and depth, as well as varied degrees of scaffolding, support, and direction for the student. In a tiered assignment, all students are working on the same goal or objective, but the assignment is tiered based on readiness and performance. In other words, all of the students end at the same place, but the paths they take are all different. Tiered assignments are motivating to students because they are successful at their own levels and are provided with challenges that are suited to their abilities.



Teaching Procedures Overview *(cont.)*

Strategies for Practicing Procedures *(cont.)*

Strategies for implementing focused practice of the procedure *(cont.)*

Tiering Assignments *(cont.)*

For example, when students are working on the procedure of classifying two-dimensional shapes, students can be asked to complete different amounts of classifications as well as different analysis questions about the classification "rules" they used.

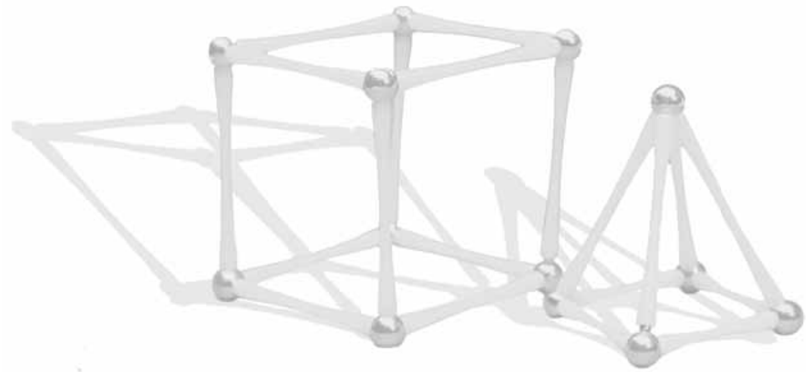
"Group Thinking"—Group Thinking is a strategy in which students must practice procedures as a group. The success of the activity depends on the whole group's understanding of the concept. In other words, all students must understand the procedure accurately in order for the group to be able to successfully complete the activity.

One example of Group Thinking is an exercise called "Pass the Pen." In this exercise, each group of 3–4 students is given a pen and a sheet of paper. Students are then asked to sit in a circle around the paper, and record the steps of a procedure. To do that, one student starts with the pen and paper and records the first step of the procedure. The rest of the group may not tell the writer what to write or give any nonverbal cues. Then, the first person passes the pen and paper. The next person may change any incorrect information in the previous step or write the next step. The pen and paper continue to be passed around the circle until all of the steps of the procedure are recorded.

Another example of Group Thinking is an exercise called "Team Response." In this exercise, the class is divided into teams. Each team sits in a different part of the room. One at a time, groups are asked questions. Each person in the team must record an answer on his or her piece of paper. The team then decides on a correct answer to the question and makes sure that each team member understands the answer completely. As a group, the team then provides an answer to the question. Then, the teacher randomly asks one student from the team to explain the answer. The team only receives a point if the chosen student adequately provides an explanation of the answer.

Summarizing—There are many ways in which students can summarize procedures as a method of practicing procedure. For verbal/linguistic students, discussing the procedure in groups or with pairs would be effective practice. Summarizing the procedure can also be written in journals as a free or focused response.

For example, after learning the procedure for multiplying fractions, a free-response journal topic could be as follows: *In your own words, explain how to multiply fractions.* A focused journal topic on the same procedure could be as follows: *Show an example of multiplying fractions and create a diagram or list explaining the procedure.*



Teaching Procedures Overview *(cont.)*

Strategies for Practicing Procedures *(cont.)*

Strategies for implementing focused practice of the procedure *(cont.)*

Graphic Organizers—In addition to scaffolding, graphic organizers can also be used to practice procedures. There are many ways in which graphic organizers can be used for focused practice of procedures. The same graphic organizer that is used to scaffold the procedure can also be completed during practice as reinforcement to the procedure. Or, a new graphic organizer can be presented to the students in which they have to apply their knowledge of the taught procedure to complete it. The most important concept to remember is that the graphic organizer used needs to fit the type of procedure adequately.

For example, students could create a time order map to sequence the procedure of multiplying binomials using F.O.I.L. The figure below demonstrates how to use a time order map for this scenario.

F.O.I.L. $(4x + 8)(2x - 3)$

Multiply the **FIRST** two terms: $4x(2x) = 8x^2$

↓

Multiply the **OUTER** two terms: $4x(-3) = -12x$

↓

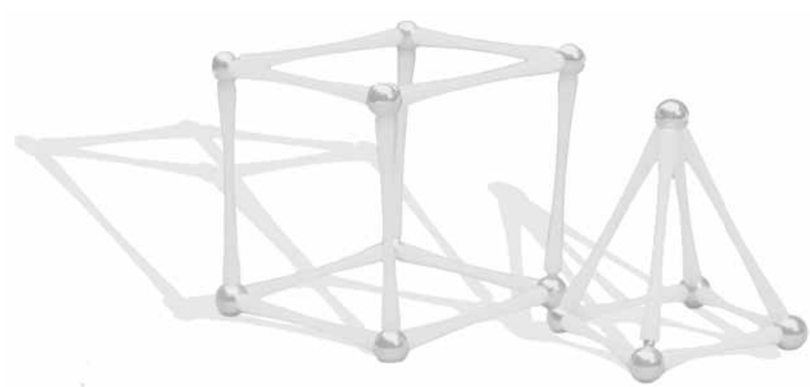
Multiply the **INNER** two terms: $8(2x) = 16x$

↓

Multiply the **LAST** two terms: $8(-3) = -24$

↓

Combine like terms: $8x^2 - 12x + 16x - 24 = 8x^2 + 4x - 24$



Teaching Procedures Overview *(cont.)*

Strategies for Practicing Procedures *(cont.)*

Strategies for implementing focused practice of the procedure *(cont.)*

Graphic Organizers *(cont.)*

Students could also use a time order map for practicing the procedure for creating fact families.

Making Fact Families

Start with a number sentence: 5 + 2 = 7

↓

Show the same number sentence a different way: 2 + 5 = 7

↓

Make the inverse of the first number sentence: 7 – 2 = 5

↓

Make the inverse of the second number sentence: 7 – 5 = 2

↓

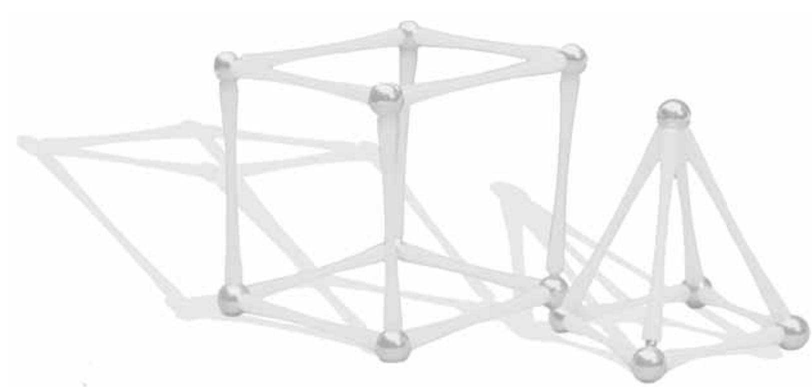
Put the number sentences together to show the fact family:

5 + 2 = 7

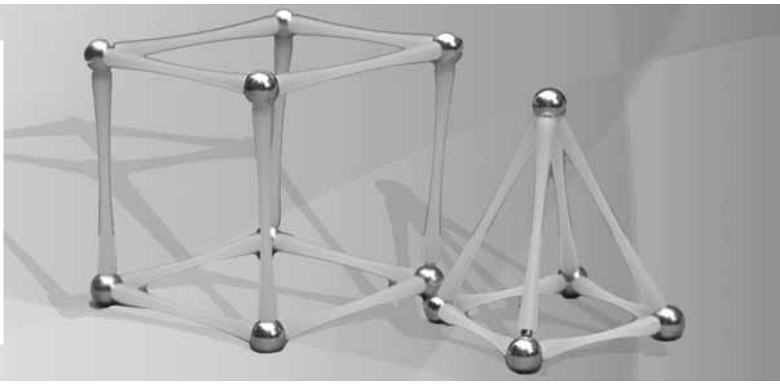
2 + 5 = 7

7 – 2 = 5

7 – 5 = 2



Alternative Algorithm for Addition



Standard

- uses basic and advanced procedures while performing the processes of computation

Grades K–2 Lesson

Materials

- interlocking cubes
- *Addition Practice* (page 138; page138.pdf)

Strategies Used in This Lesson

- Modeling
- Visual Aids
- Concrete Materials
- Graphic Organizer

Procedure

1. Review place value with the students by asking them how many tens and ones are in different numbers.
2. For example, write 27 on the board or overhead. Ask students how many tens are in this number (2) and how many ones are in this number. (7) Then ask them what 2 tens represents (20) and what 7 ones represents. (7) Repeat this for other numbers.
3. Next try this activity without writing the number on the board or overhead. Encourage students to visualize the number in their heads and break it apart.
4. Divide the students into pairs and have them practice this together, breaking apart numbers into tens and ones. Instruct students to write down each number they practice, the number of tens and ones, and what they each represent.
5. Explain to students that they will be learning how to use the partitioning method of addition. In this method, they will break apart the numbers into tens and ones and then add.
6. Write the addition problem 17 + 22 on the board or overhead. Show students how to write each number in expanded form using a T-chart. (shown below)

	Tens	Ones
17 =	10	7
22 =	20	2

7. Have students build each number with interlocking cubes and lay them out on their desks just like they see on the T-chart. Build each number as a model as well. (The number 17 should be represented by 1 stick of 10 cubes and 7 individual cubes, and 22 should be represented by 2 sticks of 10 cubes and 2 individual cubes.)

Alternative Algorithm for Addition *(cont.)*

Procedure *(cont.)*

8. Ask the students how they might add the numbers. Discuss the various ideas students come up with.
9. Using your cube model of the numbers, demonstrate how they can first combine the tens. Have students also complete this on their desks. Ask them how many sticks of ten they have *(3)* and how much that represents. *(30)* Do the same with the ones.
10. Demonstrate this on the board or overhead.

	Tens	Ones		
17 =	10	7		
22 =	20	2		
	30 +	9	=	39

11. Reflect on the procedures of this algorithm as a class. Create a class flow chart of the procedures. Allow students to come to the chart and write the steps and examples. The steps in the class flow chart should be similar to the list below. Be sure to include visual examples of each step in the chart as well.
    - First, write the problem.
    - Second, write each number in the problem in expanded form.
    - Third, add the tens.
    - Fourth, add the ones.
    - Last, add the tens and the ones.
12. Repeat the procedures with new addition problems. Refer to the flow chart as you work the problems together. Depending on the level of your students, you may also want to show them problems where they would have to move a group of tens over from the ones place.
13. Distribute copies of *Addition Practice* (page 138) to the students and have them complete it according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete problems 3–5 independently. Then in pairs, have them create a procedure for completing the following problem: 63 + 45.

Below-Level Learners

Allow students to complete this activity in groups of two or three. Encourage them to use the interlocking cubes as a way to check their work.

English Language Learners

In a small group, review the concepts of place value and addition and the vocabulary associated with them. Also review how the cubes can be used to demonstrate those vocabulary words. Allow students to complete the activity in pairs.

Name ______________________________

Addition Practice

Directions: Solve the following addition problems by completing the tables and then adding the tens and ones.

1. 15 + 42 = ☐

TENS	ONES

2. 26 + 53 = ☐

TENS	ONES

3. 45 + 31 = ☐

TENS	ONES

4. 56 + 38 = ☐

TENS	ONES

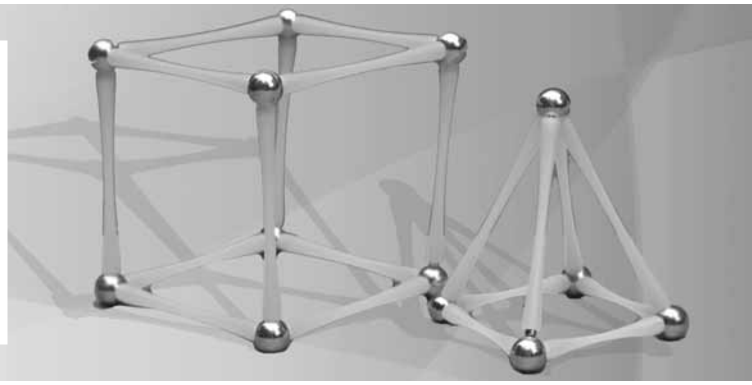
5. 29 + 47 = ☐

TENS	ONES

Alternative Algorithm for Subtraction

Standard

- uses basic and advanced procedures while performing the processes of computation



Grades K–2 Lesson

Materials

- hundred charts
- counters
- *Practicing Subtraction* (page 141; page141.pdf)

Strategies Used in This Lesson

- Modeling
- Visuals
- Concrete Materials
- Summarizing

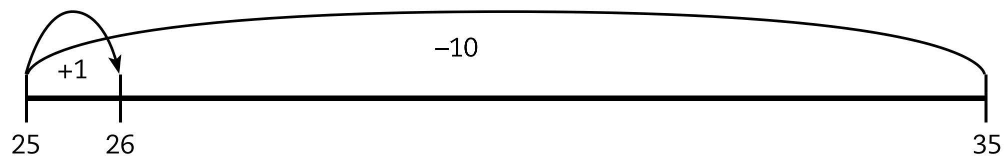
Procedure

1. Tell students that they will be learning how to use the compensation method for subtraction. In this method, students subtract too much and then add back on.
2. Distribute a hundred chart and a counter to each student and display a large chart on the board or overhead.
3. Place the counter on the number 35 and have students do the same. Then move the counter down one row to 45. Let students watch you first and then mirror what you do.
4. Ask students what they did by moving the counter down one row. *(added 10 to the original number)*
5. Place the counter back on the number 35 and have students do the same. Then move the count up one row to 25. Let students watch you first and then mirror what you do.
6. Ask students what they did by moving the counter up one row. *(subtracted 10 from the original number)*
7. Place the counter back on the number 35 and have students do the same. Model how to subtract 9 by first subtracting *(moving up one square)* and then adding 1. *(move to the right one square)* Have students watch you first and then mirror what you do.
8. Ask students what was done to subtract 9 and have them discuss their ideas with a partner. Discuss their ideas as a class.
9. Help students create an equation that mirrors what they did with their counters. *(35 – 9 = 26)* Write this equation on the board or overhead.

Alternative Algorithm for Subtraction *(cont.)*

Procedure *(cont.)*

10. Demonstrate how this can also be shown on a number line.



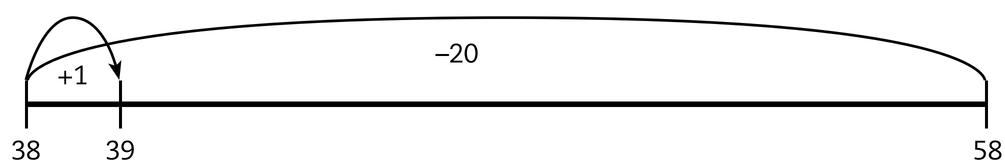
11. Complete this procedure several times. Then in pairs, have students write a summary of the procedure and allow some pairs to share out loud with the class.

12. Now have students place their counters on the number 58. This time, they will subtract 19. Tell students that 20 is one more than 19, so they will first subtract 20. Model to students how to move up two rows on the hundred chart to 38, and then have them do the same.

13. To add back the extra amount you subtracted, move the counter to the right one square. Model this for the students and then have them do the same.

14. Help students create an equation that mirrors what they did with their counters. *(58 – 19 = 39)* Write this equation on the board or overhead.

15. Demonstrate how this can also be shown on a number line.



16. Complete this procedure several times. Then in pairs, have students write a summary of the procedure and allow some pairs to share out loud with the class.

17. Distribute copies of *Practicing Subtraction* (page 141) and have students complete it according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 5–8 individually. In pairs, have them work on the challenge section of the activity sheet.

Below-Level Learners

Instruct students to complete questions 1–5. Allow them to use their hundred charts and counters to solve the problem first before completing it on the number line.

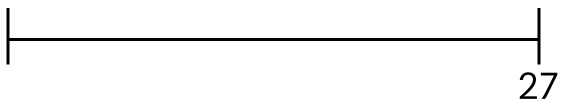
English Language Learners

Work with the students in a small group to complete questions 1, 2, 5, and 6. Review the vocabulary and procedures as you complete each problem together. Have students complete the rest of the activity sheet in groups of two or three.

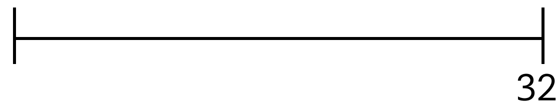
Name ____________________________________

Practicing Subtraction

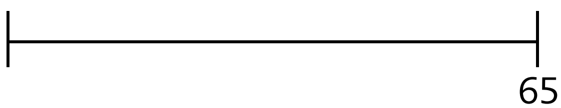
Directions: Use the number lines below to help you subtract using the compensation method.

1. 

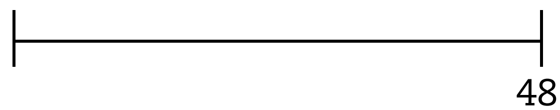
27 – 9 = ☐

2. 

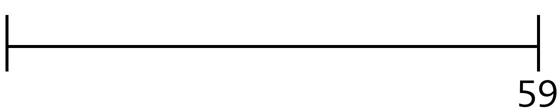
32 – 9 = ☐

3. 

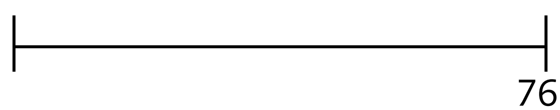
65 – 9 = ☐

4. 

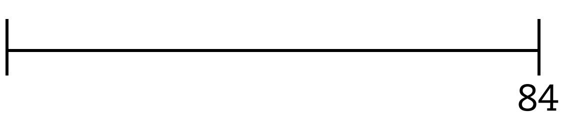
48 – 9 = ☐

5. 

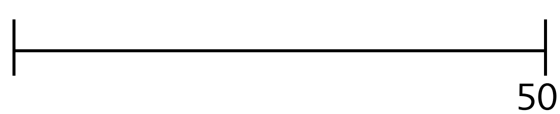
59 – 19 = ☐

6. 

76 – 19 = ☐

7. 

84 – 19 = ☐

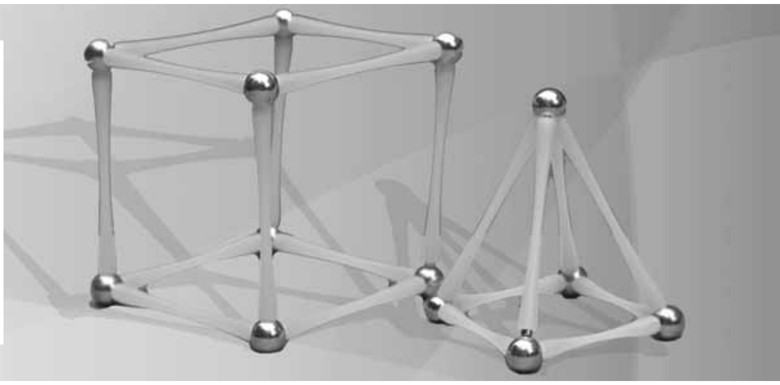
8. 

50 – 19 = ☐

Challenge

How could you subtract 8 from any number? How could you subtract 18 from any number?

Alternative Algorithm for Multiplication



Standard

- uses basic and advanced procedures while performing the processes of computation

Grades 3–5 Lesson

Materials

- paper
- *Multiplication Grids* (page 144; page144.pdf)

Strategies Used in This Lesson

- Modeling
- Visual Aids
- Acronyms

Procedure

1. Tell students that they will be learning how to use the grid method for multiplication. In this method, students break one of the numbers into tens and ones and then multiply.
2. Distribute a piece of paper to each student. Instruct students to fold their papers into four equal sections.
3. Review multiplication by having students use mental math to multiply single-digit numbers by 10.
4. Write the problem 24 × 7 on the board or overhead. Below it, draw a grid like the one shown below. Have students set up the problem in the top left box on their papers.

24 × 7

5. Have students identify the single-digit number and the two-digit number. Tell them that in the grid method of multiplication, the single-digit number belongs on the left side of the grid.
6. Next, have students break the two-digit number into tens and ones. *(20 and 4)* Tell students that if you add the two pieces together, the sum is still 24; 20 + 4 is simply a different way to represent the number 24. Place the addition sentence above the grid as shown below. Have students do the same on their papers.

24 × 7

	20 +	4
7		

Alternative Algorithm for Multiplication *(cont.)*

Procedure *(cont.)*

7. Tell students that the next step is to multiply 7 by each part of 24 and place the quotient in the appropriate boxes. Have students complete the grid with you as you model multiplying 7 by each piece of the equation.

	24 ×	7
	20 +	4
7	140	28

8. Then have students add the two numbers in the grid to find the product of 7 and 24. The completed grid should look like this:

	24 ×	7	
	20 +	4	
7	140	28	= 168

9. Now write the acronym **B**efore **L**unch **M**om **F**ed **M**e **S**ome **A**wful **B**reakfast on the board or overhead. Tell students that this acronym will help them remember the procedure for using the grid method of multiplication. The **B** stands for **b**reak apart the numbers. The **L** stands for **l**ay out the problem using the grid. Then you **M**ultiply the number from the **F**irst box, and then **M**ultiply the numbers for the **S**econd box. Finally, you **A**dd **B**oth numbers in the grid together. Have students write this acronym across the top of their papers.
10. Repeat this procedure three more times, using the acronym as a way to help the students remember the procedure. Have students complete each new problem in a separate square on their papers. Use the following problems: 29 × 3; 34 × 5; 38 × 2.
11. Distribute copies of *Multiplication Grids* (page 144) and have students complete it according to the differentiation strategies below.

Differentiation

Above-Level Learners

Have students complete questions 7–13 individually. Then have students correct their work with partners. Instruct students to also complete the challenge activity with their partners.

Below-Level Learners

Have students complete questions 1–6 and 11 in pairs. Remind students to write the acronym on the tops of their papers to help them remember the procedures. With remaining time, allow students to choose two more problems from the paper to practice. Encourage them to choose problems they think may be a little difficult for themselves.

English Language Learners

In a small group, review the acronym and create a flow map detailing the procedures that correlate with each part of the acronym and include the grid diagram at each step. Then instruct students to complete questions 1–8 and 11 in groups of two or three.

Name ____________________________________

Multiplication Grids

Directions: Complete each problem using the grid method of multiplication.

1. 21×6 | | | =

2. 25×4 | | | =

3. 31×2 | | | =

4. 42×3 | | | =

5. 37×5 | | | =

6. 48×3 | | | =

7. 52×6 | | | =

8. 68×2 | | | =

9. 57×3 | | | =

10. 85×5 | | | =

11. Louis is buying boxes of pens to sell at the student store at school. There are 24 pens in each box. Louis buys 5 boxes. How many pens did Louis buy?

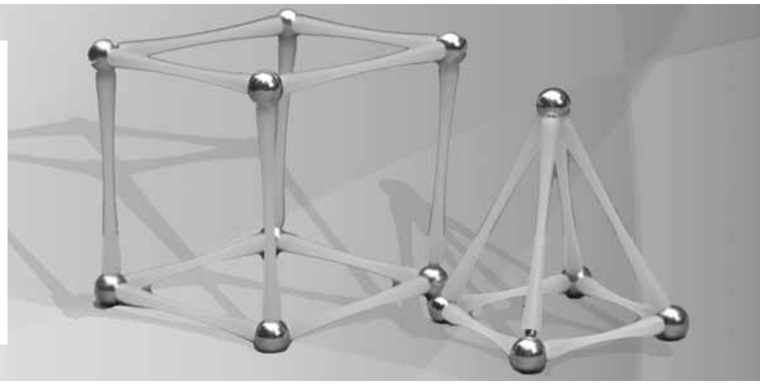
12. Kiko is helping her mom organize the shoes in their closets. They have 4 shoe organizers that each hold 32 pairs of shoes. How many shoes will fit in the organizers altogether?

13. Nori went to the pet store with his parents to buy a new fish. There are 6 fish tanks at the pet store. Each tank holds 48 fish. How many fish does Nori have to choose from?

Challenge

Use a Venn diagram to compare and contrast the grid method of multiplication with another method of multiplication that you know.

Alternative Algorithm for Division



Standard

- uses basic and advanced procedures while performing the processes of computation

Grades 3–5 Lesson

Materials

- paper
- interlocking cubes (300 per group of four)
- *Division Using Multiples* (page 147; page147.pdf)

Strategies Used in This Lesson

- Modeling
- Concrete Materials
- Group Think

Procedure

1. Tell students that they will be learning how to use the multiples of the divisor method for division. In this method, students subtract multiples of the divisor from the dividend.
2. Review the terms *multiple*, *dividend*, and *divisor*.
3. Divide the students into groups of four. Provide each group with 300 cubes. It will help if these are distributed already in sticks of 10. (Each group gets 30 sticks of 10 cubes.)
4. Write the problem 250 ÷ 9 on the board or overhead. Have students place 250 cubes in front of them and put the other 50 away.
5. Tell students that dividing 250 by 9 is like taking away groups of 9 over and over again. Because 250 is a large number, we can subtract large sets of 9 at one time. Tell students that they will first take away 10 groups of 9. Have groups do this with their cubes and put them in one pile on the side of their work space.
6. Show students what they have done by writing the first three lines of the example problem on page 146 on the board or overhead.
7. Ask students if another set of 90 can be taken away from 160. *(Yes)* Have students take away another set of 90 cubes and put it with the other set. Then write the next two lines from the example on page 146.
8. Have students count their piles to see if they have 70 cubes left over like the example on the board or overhead shows.
9. Ask students how many groups of 9 could be taken from 70. (7) Have students take that many sets from their pile. Students will have to break apart a stick of 10 to be able to complete this step. Then write the next two lines from the example.

Alternative Algorithm for Division *(cont.)*

Procedures *(cont.)*

10. Ask students to count how many cubes they have left in front of them to verify that the equation on the board or overhead is correct.

11. Now have students add the number of 9s that were taken from 250 to find the answer. *(10 + 10 + 7 = 27)* Have students verify this by redistributing the interlocking cubes to make sticks of 9. Students should have 27 sticks. There were also 7 cubes leftover, so the answer becomes 27 r7. Students should also be able to verify this by seeing the remaining 7 loose cubes. Record this in the equation on the board or overhead as shown below.

250 ÷ 9

250	
– 90	(9 x (10))
160	
– 90	(9 x (10))
70	
–63	(9 x (8))
7	= 27 (r7)

→ 10 + 10 + 7 = 27 with 7 left over

12. Write the problem 275 ÷ 8 on the board or overhead. Distribute a piece of paper to each group. Have each group complete a "Pass the Pen" activity to complete this problem. (See page 133 for how to complete this activity.) Students may use the cubes to complete the problem, but they may not talk while doing this. When finished, allow one group to share its work and discuss the problem with the class.

13. Distribute copies of *Division Using Multiples* (page 147) to students and have them complete the activity according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 6–12 independently. With remaining time, allow students to switch papers to solve the problem in question 12.

Below-Level Learners

Have students complete questions 1–6 and 12 in groups of two or three, and allow them to use the cubes. With remaining time, have groups switch papers to solve the problem in question 12.

English Language Learners

Review the necessary vocabulary and the procedures as a small group. Have students create a diagram of the procedures with any notes that are helpful to them. Then have students complete questions 1–7 and 12 in groups of three or four. With remaining time, have groups switch papers to solve the problem in question 12.

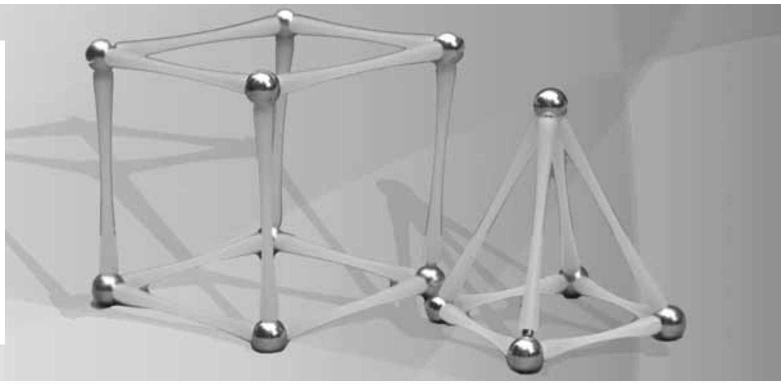
Name ____

Division Using Multiples

Directions: Use the multiples of the divisor method of division to complete the problems below. Show your work. Create your own problem in question 12. Do not solve it.

1. 137 ÷ 6	**2.** 165 ÷ 9	**3.** 154 ÷ 8
4. 239 ÷ 5	**5.** 252 ÷ 3	**6.** 189 ÷ 9
7. 261 ÷ 8	**8.** 297 ÷ 7	**9.** 224 ÷ 4
10. 342 ÷ 7	**11.** 425 ÷ 9	**12.** ____________

Adding Fractions



Standard

- adds simple fractions

Grades 3–5 Lesson

Materials

- scissors and crayons or colored pencils
- blue and yellow copy paper
- *Eighths* printed on blue paper (Teacher Resource CD *eighths.pdf*)
- *Twelfths* printed on yellow paper (Teacher Resource CD *twelfths.pdf*)
- *Working with Fractions* (page 150; page150.pdf)

Strategies Used in This Lesson

- Modeling
- Visual/ Concrete Materials
- Multiple Exposures
- Summarizing

Procedure

1. Divide the students into pairs. Give each pair two pages of *Eighths* (eighths.pdf) and two pages of *Twelfths* (twelfths.pdf).
2. Discuss the figures on the blue and yellow papers. Tell students that the circles represent pizzas and the strips represent snakes. Have students look at both papers to see the difference between them. Have each pair cut out one of each pizza and snake into its eight or twelve pieces. The other two identical papers will be used as a working mat.
3. Ask the pairs of students to look only at the blue paper and figures. Ask students what fraction of the pizza is 3 pieces. Have the students discover the answer by placing pieces of pizza on their whole pizza. Have one pair model the answer for the class. Then write $\frac{3}{8}$ pizza on the board and review the meaning of numerator and denominator. Emphasize that the denominator is the number of equal pieces that make up one whole.
4. Ask students what fraction of the pizza is 2 pieces. Have the students discover the answer by placing pieces of pizza on their whole pizza. Have one pair model the answer for the class. Then write $\frac{2}{8}$ pizza on the board.
5. In pairs, have one student pretend to eat 3 pieces and one pretend to eat 2 pieces. Ask students what fraction of the pizza the two of them ate combined. Have one pair model their process for the class. Then write $\frac{3}{8}$ pizza + $\frac{2}{8}$ pizza = $\frac{5}{8}$ pizza on the board to connect the model to the equation.

Adding Fractions *(cont.)*

Procedure *(cont.)*

6. Ask students, "When you compare the sum with the addends, what part of the fractions changed in the sum and what part stayed the same? Why?" After this is discussed, ask the students to model $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$ with their blue snake. Emphasize again the meaning of *numerator* and the meaning of *denominator*.
7. Then ask students why it is incorrect to write $\frac{3}{8} + \frac{2}{8} = \frac{5}{16}$. After discussion, point out that they have explained why the two denominators must be the same when adding fractions and that the denominator in the answer must also be the same due to of the definition of denominator.
8. Ask students if they can show $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$ with the yellow pizza. Get the students to be able to explain that the yellow pizza is not cut to model $\frac{3}{8} + \frac{2}{8}$, because in $\frac{3}{8} + \frac{2}{8}$, it takes 8 equal pieces to make a whole unit, but the yellow pizza is cut so that it takes 12 equal pieces to make a whole pizza. Have them solve the problem $\frac{3}{12} + \frac{2}{12}$ with both the yellow pizza and the yellow snake and have a pair model its procedures to the class. Continue to emphasize why the denominator does not change.
9. Ask the students to look at their model of $\frac{5}{8}$ and to look at their model of $\frac{5}{12}$. Discuss the differences in the two fractions.
10. Write the following expressions on the board: $\frac{6}{8} + \frac{1}{8}$, $\frac{2}{12} + \frac{9}{12}$, and $\frac{6}{12} + \frac{4}{12}$. Ask the pairs to solve these expressions with either the pizza or the snake. Allow pairs to model their procedures and discuss the answers as a class. Then have pairs draw pictures of the problems.
11. Write the following equations on the board: $\frac{4}{11} + \frac{5}{11} = \frac{9}{22}$ and $\frac{4}{11} + \frac{5}{11} = \frac{9}{11}$. Have pairs discuss which equation is correct and why. After they have discussed the answer and reasoning among themselves, give pairs some time to share their thoughts and methods with the class.
12. Distribute *Working with Fractions* (page 150) to the students and allow them to complete the activity sheet according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 5–12 individually. Then allow students to work in pairs to write at least two word problems similar to the ones on the activity sheet.

Below-Level Learners

Allow students to complete the activity sheet in pairs. Encourage them to create fraction snakes or draw pictures to help them solve the problems that do not have denominators of 8 or 12.

English Language Learners

In a small group, review the definition of numerator and denominator using the fraction pieces from the lesson. Students may complete the activity in small groups. Students may also verbally explain their answers for questions 9–12 to a teacher if necessary.

Name ____________________________________

Working with Fractions

Directions: Use your eighths and twelfths models to find the value of each expression

1. $\frac{5}{12} + \frac{4}{12}$

2. $\frac{2}{8} + \frac{4}{8}$

3. $\frac{7}{12} + \frac{4}{12}$

4. $\frac{3}{8} + \frac{5}{8}$

Directions: Draw pictures to help find the value of each expression.

5. $\frac{3}{9} + \frac{4}{9}$

6. $\frac{5}{10} + \frac{2}{10}$

7. $\frac{4}{7} + \frac{2}{7}$

8. $\frac{5}{11} + \frac{3}{11}$

Directions: Use pictures and clear explanations to answer the question.

9. Does $\frac{1}{3} + \frac{1}{3} = \frac{2}{6}$? Explain. If this is not the correct answer, explain how to arrive at the correct answer and why you take the steps that you do.

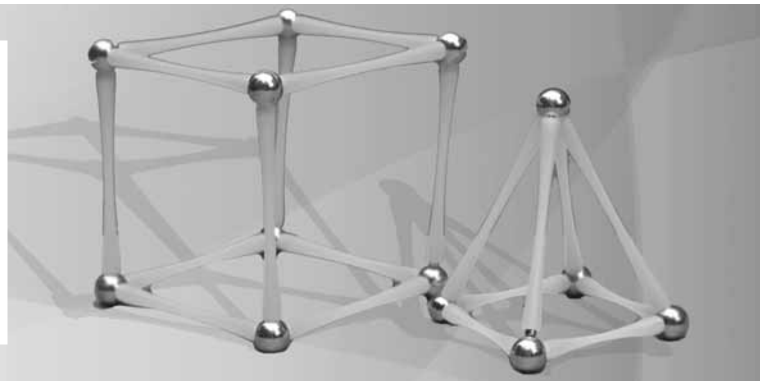
Directions: Write an expression for the situation. Add to find the answer.

10. After jogging $\frac{4}{8}$ of a mile, Samuel jogged $\frac{3}{8}$ of a mile farther. What is the total distance Samuel jogged?

11. Keera, Tamisha, and Courtney each ate $\frac{2}{8}$ of a pizza. What is the total amount of pizza that was eaten?

12. Quentin hiked $\frac{6}{15}$ of a mile up a hill before taking a break. Then he hiked $\frac{7}{15}$ of a mile. How far did Quentin hike?

Missing Addends



Standard

- solves simple open sentences involving operations on whole numbers

Grades K–2 Lesson

Materials

- counters
- paper and markers or small dry erase boards and dry erase markers
- *Missing Numbers* (page 153; page153.pdf)

Strategies Used in This Lesson

- Think Aloud
- Concrete Materials
- Summarizing
- Multiple exposures

Procedure

1. Display the equation 3 + 5 = 8. Review the other equations in the fact family that can be generated from this number sentence. *(5 + 3 = 8; 8 – 3 = 5; 8 – 5 = 3)*
2. Distribute paper and markers (or dry-erase boards and dry-erase markers) to pairs of students. In pairs, have students complete a fact family for the equation 7 + 2 = 9. When finished, have them display their answers above their heads and discuss them as a class.
3. Based on these fact family relations, lead a discussion with students about how addition and subtraction are related.
4. Distribute 20 counters to each student. Display the number sentence 8 + □ = 12. Tell students that sometimes a number is missing from equations, and fact family relationships can be used to find the missing number. The number is called a *missing addend*.
5. Have students place 12 counters in front of them. Draw 12 counters on the board or display them on the overhead. Then read the think aloud below to model finding the missing addend.

   Think Aloud: To find the missing number in this equation, I can use fact families. My original equation is 8 + □ = 12. I also know that □ + 8 = 12. (As you discuss the equations, write them one below the other on the board or overhead.) The other two equations in the fact family are 12 – □ = 8 and 12 – 8 = □. Now I need to find the equation that can best help me solve the problem. The equation that helps me find the missing number is 12 – 8 = □ because the box is by itself. Now I can use counters to complete the problem. I put the 12 counters out, take 8 counters away, and then count what is left over. (Count the counters aloud.) I have 4 counters left, so the missing number is 4. I can also check this by using counters in the original equation. First I put out 8 counters. Then I put out 4 counters. When I add them together, it equals 12. I found the correct missing number.

Missing Addends *(cont.)*

Procedure *(cont.)*

6. Have students work through the problems below with you. Model the procedures as you work through each equation.
   - 9 + ☐ = 17
   - 6 + ☐ = 14
   - 4 + ☐ = 16
   - ☐ + 5 = 18
7. Once those problems are finished, have students work with a partner to write and/or draw a summary of the procedure for finding missing addends. Discuss the procedure as a group.
8. In pairs, have students create a problem of their own. Then, in groups of four, have students switch problems and solve them, using their procedure summaries. Once finished, students should share their problems and revise their summaries if necessary.
9. Distribute copies of *Missing Numbers* (page 153) to the students and have them complete it according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students work independently to complete problems 5–8 and the challenge problem. In pairs, have students share their alternate algorithms for finding the missing number.

Below-Level Learners

Have students work in pairs to solve problems 1–6. Allow students to use the counters to help them check their work. In groups of four, allow students to work on the challenge problem.

English Language Learners

Have students work in groups of three to complete problems 1–6, using counters to check their work. Bring the students together to work on the challenge problem. Scaffold students' vocabulary and provide them with leading questions as they work on the challenge problem.

Name ______________________________

Missing Numbers

Directions: Find the missing numbers in the equations below by using fact families.

1. $8 + \square = 11$

2. $4 + \square = 12$

3. $5 + \square = 15$

4. $7 + \square = 16$

5. $9 + \square = 19$

6. $\square + 8 = 17$

7. $\square + 4 = 15$

8. $\square + 6 = 18$

Challenge

Directions: Look at the equation below. Write about a different way to find the missing number.

$$5 + \square = 12$$

____

____

____

____

____

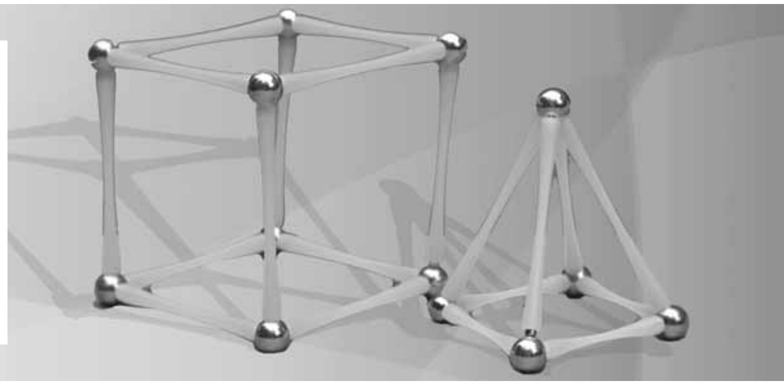
____

____

Collecting Like Terms

Standard

- simplifies expressions by combining like terms



Secondary Lesson

Materials

- 6 rulers
- 8 pencils
- square piece of construction paper
- class set of algebra tiles
- *Simplifying by Combining* (page 156; page156.pdf)

Strategies Used in This Lesson

- Visual/ Concrete Materials
- Modeling
- Chunking
- Summarizing
- Multiple Exposures
- Grouping (Pairing)

Procedure

1. Review the commutative property and the associative property of addition by using the expressions 2.6 ft. + 3.1 ft.; 3.1 ft. + 2.6 ft.; (4.2 ft. + 3 ft.) + 5.07 ft.; 4.2 ft. + (3 ft. + 5.07 ft.). Help the students see which are equivalent.
2. Ask four students to stand and hold 1 ruler each. Ask three students to stand and hold 1 pencil each. Ask the pencil and ruler students to stand in a group. Write *4 rulers and 3 pencils* = $4r + 3p$ on the board.
3. Ask two students to stand and hold 1 ruler each. Ask five students to stand and hold 1 pencil each. Ask those students to stand in a group away from the first group. Write *2 rulers and 5 pencils* = $2r + 5p$ on the board. Have the students with rulers stand together and the students with pencils stand together. Have a student not standing state the total number of rulers and the total number of pencils. Have everyone sit down.
4. Write *6 rulers and 8 pencils* = $6r + 8p$ on the board. Discuss why $6r + 8p$ cannot be written as $14rp$. Point out that only the same types of objects were combined. Write $(4r + 3p) + (2r + 5p)$. Explain that first the students regrouped themselves making $(4r + 2r) + (3p + 5p)$. Then they combined the like objects making $6r + 8p$. Explain that the terms $4r$ and $2r$ were combined to give $6r$ and the terms $3p$ and $5p$ were combined to give $8p$.
5. Go further to explain that the $4r$ and $2r$ were able to be combined because they were like terms (rulers) just as the terms $3p$ and $5p$ (pencils) were able to be combined because they were like terms. Explain that $6r$ and $8p$ are not like terms because the variables are different.

Collecting Like Terms *(cont.)*

Procedure *(cont.)*

6. Have the students look at 2.6 ft. + 3.1 ft. = 3.1 ft. + 2.6 ft. again. Point out that in the expression 2.6 ft. + 3.1 ft., just as in the expression 3.1 ft. + 2.6 ft., the like terms were able to be combined to give 5.7 ft. The same idea of combining like terms worked for (4.2 ft. + 3 ft.) + 5.07 ft. and 4.2 ft. + (3 ft. + 5.07 ft.) as well, giving 12.27 ft.
7. Now ask students if x^2 and x are like terms. Hold up the square piece of construction paper and point out that since we do not know the length of the square, we can call it x. Draw a line on the board to represent the length. Next, show that the area may be represented as x^2 and draw a square on the board to represent the x^2. Ask if those are the same or different. Point out that just as a line is clearly different from a square, x and x^2 are not like terms.
8. Give each student a set of algebra tiles. Identify each type as x^2, x, 1, $-x^2$ $-x$, or -1 and review this identification with the students. Write $(3x^2 + x + 4) + (2x^2 + 2x + 1)$ and help the students use algebra tiles to model this expression. As they combine like terms with the tiles, write $(3x^2 + 2x^2) + (x + 2x) + (4 + 1)$. As they simplify, write $5x^2 + 3x + 5$. Review what is written on the board and have the students write the steps as they are reviewed.
9. Have the students independently model and simplify $(2x^2 + x + 3) + (3x^2 + x) + (4x + 1)$. Review the steps once students have completed the work.
10. As in step 8, work the expressions below. You may also choose to have students work some of the problems in pairs instead of as a class.
    - $(4x^2 - 3x + 2) + (-4x - 5) + (-x^2 + 2x + 7)$
    - $(-7x + 1) + (3x^2 - 2x) + (5x - 10)$
    - $-(2x^2 + 3xy - 4) + (2xy + 2x + 1)$
    - $(-2x^2 + 7) - (3x^2 - 4x + 12) - (-x^2 + 2)$
11. Have students complete *Simplifying by Combining* (page 156) according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 3–6 and 10–16 only. You may choose to have students complete the problems individually or in pairs.

Below-Level Learners

Have students complete problems 1–8 in pairs and 9–12 individually. You may choose to have students complete all of the listed problems in pairs, depending on each student's understanding of the concept.

English Language Learners

Pair each English language learner with a student of the same mathematical ability to complete questions 1–12. Allow these students to summarize more with underlining, highlighting, and short phrases than by longer explanations in questions 3–6.

Name ________________________________

Simplifying by Combining

Directions: Use algebra tiles to model, and then simplify each expression.

1. $(2x^2 - 3x + 5) + (x^2 - 2x - 3)$
2. $(4x^2 - 2x + 6) - (2x^2 - 7x - 4)$

Directions: For each question, write your answer, discuss with a partner, and then make adjustments to your answer as needed.

3. Consider $-3x^2$, $4xy$, $-7x^2y$, $8xy$, $-6x^2y$, $8x^2$. Which are like terms? Explain.

   ____

4. The sum of $(3ab^2 + 4c - 5b^2) + (8ab^2 + c - b^2)$ is $11ab^2 + 5c - 6b^2$. Explain how to arrive at that answer.

   ____

5. The difference of $(3xy - 2y^2) - (8x^2 + 4xy - 7y^2)$ is $-8x^2 - xy + 5y^2$. Explain how to arrive at that answer.

   ____

6. In general, what does it mean to "combine like terms?"

   ____

Directions: Simplify. Show your work.

7. $2a + 7a + 5b$
8. $-m + 8n + 2n + 4$
9. $(3a - 6b - 8) + (-8a - 5b + 2) + (2a + 7b)$
10. $(7a + 2ab + 6c) + (-4a + 2c) + (3ab + 3c)$
11. $4b + 7 - (-2a - 3b + 4)$
12. $(2a + 7b + 4) - (12a - 3b + 10)$

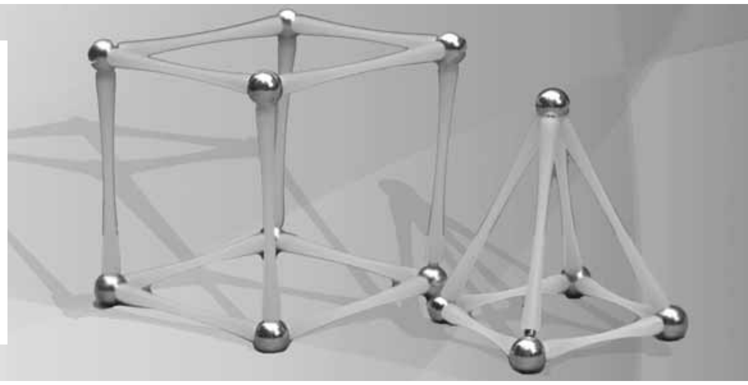
Directions: Follow the instructions to answer each question.

13. Find the perimeter of a triangle that has sides with lengths of $6a + 2b$, $12a - 3b$, and $5a + b$.
14. Use algebra tiles to model $3(2x^2 + 4x + 2)$. Draw your results. Explain how you can write an expression for the results without parentheses and how you could have found these results without the algebra tiles.
15. Use what you learned in question 14 to simplify $3(5x^2 - 2x + 1) + 2(x^2 + 4)$.
16. Simplify $4(2a + b) + 3(5a + 2ab + 5b)$.

Linear Equations

Standard

- solves linear equations using concrete, informal, and formal methods



Secondary Lesson

Materials

- Algebra tiles (one set per student)
- calculators *(optional)*
- *Practicing Two-Step Equations* (page 159; page159.pdf)

Strategies Used in This Lesson

- Note Taking
- Visual/ Concrete Materials
- Think Aloud
- Summarizing
- Chunking
- Group Thinking

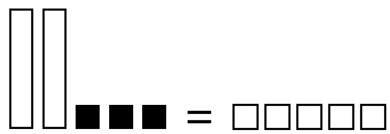
Procedure

1. In this lesson the students will solve two-step equations. Have them record their work in notes so that they have something to refer to. Do this by displaying on the board or overhead the notes you would like them to write.
2. Have students write the title "Two-Step Equations" on their papers for notes. Also distribute a set of algebra tiles to each student.
3. Review the value of each algebra tile: x, $-x$, 1, and –1 and the steps of solving one-step equations. Have students take notes on both of the concepts. Practice the concepts by using algebra tiles to solve the first two problems below with the class and then by solving the last two problems with the class without the aid of the tiles. In each case, have each student write the problem, show the steps, and answer the question in his or her notes.
   a. Once the temperature is increased by 4° the new temperature is –3°. What was the original temperature?
   b. After the number of volunteers is tripled, the new number of people volunteering is 12. How many people originally volunteered?
   c. A number decreased by 32.07 is equal to –5.3. What is the number?
   d. A number divided by 2.7 is equal to 3. What is the number?
4. The following is a sample think aloud to use with the first problem: "I need to get the variable by itself. Right now the 4 is on the same side as the variable. I need to move the 4. To do this I subtract 4, because it is the opposite of adding 4. But, I must do this to both sides to keep the equation balanced. What I do to one side I must do to the other." (This sample think-aloud can be used with each problem.)

Linear Equations *(cont.)*

Procedure *(cont.)*

5. Now have the students practice independently by asking them to use algebra tiles to solve $x - 4 = -2$ and $2x = 6$. Have them also solve each problem without the tiles. Have them then describe their steps to one another.
6. Present the next problem by having the students set up their algebra tiles as shown below.



7. Ask the students to translate the problem shown above into an equation. ($2x - 3 = 5$) Discuss what should be done first to the equation. After discussion, demonstrate the step of adding 3 to each side and have the students do so with their algebra tiles. Point out that the resulting equation, $2x = 8$, now only needs division to be solved. Solve the problem as a class.
8. Ask the students to draw and to write the steps in their notes with your guidance. Discuss why first adding both sides by 3 is more reasonable than first dividing both sides by 2.
9. Ask the students to model, draw, and then solve the problems $3x + 4 = -11$ and $2x - 8 = 4$ by hand. Have them work in pairs. When the work is completed, have each student work with a new partner. In these new pairs, one student is to explain his or her steps for the first problem and then the other student is to explain his or her steps for the second problem.
10. Distribute *Practicing Two-Step Equations* (page 159). Assign groups of three for the middle portion of the assignment. Allow students to complete the activity sheet according to the differentiation suggestions provided below.

Differentiation

Above-Level Learners

Have students solve questions 7–9 in groups of three. Instruct them to solve questions 10–14 independently and then discuss their answers to questions 13 and 14 in pairs.

Below-Level Learners

In a small group, work problems 1 and 2 together. Discuss the steps together and how the problems were solved, and have students write notes to themselves as reminders. Tell these students that they are to refer to the notes each time they are about to independently try a problem. Students should only complete problems 1–12. Allow these students to use a calculator if necessary.

English Language Learners

Review the notes from the lesson as a group. Allow students to ask questions about procedures, vocabulary, and problems they have difficulty with. Have students complete problems 1–12 only.

Name ______________________________

Practicing Two-Step Equations

Directions: Use algebra tiles to solve each equation for *x*. Draw pictures to show your work.

1. $4x - 3 = 5$

2. $9 = 2x + 13$

3. $3x + 7 = 16$

4. $2x + 12 = 2$

5. $4x - 2 = -14$

6. $4x + 4 = 12$

Directions: As a group, solve each problem without the cups and chips. Write each problem below on a separate sheet of paper. Your group will share one paper for these problems. Follow the steps below to solve the problems together.

- The first person writes the problem.
- The next person shows and completes the first step.
- The next person either corrects the work of the previous person, if needed, or works the second step.
- This process continues until the problem is completed. At that point the person next in the group writes another problem and the process continues.
- There is no talking during this exercise. Everyone must pay attention to each step.

7. $4x - 5 = -37$

8. $-9 = 3x + 18$

9. $12x + 23 = 119$

Directions: Solve each equation for *x*. Show each step clearly. Algebra tiles are not required.

10. $10x - 7 = -37$

11. $21x + 8 = -76$

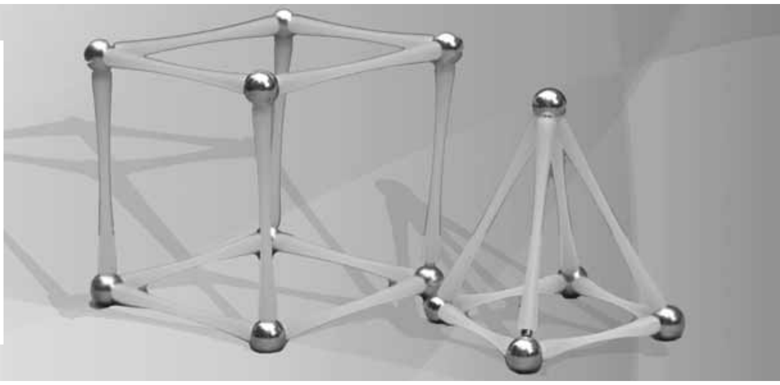
12. $2.1x - 4 = 8.6$

Challenge Problems: Solve each equation for *x*. Show each step clearly.

13. $2(x + 3) = 8$

14. $2.7(x - 3.4) = 16.2$

Calculating Area



Standard

- understands the basic measures of area.

Grades 3–5 Lesson

Materials

- centimeter grid paper
- *Looking at Area* (Teacher Resource CD *area.pdf*)
- *Working with Area* (page 162; page162.pdf)

Strategies Used in This Lesson

- Open-Ended Questions
- Visual Aids
- Graphic Organizers
- Summarizing
- Tiered Assignments

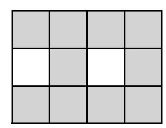
Procedure

1. Write the following problem on the board or overhead: Jayden needs to paint a billboard that is 15 feet tall and 40 feet wide. He will use a single coat of paint. He knows that he can cover 350 square feet of the billboard with a one-gallon can of paint. How many cans of paint does he need?
2. Discuss the problem in step 1 so students understand that the square footage of the billboard is needed. Explain that they will first need to find the area of the billboard.
3. Give each student a few pages of centimeter grid paper. Have them shade one small square. Explain that the square is 1 cm by 1 cm. The area is 1 square cm. That is, the amount of surface in the closed region is 1 cm^2. Next to the first square, have the students shade a second square. Ask students how many square centimeters are shaded. Continue this process until the students have shaded a 1 cm × 12 cm rectangle.
4. Have the students then shade a 2 cm × 6 cm rectangle and ask them how many square centimeters are shaded and what the area of the rectangle is. Now have the students shade a 3 cm × 4 cm rectangle and ask what the area of the rectangle is. Define area and point out that it is measured in square units.
5. Have the students go back and label the length and width on each of the three rectangles. On the board, make a chart with three columns. Label one *Length*, another *Width*, and the last *Area*. Help the students complete the three rows with the information from the rectangles drawn so far. Have each student draw and fill in a chart to match what is on the board. A copy is provided on the Teacher Resource CD to help differentiate instruction (*Looking at Area* filename: *area.pdf*).

Calculating Area *(cont.)*

Procedure *(cont.)*

6. Have the students draw, shade, and label the following rectangles:
    - length 16 cm, width 1 cm
    - length 4 cm, width 4 cm
    - length 16 cm, width 1 cm
    - length 3 cm, width 3 cm
    - length 8 cm, width 2 cm
    - length 8 cm, width 2 cm
    - length 5 cm, width 4 cm
    - length 10 cm, width 7 cm
7. Then have them use the information about the rectangles to continue the chart. When all are finished, have the students check their work as you complete the class chart with student input.
8. Ask the students to write the answers to the following questions in their own words: What is area?; How do you find the area of a rectangle?; Why does area have to be described with labeling of square units? When all are finished, have the students share their answers with one another. Have the students do this in pairs, but with a total of three different partners each. As a whole class, discuss some of the answers.
9. Refer to the original problem in the first step. With each student still with his or her last partner, have the students find a solution to the problem. After a few minutes, discuss the solution and the process as a whole class.
10. Draw the figure to the right on the board. Ask the students to identify the area of the shaded region. Once the area of 10 square units is identified, remind them that in this lesson, they first looked at area by counting squares (such as what was done in this step), and they then looked at the special case of the rectangle. Remind them of the definition of area and of the fact that area must be identified in square units.



11. Distribute *Working with Area* (page 162). Have your English language learners work with those of similar mathematical abilities. Allow class time for completion of the assignment.

Differentiation

On/Above-Level Learners

Have above-level students work in pairs to complete problems 2, 4, and 8–12. Assign problems 1–10 to your on-level students and allow them to work in groups of three.

Below-Level Learners

Have these students work in groups of three to complete problems 1–8. Encourage students to draw the pictures needed to understand the problems in the assignment.

English Language Learners

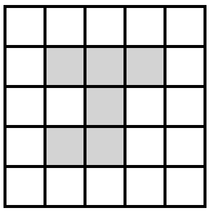
In a small group, help students create a reference sheet for the vocabulary and the concepts discussed in this lesson. Students should use this sheet while solving the problems. Assign them to work with students at similar ability levels after creating the reference sheet.

Name ______________________________________

Working with Area

Directions: Find the area of each shaded region.

1.



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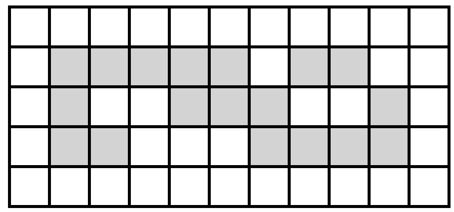
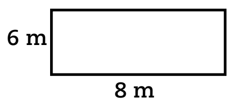
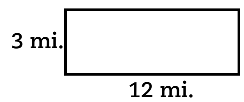


Directions: Find the area of each rectangle.

3.



4.



Directions: Draw pictures and show your work as you determine each answer.

5. One flower garden is planted. It has dimensions of 8 feet by 5 feet. What is the area of the garden?

6. One flower garden is planted. It has an area of 52 square feet. If it cost $4 per square foot to prepare and plant a flower garden, how much does it cost to prepare and plant this garden?

7. What is the width of a rectangle with an area of 14 square units and a length of 7 units?

8. What is the length of a rectangle with an area of 21 square units and a width of 3 units?

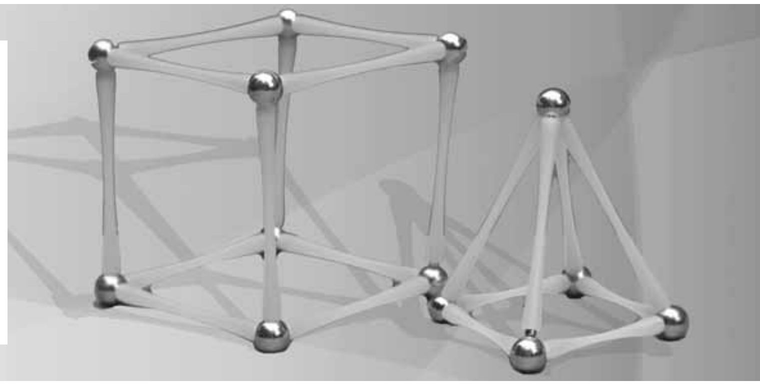
9. What is the width of a rectangle with an area of 30 square units and a length of 6 units?

10. What is the width of a rectangle with an area of 132 square units and a length of 12 units?

11. How do you find the width of a rectangle when the length and area are given? How do you find the length of a rectangle when the width and area are given? Give a new example of each case.

12. If a backyard is 60 feet by 70 feet and a 20-foot by 30-foot hole is dug for the pool, what is the area of the remaining portion of the backyard?

Calculating Perimeter



Standard

- understands the basic measures of perimeter

Grades 3–5 Lesson

Materials

- 35 toothpicks per student
- glue
- straightedge
- construction paper
- graph paper *(optional)*
- *Understanding Perimeter Notes* (Teacher Resource CD *perimeter.pdf; perimeter.doc*)
- *Working with Perimeter* (page 165; page165.pdf)

Strategies Used in This Lesson

- Concrete Materials
- Acronyms
- Graphic Organizer
- Open–Ended Questions
- Visual Aids

Procedure

1. Give each student 20 toothpicks and a piece of construction paper. Explain that they will each create a design with the toothpicks. Their toothpicks must lie end to end, they may not cross, and the figure must close. Allow each student time to create a design and to glue the toothpicks to the paper once a design is determined.
2. Once completed, have all students view each other's pictures. Then ask students how many toothpicks they used. Lead them to understand that since 20 toothpicks were used, the perimeter of their shapes is 20 toothpicks and that addition is used to determine this.
3. Write *Perimeter—the distance around the sides of a closed figure on the board*. Then write **PASS**—*to find the* ***P****erimeter,* ***A****dd for the* ***S****um of the* ***S****ides*. Discuss both the definition and the mnemonic acronym with the students.
4. Guide students in taking additional notes on perimeter. The completed notes, *Understanding Perimeter Notes*, are available on the CD for your reference *(perimeter.pdf)*, and a partially completed sample is also available to distribute to students who need extra scaffolding while taking notes.

Calculating Perimeter *(cont.)*

Procedure *(cont.)*

5. To continue the notes, give each student 14 more toothpicks. Have each student prepare a chart with four columns labeled *Figure*, *Work*, *Perimeter*, and *Notice*. Ask the students to use three toothpicks to form a triangle. Have them draw and label their triangles in their charts under *Figure*. Under *Work* have them show 1 + 1 + 1. Under *Perimeter* they should write 3 toothpicks. Leave the *Notice* column alone until the rest of the chart is completed. Continue the chart with the following shapes and side lengths:
   - triangle—sides 2, 2, 2
   - triangle— sides 1, 2, 2
   - triangle—sides 3, 4, 5
   - square—sides 1, 1, 1, 1
   - square—sides 2, 2, 2, 2
   - square—sides 3, 3, 3, 3
   - rectangle—sides 1, 2, 1, 2
   - rectangle—sides 3, 4, 3, 4
6. As a class, discuss what students notice about the two equilateral triangles. After discussion, write what was noticed in the chart (especially $3 \times 1 = 3$ and $3 \times 2 = 6$). Do the same for the squares (e.g., $4 \times 1 = 4$). Do the same for the rectangles [e.g., $(2 \times 3) + (2 \times 4) = 6 + 8 = 14$]. Discuss that while the perimeter may be found by adding the sum of the sides (PASS acronym), there are special cases where other methods also work. Help students understand the formulas for perimeter by writing *Perimeter of Squares* = 4 × *length of sides* = $4s$ and *Perimeter of Rectangles* = 2 × *length* + 2 × *width* = $2l + 2w$ on the board.
7. Ask students how those who are involved in home or business design or construction might use perimeter. Discuss their answers. Then, on the board, draw basic plans for a small store (from *Understanding Perimeter Notes*). Have the students include this drawing in their notes. Include three areas: the area for customers, the storage area, and the bathroom. Ask the students to determine the perimeter of each area as well as the perimeter of the entire store.
8. Distribute copies of *Working with Perimeter* (page 165). Have students complete the activity according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 7–10 on the activity sheet. For question 10, have students also include a playroom, a sitting room, and two additional bedrooms.

Below-Level Learners

Give these students the partially completed copy of the notes from the Teacher Resource CD. Students should complete the activity sheet in pairs. For question 10, make the following rooms optional: dining room and garage.

English Language Learners

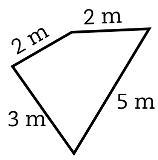
Give these students the partially completed copy of the notes from the Teacher Resource CD. Review the necessary vocabulary for the activity in a small group. Allow students to use their notes as they complete the activity sheet in pairs.

Name ______________________________

Working with Perimeter

Directions: Find the perimeter of each figure shown.

1.



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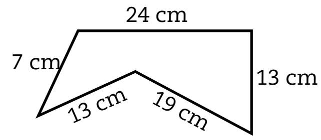
2.



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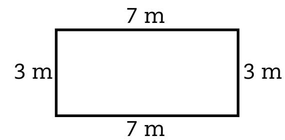
3.



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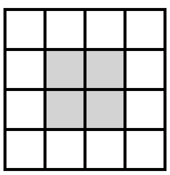
Directions: Find the perimeter of each shaded region shown.

4.



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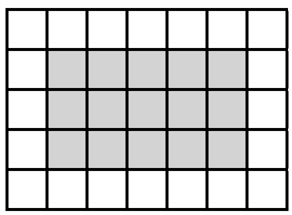
5.



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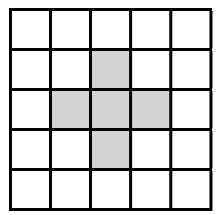
6.



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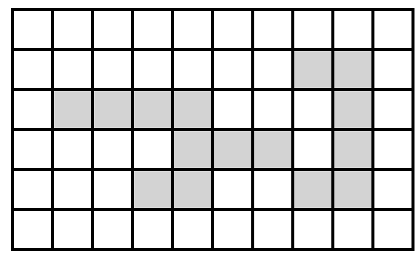
7.



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Directions: Use a straight edge to draw the figure. Find the perimeter.

8. square with side length of 3 cm

9. rectangle with a width of 2 cm and a length of 6 cm

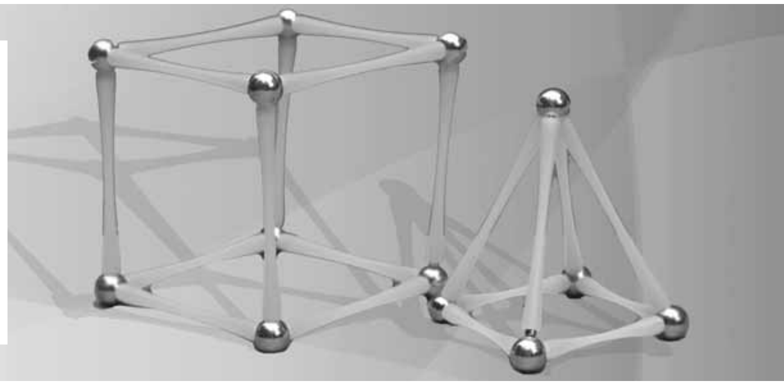
Directions: Complete the requirements for question 10 on separate paper.

10. Design the floor plan of a one-story house. Include at least 2 bedrooms, a kitchen, a dining area, a living room, a bathroom, and a garage. More rooms may be included if you like. Label the measurements. Find the perimeter of each room and of the entire house.

Calculating Volume

Standard

- understands the basic measures of volume



Grades 3–5 Lesson

Materials

- nursery rhyme "Peter Piper"
- a box in the shape of a rectangular prism
- centimeter cubes
- poster board and markers
- *Working with Volume* (page 168; page168.pdf)

Strategies Used in This Lesson

- Children's Literature
- Visual/ Concrete Materials
- Graphic Organizers
- Summarizing
- Tiered Assignments

Procedure

1. Give each student 50 centimeter cubes. Hold up the box and 1 centimeter cube and ask how many of these small cubes this box will hold. Have students make predictions. Point out that their predictions are showing the volume of the box.
2. On the board write *Volume—the amount of space an object fills*. Have each student create a four-columned chart titled, "Volume of Rectangular Prisms" with column labels of *Length*, *Width*, *Height*, and *Volume*. Have each student select one centimeter cube. Help the students identify the length, width, and height of the cube and have each student write the information on his or her chart. Explain that the volume of the cube is 1 cubic centimeter, that each side is 1 centimeter in length, and have them record that information in their charts.
3. Now have the students place two cubes together so that the resulting figure has a length of 1 cm, a width of 2 cm, and a height of 1 cm. Ask students what the volume of the new figure is. Once the answer is recorded, help the students complete the rest of the chart.
4. Then have students work in pairs to determine if there are other ways to arrange the cubes to keep the volume as 2 cm. *(length 2 cm, width 1 cm, height 1 cm; length 1 cm, width 1 cm, height 2 cm)* Discuss their findings and record them in the chart as a class. Also discuss how multiplication is used to find volume.

Calculating Volume *(cont.)*

Procedure *(cont.)*

5. Help the students use the centimeter cubes to form and record information for the following rectangular prisms: 2 cm × 3 cm × 1 cm; 2 cm × 2 cm × 2 cm; 3 cm × 2 cm × 3 cm; 5 cm × 1 cm × 3 cm; 4 cm × 3 cm × 2 cm. Ask the students to determine each volume. Then, confirm the answers, explain as needed, and make sure the volume is given in cubic centimeters. Make sure that students who provide answers explain their thinking. After the discussion, have all students write a summary of what volume is and the procedure for finding the volume of a rectangular prism.
6. Place students in homogeneous groups of up to four students. Have the students share their summaries with one another. While students share, write the following on the board: $l = 8$ *cm*, $w = 2$ *cm*, $h = 2$ *cm* and $l = 6$ *cm*, $w = 4$ *cm*, $h = 2$ *cm*. Tell all groups to create a model of the rectangular prisms described on the board with the centimeter cubes and identify each volume.
7. As groups begin working, tell the below-level groups to draw the rectangular prisms on their poster board. Tell the on-level groups to demonstrate on their poster boards as many other rectangular prisms as possible with the same volumes as the originals but with different dimensions. Tell the above-level groups to create two word problems, one per prism, in which one of the three dimensions is unknown. The problems, work (without using cubes), and answers should be displayed on the poster board.
8. Have each group share its first prism. When all groups have shared, have the students write summaries of what they learned from each group. Complete the same process after all of the groups share their second problems.
9. Read the nursery rhyme "Peter Piper." Explain that a peck is a unit of volume. Tell students that they will be investigating this idea in the next activity. Distribute copies of *Working with Volume* (page 168) and have students complete it according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have these students complete problems 7–14 independently. Then have students work in pairs to determine the amount of cubic centimeters in 6 cubic meters. Allow them to draw a diagram and write an explanation of their thinking process.

Below-Level Learners

Have students complete problems 1–13 on the activity sheet in pairs. Encourage students to use the centimeter cubes.

English Language Learners

In a small group, have each student draw a cube, label the length, width, and height, and write the formula for volume. Discuss what each word means and allow students to write any additional notes they need. Students should use their papers as they complete the activity sheet.

Name ____

Working with Volume

Directions: Use centimeter cubes to form the rectangular prism described in each column. Draw the prism on a separate sheet of paper. Identify the volume.

1. length: 3 cm	**2.** length: 4 cm	**3.** length: 3 cm
width: 4 cm	width: 2 cm	width: 5 cm
height: 1 cm	height: 5 cm	height: 3 cm
volume: ______________	volume: ______________	volume: ______________

Directions: Complete the chart below. Use centimeter cubes with at least the first three.

	Length	Width	Height	Volume
4.	5 cm	2 cm	3 cm	
5.	2 cm	6 cm	2 cm	
6.	7 cm	3 cm	2 cm	
7.	6 cm	5 cm	8 cm	
8.	5 cm	4 cm	12 cm	
9.	2 cm	4 cm		16 cm
10.	9 cm		3 cm	54 cm
11.		5 cm	3 cm	120 cm
12.		15 cm	4 cm	1,200 cm

Directions: Read the nursery rhyme below. Answer questions 13 and 14.

Peter Piper picked a peck of pickled peppers;

A peck of pickled peppers Peter Piper picked.

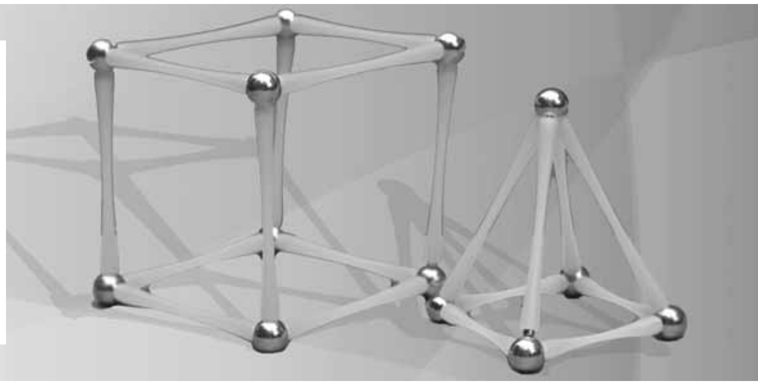
If Peter Piper picked a peck of pickled peppers,

Where's the peck of pickled peppers Peter Piper picked?

13. A peck in U.S. customary measure is equivalent to about 8,810 cubic centimeters. Would Peter's peck of pickled peppers fit in a box with the dimensions of 12 centimeters in length, 10 centimeters in width, and 20 centimeters in height? Explain.

14. If Peter places his peck of pickled peppers in a 9,000 cubic centimeter box that has a length of 15 centimeters and a width of 20 centimeters, what is the height of the box?

Finding Missing Angle Measures



Standard

- understands the defining properties of triangles and quadrilaterals

Secondary Lesson

Materials

- class set of rulers and protractors
- *Triangles and Quadrilaterals* (Teacher Resource CD *angles.pdf*)
- *Looking at Triangles and Quadrilaterals from All Sides and Angles* (page 171; page171.pdf)

Strategies Used in This Lesson

- Note Taking
- Visual/Concrete Materials
- Graphic Organizer
- Grouping

Procedure

1. Give each student a ruler and protractor. Guide them in using these tools to draw ΔABC where $m\angle A = 60°$, $m\angle B = 50°$, $m\angle C = 70°$, $AB = 12$ cm, $BC = 11$ cm, and $AC \approx 10$ cm. Then, ask the students to turn their papers over and tell them that they will use their protractors and rulers to create and label their own triangles based on your requirements. Assign triangles according to student readiness level. After drawing their triangles, students must find any missing measurements. Assign requirements based on triangle types (scalene, isosceles, equilateral, acute, right, and obtuse). Also have students find the sum of their angles.
2. Distribute copies of the notes titled *Triangles and Quadrilaterals* (Teacher Resource CD angles.pdf). Lead students through the notes and be clear about where and how you expect the information to be written. Consider giving English language learners a copy of the notes, but have them make the drawings.
3. Before discussing triangles in the notes, ask students what was determined to be the sum of the measures of the angles of their triangles. Help the students see that the answers round to 180° (for any values not exactly 180°, mention slight errors in measurement). Discuss triangles, draw a triangle on the board or overhead to aid in the discussion, and have the students take notes through the discussion, including copying and labeling the triangle that you drew.
4. Continue guiding the students through the provided notes as you discuss each type of triangle. Once you have given the definition, have the students who made the triangles fitting that definition share their triangles. Draw a triangle with the defining information for the notes.

Finding Missing Angle Measures *(cont.)*

Procedure *(cont.)*

5. Ask the students to look at all the information on their triangles. Have the students with both acute and scalene triangles raise their hands. Ask for two examples to be shared. Have the students write as many identifying names as possible on their triangles and allow time for the sharing of results.
6. Just as you assigned the triangles to your students, do the same for the seven types of quadrilaterals listed in the notes (parallelogram, square, rectangle, rhombus, trapezoid, isosceles trapezoid, and irregular). Suggest that the lines on their lined paper be used to help with creating a set of parallel lines when needed. Students need to label all measurements.
7. Guide the students through the note taking for quadrilaterals in the same way you did for triangles. Then draw, but do not label, a Venn diagram for quadrilaterals. Describe the characteristics of quadrilaterals. Ask that all students whose figures meet those characteristics raise their hands (all should raise their hands). Do the same for the parallelogram. Emphasize the figures that are also parallelograms (e.g., squares). Do this for all the quadrilaterals and help the students fill out a Venn diagram in the notes during the discussion.
8. Guide students through the problems provided on the notes. Make sure they see connections such as finding congruent angles in isosceles triangles. Then, give the students time to review their notes and to make additional comments, underline, etc., in those notes.
9. Distribute *Looking at Triangles and Quadrilaterals from All Sides and Angles* (page 171) and have the students complete it according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 1–6 and 9–13 individually. Then allow students to create at least two problems like the ones in questions 11–16 in pairs. With remaining time, have pairs switch papers and solve the new problems.

Below-Level Learners

In pairs, have students complete questions 1–3, 7–9, and 11–14. Then allow pairs to create at least one problem like the one in questions 11–16. With remaining time, have pairs switch papers and solve the new problems.

English Language Learners

Allow students to use their notes when completing the activity sheet. Have students complete questions 1–4 and 8–16 in pairs or groups of three.

Name ________________________________

Looking at Triangles and Quadrilaterals from All Sides and Angles

Directions: On separate paper, sketch the examples in the most general way possible. Identify the figures in as many ways as possible.

1. Four congruent sides.
2. Three sides, only two of the sides are congruent; the angles measure 84°, 48°, and 48°.
3. Three sides, all three sides are congruent.
4. Side measures of 4 units, 6 units, 4 units, and 6 units. The angles all have measures of 90°.
5. Only two sides of four are parallel.
6. Three sides. None of the sides are congruent. One angle measures 90°.

Directions: Identify each statement as true or false. Explain your answer.

7. All squares are rectangles. T F

____

8. Some right triangles are scalene. T F

____

9. A triangle with two of its angles being 30° and 10° is acute. T F

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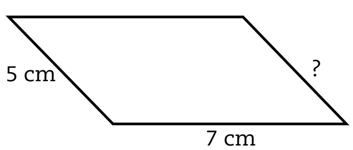
10. A triangle with two of its angles being 80° and 50° is acute. T F

____

Directions: Find the measure of the missing side or angle in each question.

11.



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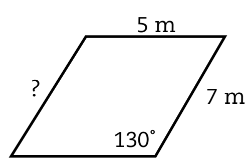

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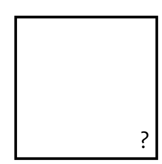
13.



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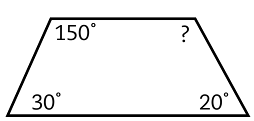

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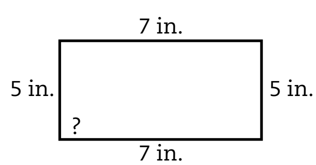

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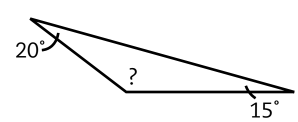

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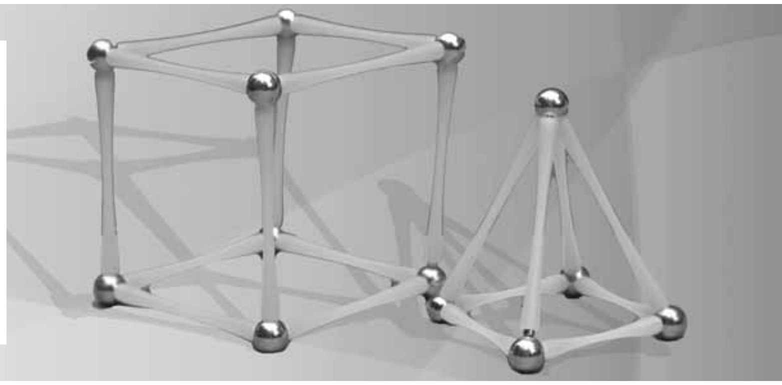
Measuring Length



Standard

- knows processes for measuring length using basic standard units

Grades K–2 Lesson

Materials

- rulers
- chart paper
- *Classroom Measures* (page 174; page174.pdf)

Strategies Used in This Lesson

- Graphic Organizer
- Concrete Materials
- Modeling
- Multiple Exposures

Procedure

1. Distribute rulers to students and tell them that rulers are used to measure length. Show students the difference between an inch and a foot. Then, ask students which distance is longer. (For teachers using the metric system, use centimeters and meters for this activity instead of inches and feet.)
2. Create a T-chart on chart paper. Label one side *inches* and label the other side *feet*. As a class, list different items that are most appropriately measured in inches. Then list items that are most appropriately measured in feet. For example, a book would be measured in inches, and the classroom wall would be measured in feet.
3. Distribute copies of *Classroom Measures* (page 174) to students. Have them look at the chart on the top of the page and write *paper* in the first row under *Item*. Model for students how to accurately measure the length of the paper using the ruler. Remind them that the end of the ruler is not the beginning of the inch marks and that the item they measure must begin at the first mark on the ruler.
4. Have pairs of students measure the length of the paper, checking to make sure that they are following the correct procedure for aligning the edge of the paper with the correct spot on the ruler. Then instruct them to write the length in the second column of the chart. *(11 inches)*
5. Have students write the word *desk* in the second row of the chart. Model for students how to mark the end of the ruler with the tip of their pencils (not leaving a mark, just using it as a placeholder) in order to measure more than one foot. Then model how to add the feet and inches to get the total length of their desks. Have students work in pairs to practice this same procedure and record their answers in their charts.

Measuring Length *(cont.)*

Procedure *(cont.)*

6. Instruct pairs to choose three other items around the room to measure. They should choose at least one item that is most appropriately measured in inches and another item that is most appropriately measured in feet.
7. Once students are finished, invite several of them to share one item they measured and its length.
8. Have students complete the rest of *Classroom Measures* (page 174) according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students work independently to complete the chart and questions 3 and 4. In pairs, have students work on the challenge problem.

Below-Level Learners

Review the procedures for measuring inches and feet with the students. Then have them work in pairs to complete the chart and questions 1 and 2. If time allows, have students work on the challenge problem.

English Language Learners

Review the procedures and vocabulary for measuring inches and feet. Have students work in groups of three to complete the chart and questions 2 and 3. If time allows, have students work on the challenge problem.

Name ________________________________

Classroom Measures

Directions: Find items in the classroom. Measure them and record the lengths in the chart below.

Item Name	Length

Directions: Choose any four items from around the classroom. Measure them and add the information to the chart.

Directions: In the space below, use a ruler to draw lines representing each length listed below.

1. 2 inches
2. 5 inches
3. 6 inches
4. 8 inches

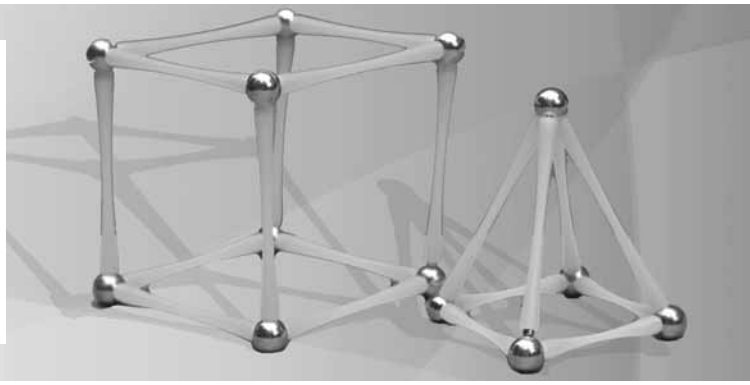
Challenge

Your paper is not 1 foot long. But, it is possible to draw a line on your paper that is 1 foot long. Turn your paper over and figure out how to do it.

Converting Measurements

Standard

- solves problems involving units of measurement and converts answers to a larger or smaller unit within the same system



Secondary Lesson

Materials

- calculators
- *Conversion Chart* (Teacher Resource CD *conversions.pdf*)
- *Conversions: All About Changing the Names* (page 177; page177.pdf)

Strategies Used in This Lesson

- Think Aloud
- Chunking
- Open-Ended Questions

Procedure

1. Write the following fractions on the board: $\frac{305}{305}$; $\frac{5}{5}$; $\frac{125}{125}$; $\frac{(2 + 2)}{4}$; $\frac{12}{12}$; $\frac{1.2}{(0.7 + 0.5)}$; $\frac{(2 \times 2)}{(2 \times 2)}$
2. Ask students what each of these fractions equal. After it is clear that each answer is 1, ask, “What is 2 × 1?” Then move to $2 \times \frac{5}{5}$ and $2 \times \frac{12}{12}$. Continue with the other forms of 1 written on the board.
3. Ask students how many inches are in 1 foot. Then write $\frac{12 \text{ inches}}{1 \text{ foot}}$ and $\frac{1 \text{ foot}}{12 \text{ inches}}$ on the board. Discuss how these fractions are both equal to 1.
4. Introduce and discuss the term *unit ratio* as a “fancy form of 1.” Ask the students to find the value of $2 \times \frac{12 \text{ inches}}{1 \text{ foot}}$ and $2 \times \frac{1 \text{ foot}}{12 \text{ inches}}$. Show that each expression simplifies to 2 × 1 = 2.
5. Remind the students there are 12 inches in a foot. Then ask how many inches are in 5 feet and how many feet are in 48 inches. *(60 inches; 4 feet)* Discuss how each answer was found.
6. Have students raise their hands if they have ever multiplied when the problem called for division or divided when the problem called for multiplication. Explain that you are going to show them a way to avoid such confusion. Use the think-aloud on the next page to show the students how to convert 5 feet to inches.

Converting Measurements *(cont.)*

Procedure *(cont.)*

7. Write the regular text on the board and say the italicized text.

5 feet	*I am starting with 5 feet and I want to convert it to inches.*
1 foot = 12 inches	*The conversion 1 foot equals 12 inches has the units I need.*
$\left(\frac{5 \text{ feet}}{1}\right)\left(\frac{\quad}{1 \text{ ft.}}\right)$	*My first value needs to become a fraction. What unit of measurement will I get rid of? I need to set up for canceling.*
$\left(\frac{5 \text{ ft.}}{1}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft.}}\right)$	*Now I need to form a unit ratio, or give myself a "fancy form of one."*
$\left(\frac{5 \cancel{\text{ft.}}}{1}\right)\left(\frac{12 \text{ in.}}{1 \cancel{\text{ft.}}}\right) = 60 \text{ in.}$	*Now I cancel and multiply the fractions.*

8. Distribute the *Conversion Chart* (Teacher Resource CD conversions.pdf). With your help as needed, have the students work in pairs to convert 2.5 years to months *(30 months)*, 23 pints to quarts *(11.5 quarts)*, 3.2 cups to ounces *(25.6 ounces)*, and 3696 feet to miles *(0.7 miles)*.
9. Show students that to convert within the metric system, you must move decimal places appropriately. Remind the students of this process by having them convert 2 decameters to centimeters *(2,000 cm)* and 42,532.5 millimeters to meters *(42.5325 m)* by moving the decimal as needed. Have pairs of students confirm the two answers by using unit ratios.
10. Demonstrate the problem below. Ask students what situations this problem may apply to and discuss the possible applications. Have the students work in pairs to create and solve one application problem each in which the students must use conversions in some way. Discuss these problems as a class.
    13 pounds 4 ounces – 5 pounds 20 ounces = 12 pounds 9 ounces – 5 pounds 9 ounces = 7 pounds 11 ounces
11. Give each student a copy of *Conversions: All About Changing the Names* (page 177) and have them complete it according to the differentiation suggestions below.

Differentiation

Above-Level Learners

Have students complete questions 7–14 individually. Then, in pairs, have students investigate the conversion of squared and cubed units. Give the example of $900cm^2 = 0.09m^2$. Have the students determine why the example is true and then have them convert 36ft.^{2} to yd.^{2} and 5yd.^{3} to ft.^{3} *($4yd^2$; $135\ ft^3$)*.

Below-Level Learners

Allow students to work in pairs to complete questions 1–6 and 9–13. Have pairs share their application problems and work in groups of four to solve them.

English Language Learners

Allow students to complete problems 1–6 individually. Then, in a small group, work with students to complete problems 9–12. Write the application for problem 13 as a group.

Name ____

Conversions: All About Changing the Names

Directions: Use unit ratios to determine each equivalent amount.

1. 2.3 pints = ___________ ounces
2. 23,760 feet = ___________ miles
3. 1.7 tons = ___________ ounces
4. 200,700 mm = ___________ km

Directions: Determine each equivalent amount using a method of your choice.

5. 2.7 m = ___________ mm
6. 24 cm = ___________ km
7. 280,320 hours = ___________ decades
8. 24 hectometers = ___________ centimeters

Directions: Solve the following problems.

9. Ms. Santoy's class will make cookies for a school celebration. They know that a single recipe will make about 25 cookies. They need about 500 cookies. A single recipe calls for 2 tablespoons of honey. How many cups of honey do they need for their 500 cookies?

10. Which is the better buy, 4 pounds of apples for $3.87 or 60 ounces of apples for $3.87? Why?

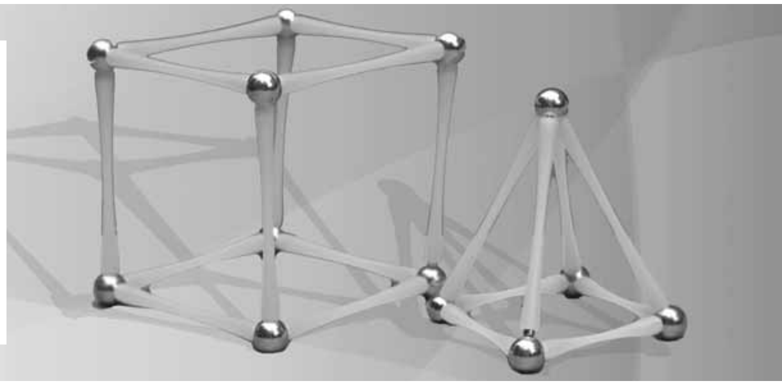
11. Megan is 4 ft. 10 inches tall. If she grows 3 inches, how tall will she be?

12. Jordan is 4 ft. 9.25 inches tall. His younger brother is 3 ft. 11.75 inches tall. What is the difference in their heights?

13. Write your own application problem using the conversion 2 cups = ___ tablespoons.

14. Demonstrate and explain how the conversion in number 13 can be performed using unit ratios.

Creating Graphs



Standard

- collects and represents information about objects or events in simple graphs

Grades K–2 Lesson

Materials

- 2 pieces of chart paper
- paper
- *My Graph* (page 180; page180.pdf)

Strategies Used in This Lesson

- Visuals
- Grouping
- Modeling
- Summarizing

Procedure

1. Before beginning the lesson, create a blank bar graph like the one on page 180.
2. Write the following question on the board or overhead: *What is your favorite type of weather?*
3. Create a chart under the question and label the first column *Weather*, the second column *Votes*, and the third column *Total*. Under the weather column write *rain*, *snow*, *sun*, *fog*, and *other*.
4. Allow each student to come to the chart and place a tally next to the type of weather that is his or her favorite. Tell students that by recording people's votes, they are gathering data.
5. Total the tallies for each row. Tell students that finding totals is part of gathering data.
6. Tell students that they are going to take this data and create a class graph.
7. Display the blank graph you prepared on chart paper before the lesson. Let students know that every graph has a title that goes above the graph. As a class, choose a title for this graph. Make sure students understand that the title of the graph should tell about what the graph shows.
8. Tell students that the choices need to be placed along the bottom of the graph. Have student volunteers write *rain*, *snow*, *sun*, *fog*, and *other* across the bottom of graph. Then write the general label *weather* across the bottom to show that the bottom of the graph is depicting the weather.

Creating Graphs *(cont.)*

Procedure *(cont.)*

9. Ask students what they think should be written on the left side of the graph and why. *(numbers; to show how many votes each type of weather received)* Have student volunteers write the appropriate numbers on the left side of the graph. Then write the general label *number of votes* along the side to show that the vertical part of the graph is depicting the number of votes each type of weather received.
10. As a class, enter the data from the tally chart into the bar graph. Then ask some questions about the contents of the graph, such as "Which type of weather was chosen the most? the least?" or "How many more does ____ have than ____?" Tell students that when you look at data on a graph, it is called *analyzing data*.
11. Display the blank piece of chart paper. Write *Steps for Graphing* across the top. Review all of the steps of the activity that was just completed, beginning with finding a question and ending with analyzing data. Have students help you summarize or draw small pictures to remind them about each step.
12. Divide the students into homogeneous groups, with three or four students per group. Tell them that they will be choosing a question of their own, polling the class to gather the data, and then recording the data on a bar graph.
13. Distribute pieces of paper and copies of *My Graph* (page 180) to students and have them complete it according to the differentiation suggestions below. Since the students will be carrying out their own graphing activities, it will be helpful for you to circulate around to the different groups and help if necessary. Refer them to the class chart as much as possible. With remaining time, have students create questions that can be answered using the information from their graphs.

Differentiation

On/Above-Level Learners

Have above-level students design a question and create 8 different choices for the voters. Have on-level students design a question and create 6 different choices for the voters. The blank paper should be used to write the question and design the table for recording votes.

Below-Level Learners

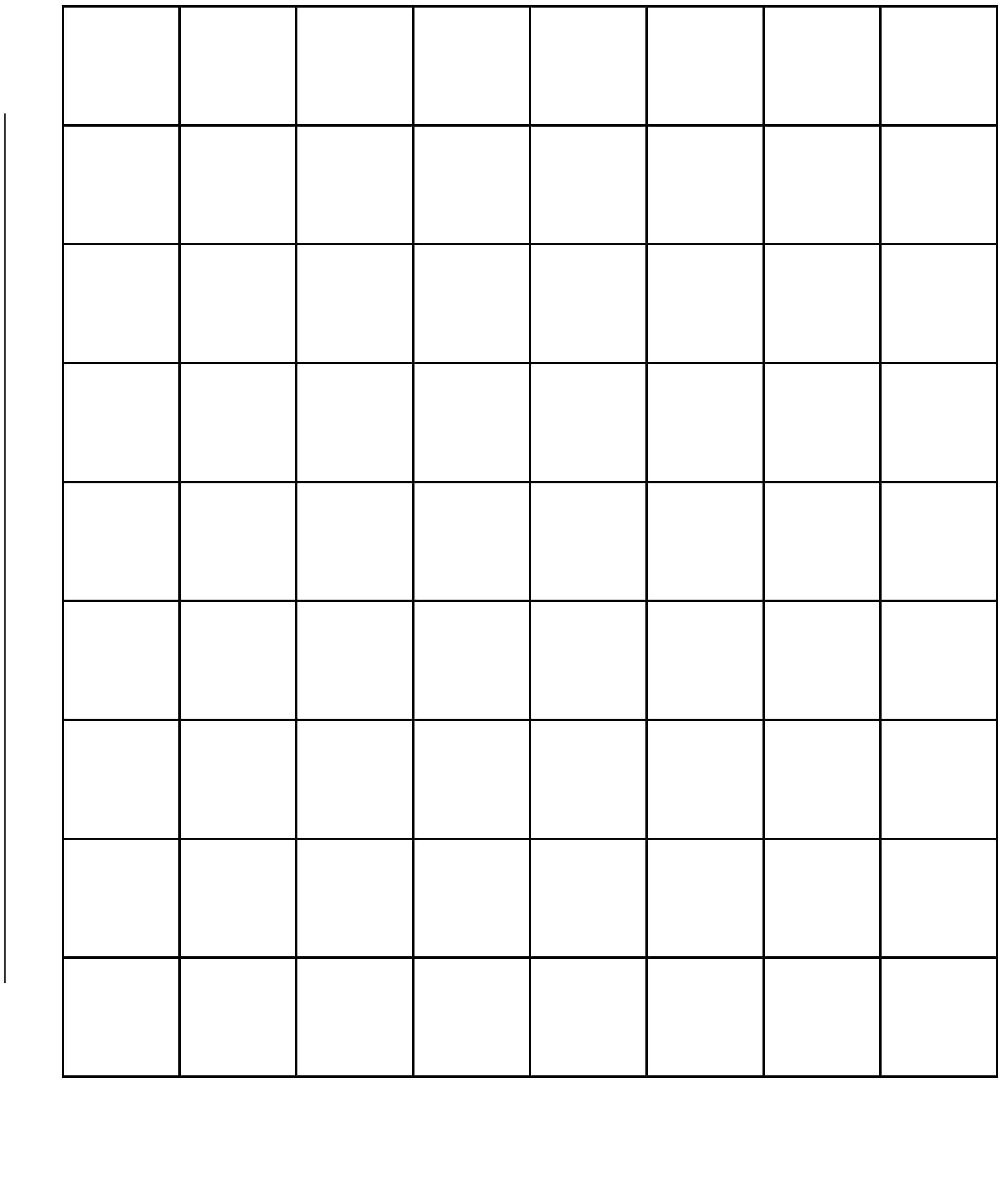
Have students design a question and create 4 different choices for the voters. The blank paper should be used to write the question and design the table for recording votes.

English Language Learners

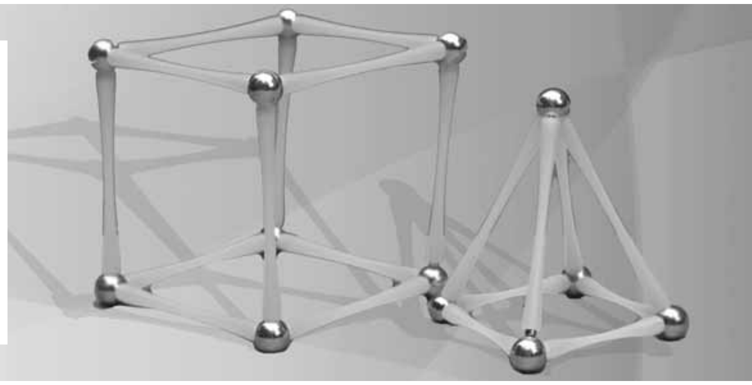
Review the procedures and vocabulary using the class-created chart as a whole group. Then divide the students into groups. Assign the number of choices the groups can create based on the students' abilities/readiness levels.

Name ____________________________________

My Graph



Finding Central Tendencies



Standard

- understands basic characteristics of measures of central tendency

Secondary Lesson

Materials

- dried beans—at least 55 per group
- small cups—at least 11 per group
- *Tendencies of Beans* (page 183; page183.pdf)

Strategies Used in This Lesson

- Visual/ Concrete Materials
- Tiered Assignments
- Graphic Organizers

Procedure

1. On the board, create a three-columned chart titled “Central Tendencies” and label the columns *Mean*, *Median*, and *Mode*. Students should also create this chart in their notes and add to it as the lesson progresses.
2. Discuss possible definitions of each term. Afterwards, write, *“The average of a set of numbers”* under *Mean*. Next, write, *“The middle number of a set of numbers written from least to greatest”* under *Median*. Finally, write, *“The number that occurs most often in a set of numbers”* under *Mode*.
3. Homogeneously group your students according to ability/readiness level. Explain that each group will study one of the three central tendencies. Distribute *Tendencies of Beans* (page 183). Explain that each group will follow the instructions numbered 1–6 for their assigned central tendency. Depending on the size of your class, you may have more than one group with any given central tendency. Distribute the beans and the cups.
4. As groups work, monitor them and check for accuracy and understanding.
5. Once the activity is complete, jigsaw the students so that new groups are formed. Each group needs to have at least one person who studied mean, at least one who studied median, and at least one who studied mode.
6. Explain to the students that each central tendency will be shared with the members of the new groups. For example, when the students who studied mean explain their tendency, they are to demonstrate and explain what they learned when completing steps 1–5 on *Tendencies of Beans*. Then the rest of the group members are to find the mean of the numbers listed on step 6. The students who studied mean are to make sure that the other students in their group can find the correct answer.

Finding Central Tendencies *(cont.)*

Procedure *(cont.)*

7. Once all of the terms are explained in the groups, the students will then make a bubble map with an example of finding the mean, median, and mode of the numbers listed in step 2 of Part II.
8. Discuss with the class the *Tendencies of Beans* activity. Review the definition of each central tendency. Add an example to each column of the chart using the set {5, 2, 8, 2, 5, 1, 5}. Talk about how this example would be done with the beans and use this as an opportunity to discuss how the concrete activity with the beans helps them understand the procedures for finding measures of central tendency. Add the steps for finding the mean, median, and mode to the chart.
9. Use the set {20, 13, 12, 12, 15, 20, 12, 20} to discuss special cases. The mean is 15.5, the median is 14 (so discuss the need to average 13 and 15), and the mode is 12 and 20. Use the set {2, 6, 4} to discuss the special case of no mode.
10. Check for understanding by having the students make a central tendencies bubble map. Use the set {7, 10, 10, 3, 8, 3, 10, 3}. Discuss the results when all are finished.
11. Allow students time to add personal reminders, diagrams, or additional explanations to their notes.

Differentiation

On/Above-Level Learners

During the lesson, have above-level students initially study mean. As an extension, they can also create an example that would show that the mean is not always the best measure of central tendency (such as any set with an outlier: example {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 45}). Have on-level students initially study median or mode.

Below-Level Learners

During the lesson, have these students initially study either median or mode. Monitor these students closely during their group work to make sure that they are developing an accurate understanding of the tendency they are studying.

English Language Learners

Pair the English language learners with other students who are native English speakers. Have these pairs stay together in both the initial grouping and in the jigsawed groups so that the native speakers can explain vocabulary and aid with comprehension of the procedures and concepts.

Name ________________________________

Tendencies of Beans

Directions: Follow the instructions listed in Part I for your assigned central tendency. Make sure to answer all questions. Once you are in new groups, follow the steps in Part II.

Part I

1. Place the following number of beans in each cup: 13, 2, 3, 2, 5, 12, 2, 4, 3, 2, 7.
2. Follow the instructions for your central tendency and answer any questions in your section.
3. Draw the steps you followed for your central tendency.
4. Show how to find your central tendency without cups or drawings.
5. List the steps you followed to find your central tendency.
6. Find your central tendency for the set {5, 3, 9, 7, 3, 7, 3, 9, 8}.

Mean: Combine all the beans. Place the beans back in the cups so that all of the cups now hold an equal number of beans. This represents the mean. What is the mean of the set of beans in this example? What mathematical operation represents combining the beans? What operation represents the even distribution of the beans? You have now represented the steps to finding the mean. Those steps are: (1) Find the sum. (2) Divide by the number of addends.

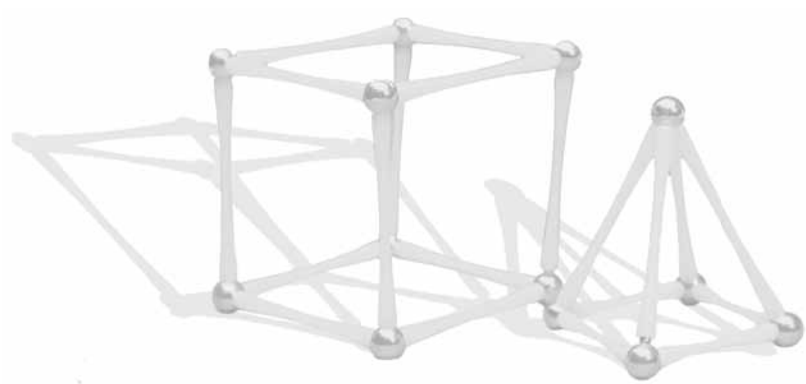
Median: Arrange the cups so that the beans are in order from the fewest number of beans to the greatest number of beans. Once the cups are in order, identify the cup in the middle. How many beans are in that middle cup? This amount is the median. You have now represented the steps to finding the median. Those steps are: (1) Arrange the numbers from least to greatest. (2) Find the amount in the middle of your arranged list.

Mode: Arrange the cups so that the beans are in order from the fewest number of beans to the greatest. The amount that occurs most often is the mode. What is the mode of the set of beans? You have now represented the steps to finding the mode. Those steps are: (1) Arrange the numbers from least to greatest. (2) Find the amount that occurs most often.

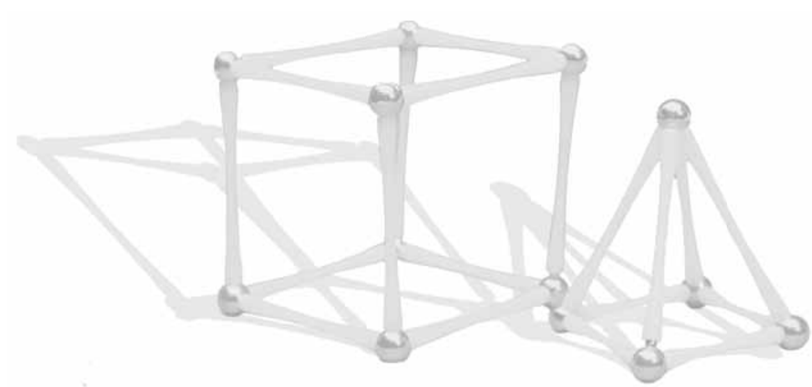
Part II

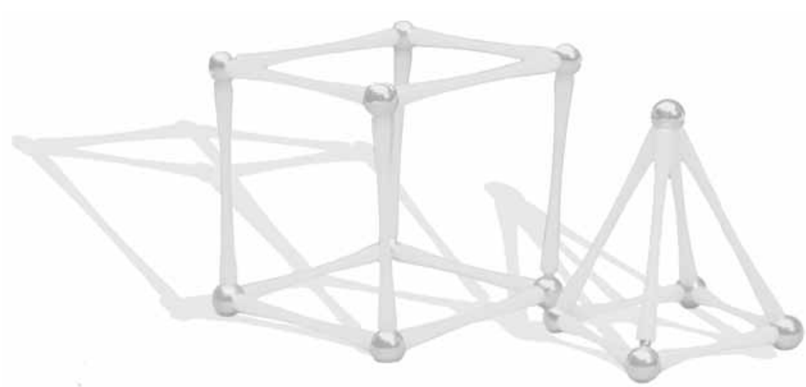
1. Each central tendency will be explained. When it is time for your central tendency:
   a. Demonstrate and explain your central tendency to your new group members. Use steps 1–5 from Part I as your guide.
   b. Give your group members the opportunity to find your central tendency for the numbers in step 6 from Part I. Make sure that they can all get to the correct answer.
2. Create a bubble map with the set {8, 9, 5, 9, 3, 6, 9} as an example. From the example set in the middle, branch out three bubbles labeled *mean*, *median*, and *mode*. Show the steps for each central tendency and briefly name each step for each central tendency. (**Hint**: Your final, and possibly third, step in each bubble could be named *answer*.)

Notes



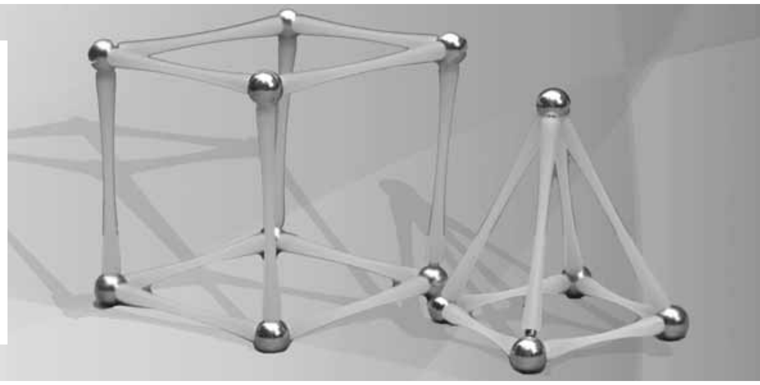
Strategies for Understanding Problem Solving





Problem-Solving Overview

Teachers must be prepared to approach mathematics instruction through questions, exploration, and problem solving.



According to research, two of the most important skills that students need in order to be prepared for 21st-century careers and citizenship are critical thinking and problem solving. Students need to be prepared to apply their knowledge and seek the right information in order to solve problems. "At the heart of critical thinking and problem solving is the ability to ask the right questions" (Wagner 2008). However, this ability is not inherent. Students must be taught how to question, and they must learn strategies that will help them solve problems, leading to more questions, more problems, and more solutions. And although the ability to vocalize their questions may not be innate, children are innately curious. This curiosity must be channeled and molded so that students can approach and solve problems in creative and meaningful ways.

According to the National Council of Teachers of Mathematics (NCTM 2000), students should be able to:

- build new mathematical knowledge through problem solving.
- solve problems that arise in mathematics and in other contexts.
- apply and adapt a variety of appropriate strategies to solve problems.
- monitor and reflect on the process of mathematical problem solving.

Teachers must be prepared to approach mathematics instruction through questions, exploration, and problem solving. In other words, problem solving is a way of teaching rather than the presentation of word problems. Exposing students to only traditional word problems is not sufficient. By doing so, they are given an unrealistic message about the way mathematics will serve them in the adult world. Most of the problems adults face require mathematical reasoning and skills that are not merely solved by translating the information given into mathematical sentences and then performing the necessary operations.

In order to function in a complex and changing society, it is necessary to be able to solve a wide variety of problems (Wagner 2008). In the real world, problems come in various shapes and forms, many of which involve mathematical concepts and applications. Often, there are numerous possible methods or strategies available to solve the problem. Students need to utilize all the resources they have developed, such as their knowledge, previous experience, and intuition. They then need to analyze, predict, make decisions, and evaluate the outcome of their solutions. For these reasons, it is extremely important that students have mathematics instruction that prepares them to become effective problem solvers.

Problem-Solving Overview *(cont.)*

Often, a greater emphasis is placed on algorithmic procedures, otherwise known as the arithmetic, because it has been more historically recognized and valued in society (Burns 2000). Arithmetic is ultimately necessary for solving many problems, but more emphasis needs to be placed on how it is used in real-life situations rather than on the actual computations. Problem solving is much more than finding answers to lists of exercises. It is, in essence, the ability to creatively approach, filter, and process information about a problem, and carry out a solution to that problem.

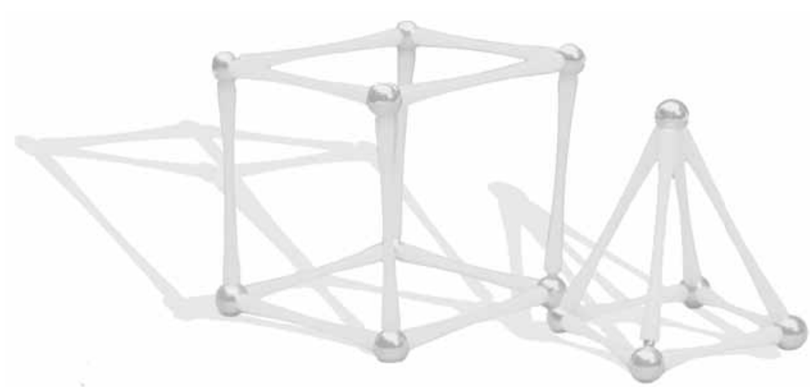
There are many classroom situations that lend themselves to illustrating real-life problems, such as collecting field-trip money, deciding on the number of buses needed for a field trip, taking attendance, and calculating grade averages. These are important and relevant ways to teach students problem-solving strategies. In addition, presenting students with contrived problem situations is also beneficial in building their problem-solving abilities because it furthers their understanding of specific problem-solving strategies. It is very challenging, but extremely important, that we offer motivating problems that spark children's natural curiosity, allowing them to use the skills they will need later (Burns 2000).

Problem-Solving Difficulty Factors

In addition to problem-solving strategies, it is important to have children recognize that the structure itself may pose a problem within the problem. There are seven difficulty factors that students must be able to recognize. Those difficulty factors are:

1. wrong order
2. key words
3. extra numbers
4. hidden word numbers
5. implied numbers
6. multiple steps
7. exact mathematical vocabulary

Each of these difficulties needs to be identified for the student, and students must have time to discover the difficulty factors within a problem. By recognizing this before attempting to solve the problem, the student is prepared to deal with the situation, and is less apt to be confused. These difficulty factors must be taught, and a list should be displayed in the classroom for all students to see.



Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

Listed below are the seven word problem difficulty factors and some recommendations when teaching them to your students.

1. Wrong Order

Numeral order is important when working subtraction and division problems. The earliest difficulty factor the learner encounters is the order in which the numerals appear in different problems. The following are examples:

Grades K–2 Example

Eric gave away 37 toy cars from his collection. He used to have 71 toy cars. How many toy cars does Eric have left?

Students often search for numbers and write them down in the order they appear in the problem. In this problem, the smaller number of 37 is given first. The student writes down this number. The second number given is 71, which the student writes under the 37. The question then calls for the operation of subtraction to be used, so the student proceeds to subtract 71 from 37. In most cases, the student does not recognize why he or she experiences difficulty solving the problem.

Grades 3–5 Example

Alexa's little brother weighs 36 pounds. Her dog, Fluffy, weighs 51.5 pounds. How many more pounds does her dog weigh than her little brother?

In this problem, the student not only is dealing with wrong order, but must also recognize that this problem involves comparison subtraction. The student assumes that the operation is addition because of the order of the numbers and the words *many* and *more*. Instead of recognizing the need to subtract, a student can easily be misled into thinking that this is a problem where something is increasing, and the comparison of the weights becomes a difficulty factor for the student.

Secondary Example

The football team drank $9\frac{3}{4}$ gallons of water at practice. The container holds 15 gallons of water. How much water is left in the container?

In this problem, the student is given the information in the opposite order in which it needs to be calculated. The student must use the larger number first in order to create the correct problem to solve.

Wrong Order Teaching Tip: Have students analyze the question first to determine what is asked. Teach students to write the equation once they understand what type of solution is needed.

Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

2. Key Words

Key words are taught in early elementary school but can be very misleading after primary grades. Students who depend on this strategy are easily fooled. The following is an example of three different situations that pose a problem to students who learn to solve word problems by merely searching for key words:

Grades K–2 Example

Maggie has 3 boxes of crayons. In every box, she has 12 crayons of all different colors. How many crayons does Maggie have altogether?

In this problem, a student who has been taught that the key words *have* and *altogether* mean *to add* is in trouble since the answer to this problem is not 15. Students need to recognize that there may be more than one operation associated with certain key words.

Grades 3–5 Example

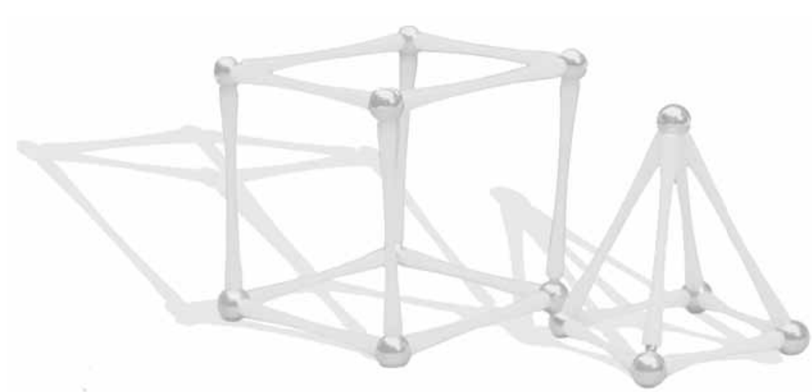
Drew planned on getting off the top of a tower by climbing down a massive 1,000-foot rope. The tower rises 865 feet straight up. How much extra rope did Drew have?

In this situation, there is not a key word to help the child identify the operation needed to solve this problem.

Secondary Example

Shen paid Ricardo $20.75 for a combination of 13 baseball cards and some basketball cards. He paid $1.25 for each baseball card and $0.75 for each basketball card. How many cards did Shen buy altogether?

Key Words Teaching Tips: Show how key words can be misleading and stress the importance of reading the entire problem before solving it. The best suggestion is that key words are most helpful when they appear in a question immediately before the question mark. Also, point out that some key words might have more than one operation associated with them. An example of this is the word *altogether*. In the primary grades, this is taught with addition, and in the intermediate grades, it could be used for multiplication.



Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

3. Extra Numbers

Problems with too many numerals can be extremely difficult for students. They are uncertain which numerals should be used. The following are examples of this difficulty factor:

Grades K–2 Example

Cindy has 15 videos, Carlos has 7 videos, and Robert has 11 videos. How many more videos does Cindy have than Robert?

Grades 3–5 Example

Lee, Ana, and Hector collect marbles. Lee has 152 marbles in his collection. Ana has 149 marbles, and Hector has 126 marbles. How many more marbles does Lee have than Ana?

In both of these problems, there are three sets of numbers; however, one set is not needed to solve the problem.

Secondary Example

At a track competition, Amal competed in the long jump. He was given three attempts. On his first jump, he went 13 feet 8 $\frac{3}{4}$ inches. On his second jump, he went 13 feet 11 $\frac{1}{2}$ inches. On his last jump, he went 13 feet 8 $\frac{1}{4}$ inches. How much farther was his longest jump than his shortest jump?

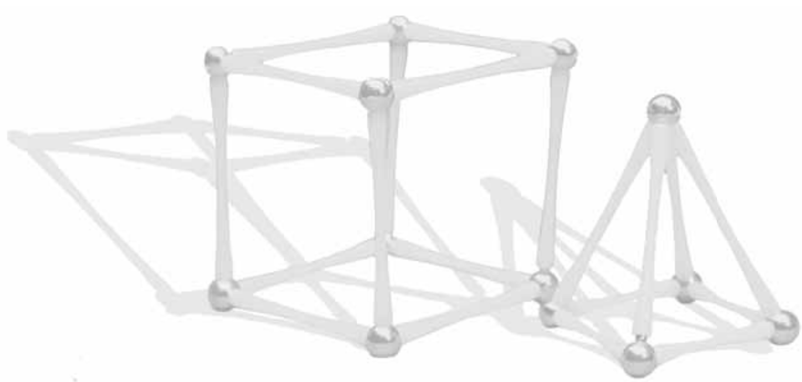
Extra Numbers Teaching Tip: Teach students to deal with this difficulty factor by talking about numbers and their relationship to the story line. Discuss why the extra number(s) should be eliminated.

4. Hidden Word Numbers

Students often look for two numerals and key words in a problem. They give the problem very little thought. Writing one of these numbers in word form complicates this strategy. The following are examples:

Grades K–2 Example

Maseo had three baseball cards. Adam had 5 baseball cards, and Tanisha had 4. How many baseball cards did they have altogether?



Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

4. Hidden Word Numbers *(cont.)*

Grades 3–5 Example

A thousand thrill seekers came to watch the mountain climber rescue Drew. If the average thrill seeker spent $14.00 for the trip to the monument, how much money was spent for travel?

Secondary Example

Julianne has $20.00 to purchase a shirt that costs $15.50. When she was checking out, the salesperson told her that the shirt is discounted twenty percent. How much change will she get after purchasing the shirt?

Hidden Word Numbers Teaching Tip: Teach number words first. Once children can identify numbers written as words, give them time to practice locating numbers in context. When they find numbers, either written as words or numerals, have them highlight the words.

5. Implied Numbers

These problems are often related to problems that contain another difficulty factor: *Exact Mathematical Vocabulary* difficulty factor. The problem might not present enough information or one of the numbers necessary for solving the problem is implied in a term, such as a measurement term. One example is using the measurement word *foot* when children need to use 12 inches to solve the problem. The following are examples of problems with implied numbers:

Grades K–2 Example

Bradon ate a dozen crackers, Mina ate 6 crackers, and Salena ate 9 crackers. How many crackers were eaten in all?

The cracker problem is difficult if students do not know the numerical value of a dozen. Often, students see the word *a* and add 1 to the sum of 6 and 9.

Grades 3–5 Example

Amy needed milk for a dessert that she was going to make for the bake sale. She went to the store and bought 1 pint of milk. She used 1 cup of milk in the recipe. How much milk was left after she made the dessert?

In this problem, the student might compare 1 cup to 1 pint and assume all the milk was used. Here, the student must know how many cups are in a pint.

Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

5. Implied Numbers *(cont.)*

Secondary Example

Mrs. Chen's science class will be planting a butterfly garden that has an area of $157\frac{1}{2}$ square feet. The length of the area is 5 yards. What is the width of the area?

Implied Numbers Teaching Tips: Teach children to look for words containing implied numbers. Practice by highlighting them and converting them to match the context of the other numbers. Always have students check first to see if the implied numbers are needed to solve the problem and are not just extra information.

6. Multiple Steps

Problems with multiple steps are difficult for students. Often, they will complete only one step; or they complete both steps correctly, but their calculations were wrong in the first step, ultimately producing an incorrect solution.

Grades K–2 Example

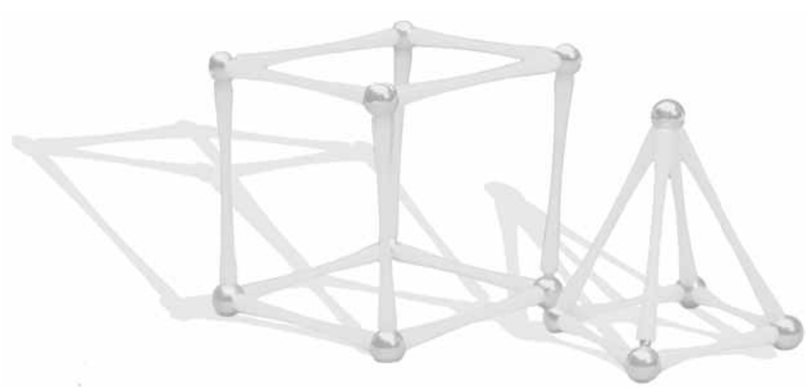
Alana has 8 gummy bears. She eats 3 of them. She then gives 2 to Bailey. How many gummy bears does she have left?

The best strategy is to act out these types of problems.

Grades 3–5 Example

Gretel picked 24 limes in the morning and 30 limes in the afternoon. Her grandmother gave her several bags and asked her to put 6 limes into each bag. How many bags will Gretel be able to fill?

Once a student identifies this as a multi-step problem, he or she must next determine the operations needed to solve it. In this problem, the student might first add all of the limes picked and divide them into groups of six or divide both numbers into groups of six and add the two quotients. Both strategies give the same result.



Problem-Solving Overview *(cont.)*

Problem-Solving Difficulty Factors *(cont.)*

6. Multiple Steps *(cont.)*

Secondary Example

Mr. Lin bought 3 picture frames for $64.50. If each picture frame cost $20.00 before tax was added, what tax rate did he pay for the 3 frames?

Multiple Steps Teaching Tip: Save problems of this type until after the students have mastered the other difficulty factors. Present the problems by demonstrating and displaying common steps needed to solve these types of problems. You may want to act out an example for clarification.

7. Exact Mathematical Vocabulary

Using exact terminology increases the difficulty of a problem. Students must be able to interpret the mathematical vocabulary to understand the situation. They must also be able to identify the possible equations associated with the terms to solve the problems. Two examples of exact mathematical vocabulary are *perimeter* and *mean*. The following are examples of problems with exact mathematical vocabulary:

Grades K–2 Example

What is the perimeter of a triangle where each side equals 3 inches?

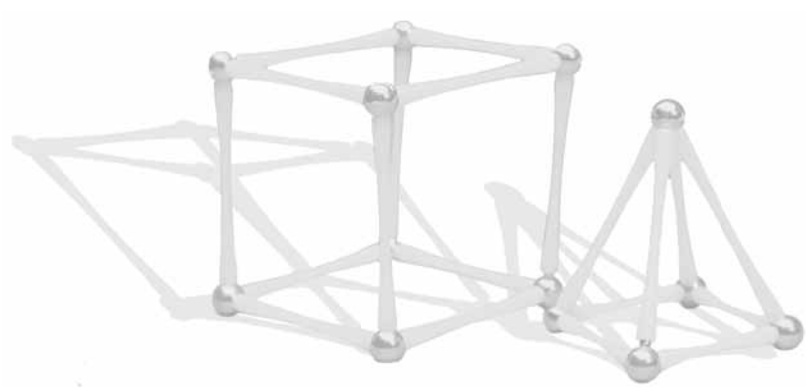
Grades 3–5 Example

Four classes raised money for a charity. Mr. Suarez's class raised $120.00, Ms. Barnes' class raised $62.00, Ms. Bard's class raised $80.00, and Mr. Feng's class raised $65.00. What is the mean amount of money raised?

Secondary Example

What is the surface area of a sphere that has a radius of 8 feet?

Exact Mathematical Vocabulary Teaching Tip: Identify common mathematical terms. Often, word problems use measurement units and geometric terms. Some examples are *circumference*, *perimeter*, and *area*.



Problem-Solving Overview *(cont.)*

Teaching About Difficulty Factors

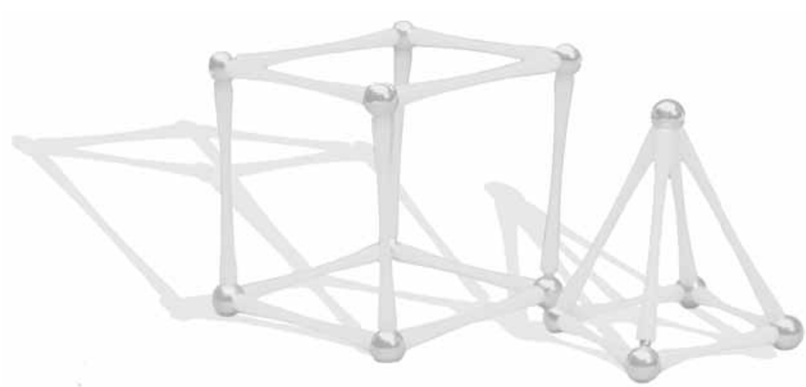
Each difficulty factor needs to be taught separately, and students need time to practice identifying which difficulty factor is present. The following procedure is a way to accomplish this:

- Introduce the difficulty factor.
- Give students several word problems, some with the difficulty factor just taught and some with no difficulty factors.
- There are two options for practice:
    1. Have students find all of the problems with the difficulty factor taught and highlight the part of the problem that has the difficulty factor. This helps them to focus their attention on the difficulty present and not necessarily on the operation to solve.
    2. Have students solve the problems with the difficulty factors only after they have identified them.
- Introduce a new difficulty factor only after students have mastered the current difficulty factor.
- As each new difficulty factor is introduced and students practice identifying them in the problem, students will then be able to handle problems that contain more than one difficulty factor. The student identifies which difficulty factor the problem has by highlighting it and writing the difficulty factor after the problem.
- For continued practice, as students enter the room each day, provide them with one problem containing a difficulty factor on a strip of paper. Have students highlight or identify the difficulty factor in the problem and then solve it. This practice is a great warm-up for your daily lesson.

Problem-Solving Process

When teaching problem solving, it is important to present students with a method of approaching all problems. For many students, the hardest part of problem solving is finding a starting point. George Polya (1973) proposed a four-stage approach to help children learn how to problem solve:

1. understanding the problem
2. devising a plan
3. carrying out the plan
4. looking back



Problem-Solving Overview *(cont.)*

Problem-Solving Process *(cont.)*

Polya's method is a systematic approach to problem solving that provides a guideline for how a problem solver moves through the process of solving many types of problems.

1. Understanding the Problem

What is it?

Understanding the problem involves interpreting what the problem means, and what questions must be answered to solve it. Students need to understand a problem thoroughly so they can determine what question is being asked in order to solve the problem and restate it in their own words.

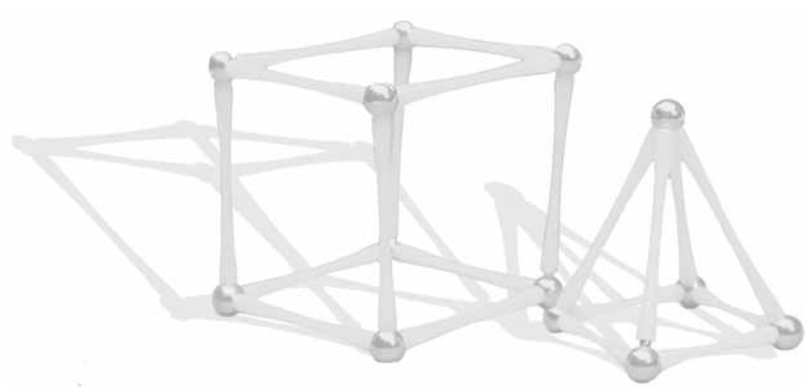
What does it look like in the classroom?

- Students should read the problem carefully until they understand what is happening in the problem and what information is needed to solve the problem.
- Students should underline or highlight unfamiliar words, then identify the meanings of those words.
- Students should locate and discard unneeded information. They also need to uncover missing information in order to solve the problem.
- Students should ask themselves questions such as, "What is the problem asking me to do?" and "What information is important for solving the problem?"
- Allow the students to have discussions with others, restating the problem in their own words if necessary.
- If students have difficulty reading or understanding the problem, they may need to dissect the problem sentence by sentence, focusing on one sentence at a time.

2. Devising a Plan

What is it?

To devise a plan means the student must choose a strategy that will help him or her solve the problem. In many problems, there is more than one applicable strategy. The student must choose the one strategy that makes sense to him or her and will help the most with solving the problem.



Problem-Solving Overview *(cont.)*

Problem-Solving Process *(cont.)*

2. Devising a Plan *(cont.)*

What does it look like in the classroom?

- Students should make a list of possible ways to solve the problem.
- Students should analyze their lists and choose one strategy they think will help them solve the problem most efficiently.
- Students should discuss their lists with others if they have difficulty choosing the best strategy.

3. Carrying Out the Plan

What is it?

To carry out the plan means to solve the problem. Strong instruction on various problem-solving strategies should be provided so that students can successfully follow through with a solution. Students must work through each part of the problem, using the strategy they chose, until they find the answer to the problem.

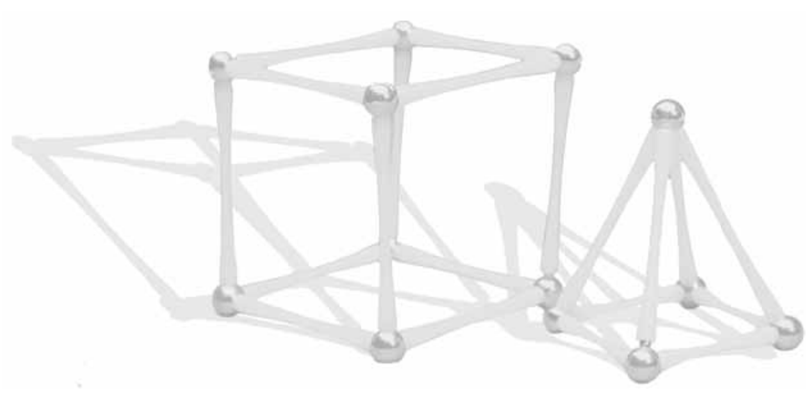
What does it look like in the classroom?

- Students should record their problem-solving steps in an organized manner so that they can see their work and decide if the strategy they selected will give them the needed results.
- If students get stuck, they can review their work to make sure they have not made any errors in their calculations.
- If the chosen strategy does not work with the problem, students should go back to their lists of strategies and choose a different approach to solve the problem.

4. Looking Back

What is it?

To look back is to examine the solution obtained from solving the problem using the chosen strategy. This step in the problem-solving process encourages students to reflect on the strategies they chose and make generalizations that could be applicable to future problems.



Problem-Solving Overview *(cont.)*

Problem-Solving Process *(cont.)*

4. Looking back *(cont.)*

What does it look like in the classroom?

- Students must reread the problem and check the solution to see if it meets the conditions stated in the problem and answers the question adequately.
- Students must ask themselves questions such as, "Does my solution make sense?" and "Is my solution logical and reasonable?"
- Students should illustrate or write down their thinking processes, estimations, and approaches. This will help them visualize the steps they took to solve the problem and make generalizations about their work.
- Allow students the opportunity to discuss with others and orally demonstrate or explain how they reached their solutions.
- Students should consider whether it is possible for them to have solved the problem in a simpler way.

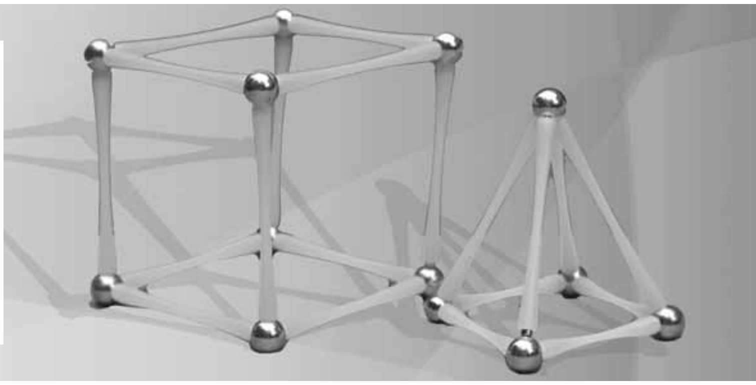
12 Strategies for Problem Solving

It is also important for teachers to provide students with specific strategies with which they can apply Polya's problem-solving approach. The following 12 strategies can be applied to a large variety of problem-solving situations:

- Drawing a Diagram
- Acting It Out or Using Concrete Materials
- Creating a Table
- Looking for a Pattern
- Guessing and Checking
- Creating an Organized List
- Working Backwards
- Creating a Tree Diagram
- Using Simpler Numbers
- Using Logical Reasoning
- Analyzing and Investigating
- Solving Open-Ended Problems

The sections that follow will provide background information on each strategy, procedures for teaching each strategy, and model problems and solutions at each grade-level range.

Drawing a Diagram



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Drawing a diagram is a visual way of processing the information in a problem. It allows students to see what is happening and relate to the situation more clearly.

This strategy often reveals aspects of a problem that may not be apparent at first. If the steps or situations being described in a problem are difficult to visualize, using a diagram may enable the students to see the information more clearly.

There are many types of diagrams that can be used with this strategy. Some of those diagrams include drawing a picture, using symbols or lines to represent objects, and using a time/distance line.

Procedure

Once it is decided that drawing a diagram is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what type of diagram will best show the information in the problem.
3. Draw the diagram according to the scenario in the problem.
4. Check your diagram and solution.
5. Record the solution.

Samples

The following skills and concepts illustrate how drawing a diagram can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

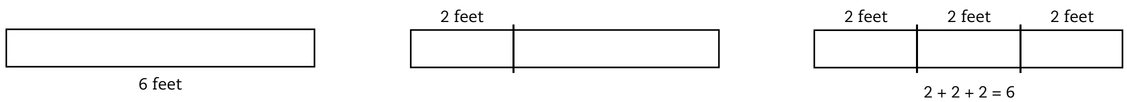
Drawing a Diagram *(cont.)*

Drawing a Picture

Drawing a picture can help students visualize the problem clearly and organize their thoughts.

Grades K–2 Sample

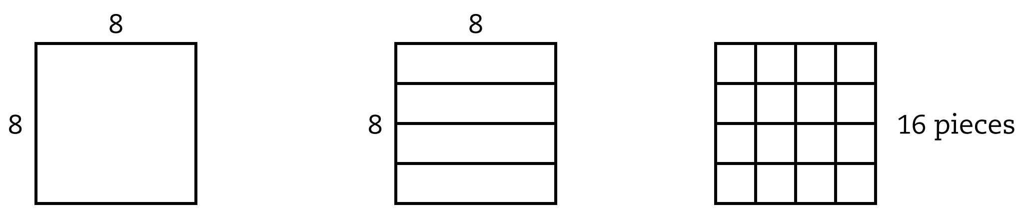
Eva has a ribbon. It is 6 feet long. She wants to cut the ribbon into pieces. She wants each piece to be 2 feet long. How many pieces of ribbon can she cut?



(step 1) (step 2) (step 3)

Grades 3–5 Sample

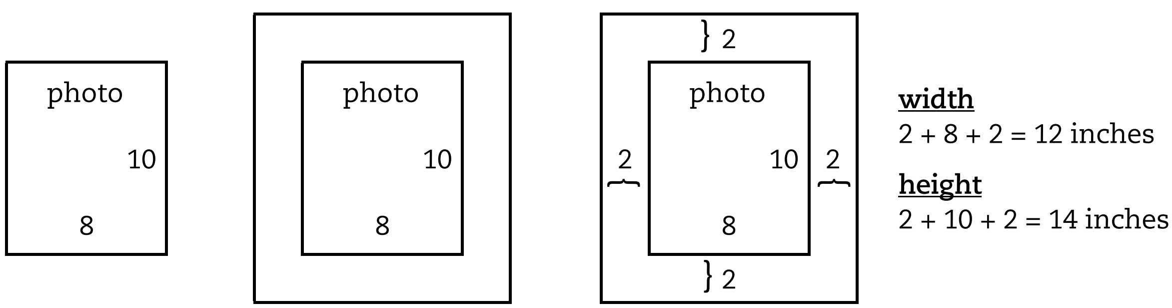
Miguel baked a cake for his mom's birthday. The rectangular pan he used measured 8" x 8". If he cuts 2-inch pieces, how many pieces can he cut?



(step 1) (step 2) (step 3)

Secondary Sample

Shen wants to place a border around a picture of his dad and himself. The picture is 8" wide and 10" tall. The border needs to be 2 inches wider and taller than the picture. What are the dimensions of the border?



(step 1) (step 2) (step 3) (step 4)

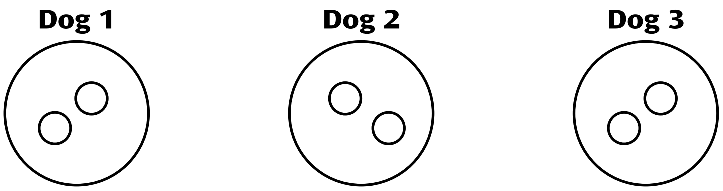
Drawing a Diagram *(cont.)*

Using Symbols or Lines to Represent Objects

Often, a problem discusses some type of object that could be difficult and time consuming to draw. Students can use lines or symbols, such as circles, squares, or triangles to represent the objects in the problem.

Grades K–2 Sample

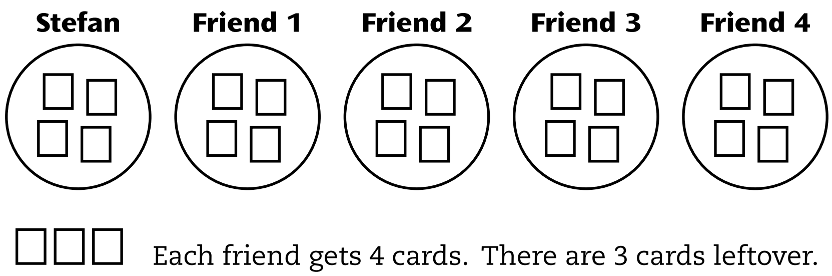
Kiara has 3 dogs. She has 6 balls that she wants to share equally among the dogs while they play. How many balls will each dog get to play with?



Each dog gets 2 balls.

Grades 3–5 Sample

Stefan has 23 trading cards that he wants to divide equally among himself and 4 friends. How many trading cards will each friend get? Are there any cards left over?

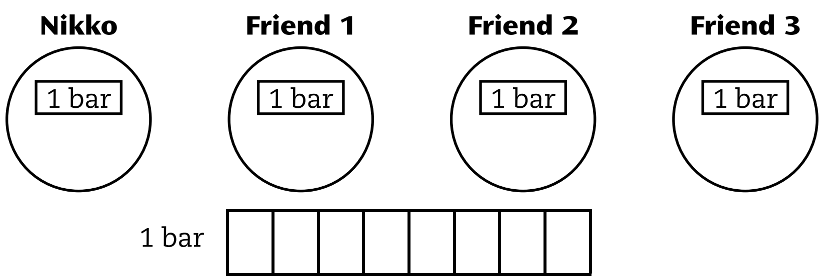


Each friend gets 4 cards. There are 3 cards leftover.

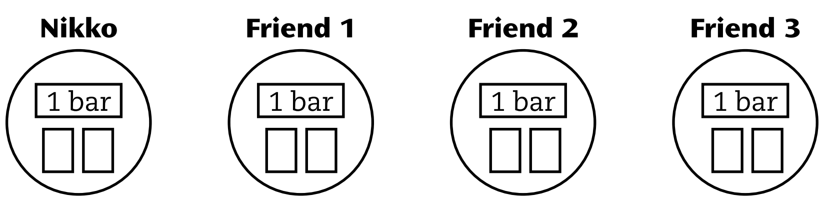
Secondary Sample

Nikko has 5 chocolate bars. Each chocolate bar can be divided into 8 parts. Nikko wants to share the chocolate evenly among himself and his 3 friends. How much chocolate does each person receive? Write a mathematical equation that could be used to check your work.

(step 1)



(step 2)



(step 3) Each person receives 1 bar and 2 pieces of chocolate. $4x = 5$

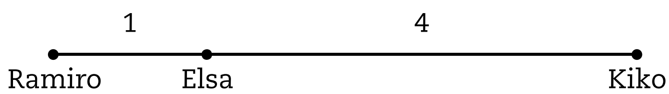
Drawing a Diagram *(cont.)*

Using a Time/Distance Line

A time/distance line helps to show distance or movement from one point to another.

Grades K–2 Sample

Ramiro, Elsa, and Kiko live on the same street. Ramiro's house is first on the street. Elsa lives 1 block down the street from Ramiro. Kiko lives 4 blocks down the street from Elsa. How many blocks is it from Ramiro's house to Kiko's house?



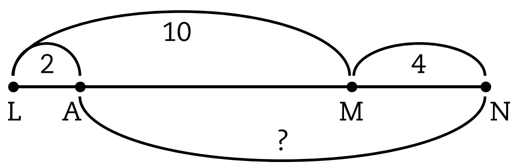
(step 1)

1 + 4 = 5 blocks

(step 2)

Grades 3–5 Sample

Lina, Marco, Nori, and Aleyah live on the same street. Lina's house is 10 blocks west of Marco's house. Nori's house is 4 blocks east of Marco's house. Aleyah's house is 2 blocks east of Lina's house. How many blocks is it from Aleyah's house to Nori's house?



(step 1)

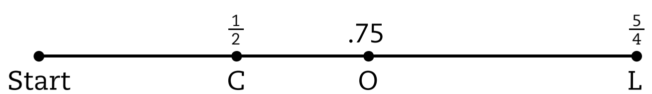
10 + 4 – 2 = 12 blocks

(step 2)

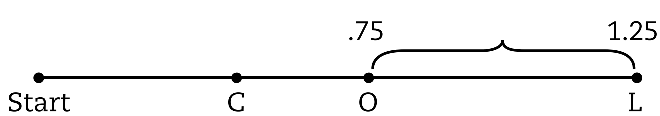
Secondary Sample

Carlos, Lilah, and Omar all had to run during P.E. Carlos ran $\frac{1}{2}$ of a mile. Omar ran .75 of a mile. Lilah ran $\frac{5}{4}$ miles. How much farther did Lilah run than Omar?

(step 1)

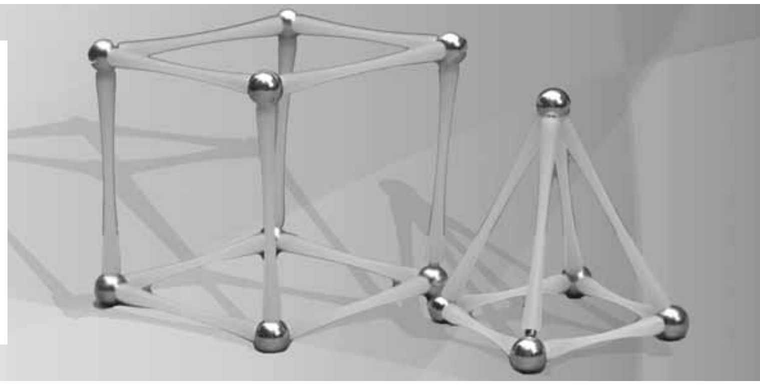


(step 2)



1.25 – .75 = .5 miles more

Acting It Out or Using Concrete Materials



Standard

- uses a variety of strategies in the problem-solving process

Background Information

This strategy is useful in helping students to gain a concrete understanding of an abstract problem. A variety of concrete objects such as counters, blocks, or beans can be used to represent the people or things in a problem. These objects can be moved or rearranged to fit the scenario or follow the steps in the problem.

There are many types of problems that can be solved using this strategy. Some of these types of problems include using amounts of money, specific quantities, position changes, and large numbers.

Procedure

Once it is decided that acting it out or using concrete materials is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide whether the problem is best solved by acting it out or by using concrete materials.
3. Gather the materials or actors.
4. Follow the steps of the problem using the materials or actors.
5. Record the solution based on the outcome of the scenario.

Samples

The following skills and concepts illustrate how acting it out or using concrete materials can be applied to many different types of problems. Students should understand and practice how to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Acting It Out or Using Concrete Materials *(cont.)*

Amounts of Money Problems

Often, problems require the use of money, giving change, or finding combinations of coins and bills that can be used to purchase items. This can be very confusing without the use of pretend money and acting out the scenarios.

Using the concrete material, pretend money, and acting out each step of the problem will help students arrive at the correct solution.

Grades K–2 Sample

Yelena had 8 quarters in her pocket. She lost 2 of the quarters while playing at recess. How much money does she have left?

Grades 3–5 Sample

Salena had a $20 bill. She exchanged the bill for two $10 bills. She took one of the $10 bills and exchanged it for two $5 bills. She took one of the $5 bills and exchanged it for five $1 bills. She took two of the $1 bills and exchanged them for 8 quarters. How many bills and coins does she have now?

Secondary Sample

Gabby works at a local toy store. The manager does not allow his employees to use calculators to give back change. A customer purchased toys totaling $13.47. The customer gave Gabby a $20 bill. How much change did the customer receive?

Specific Quantities

Sometimes quantities are given in a problem. Most often these problems can be acted out using the students in the class. However, using concrete materials may be more appropriate if the quantities are large.

Grades K–2 Sample

There were 4 birds sitting in a tree. Two of the birds flew away. Then 5 more birds came to sit in the tree. How many birds are sitting in the tree?

Grades 3–5 Sample

A city bus is traveling downtown. There are 36 people on the bus. At the first stop, one-third of the people get off of the bus and six people get on the bus. At the second stop, half of the people get off of the bus. At the third stop, eight people get off of the bus and two people get on the bus. How many people are on the bus now?

This problem has a high quantity of people and it may not be feasible to act this out using the students in class. Students can solve this problem using counters (or any other concrete material) as a model for the people on the bus.

Acting It Out or Using Concrete Materials *(cont.)*

Specific Quantities *(cont.)*

Secondary Sample

Many types of pie were served at a large holiday party. There were 10 pies at the beginning of the party. Right away, $2\frac{1}{2}$ pies were eaten because people were hungry. After dinner, half of the remaining pie was eaten for dessert. In the evening, an aunt came late to the party and brought 2 more pies. After the party, the hosts ate $\frac{1}{4}$ of a pie as a midnight snack. How much pie was left the day after the party?

Moving Positions

Problems can be confusing when the characters or objects are frequently moving around. By getting students to act out the problem as a group or by using concrete materials, the movements can be clearly visualized and organized.

Grades K–2 Sample

Lilah is walking around her backyard. She walks 10 steps forward. Then she walks 8 steps to the right. Next, she walks 6 steps backwards. Last, she walks 4 steps to the left. How many steps has she taken?

Grades 3–5 Sample

Moira is late meeting her friends at the fair. There are 40 people in line ahead of her for a ticket and she is very impatient. Each time a person in front of her is served, she cuts ahead of 3 people. How many people will be served before Moira reaches the front of the line?

Because this problem has a high quantity of people, students can use counters (or any other concrete material) as a model for the people waiting in line.

Secondary Sample

A football team started on its own 43-yard line with only a minute left in the game. They ran 5 plays before kicking a field goal to win the game. During the first possession, they gained 10 yards. The second possession, the team had a loss of 4 yards. The third possession, they gained 7 yards. The fourth possession, they had a loss of 3 yards. The fifth possession, they gained 11 yards. From which yard line did they kick the field goal?

Acting It Out or Using Concrete Materials *(cont.)*

Using Large Numbers

When a problem contains large numbers, it is often most practical for students to use concrete materials to solve it. Students can use any concrete materials, such as blocks, counters, beans, or small manipulatives, to represent the objects in the problem.

Grades K–2 Sample

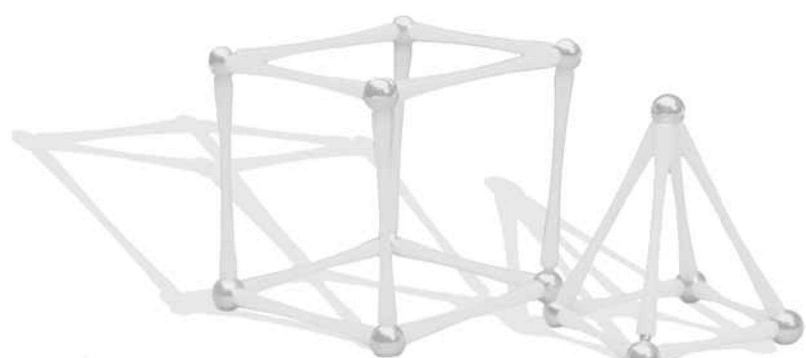
On the 100th day of school, Adrian brought in 100 cookies to share with his classmates. If there are 20 students in the class, how many cookies does each student get?

Grades 3–5 Sample

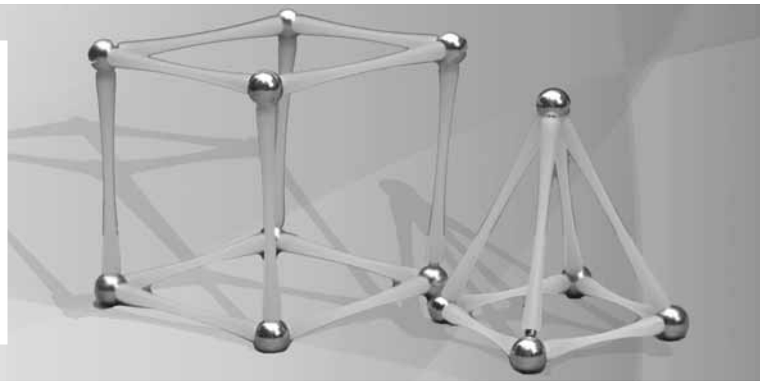
There were 60 chocolates in a row waiting to be boxed in the chocolate factory. Shen's job is to taste the chocolates to make sure they taste right. Starting with the 8th chocolate, he tastes every 9th one. How many chocolates does Shen taste?

Secondary Sample

Mrs. DeLeon has 14 students that play in a tennis tournament two times a week after school. Each student will be paired with one of the other students to play a game. The games will continue until every student has played each of the other students one time. How many games will be played in the tournament?



Creating a Table



Standard

- uses a variety of strategies in the problem-solving process

Background Information

A table often helps to organize information from a problem so that it can be easily understood and relationships can be clearly seen. This is a good strategy to use when a problem contains information with more than one characteristic. By creating a table, it is easy to see what information you have and what information is missing. Using a table can also reduce the possibility of making mistakes or repetitions.

There are many types of problems that can be solved using this strategy. Some of these types of problems include calculating multiples and following patterns.

Procedure

Once it is decided that creating a table is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what information is known in the problem.
3. Use the information from the problem to choose the number of columns needed for the table and to decide the headings for each column.
4. Use the information from the problem to choose the number of rows needed for the table.
5. Create the table.
6. Fill in the known information in the table.
7. Continue filling in information in the table until the solution to the problem is reached.
8. Check the work and record the solution.

Samples

The following skills and concepts illustrate how creating a table can be used with many different types of problems. Students should be comfortable with these skills to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Creating a Table *(cont.)*

Calculating Multiples

Sometimes problems use multiples of numbers. When this information is put into a table, the pattern can be quickly seen. Then, the information can be easily used to solve the problem.

Grades K–2 Sample

Mr. Lin's class is making holiday cards to give to people at the local retirement home. Each day, the class makes 5 cards. How many cards will they have after 10 days?

Day	1	2	3	4	5	6	7	8	9	10
Cards	5	10	15	20	25	30	35	40	45	50

Grades 3–5 Sample

Maria and Celine planted a flower garden at the school. On Monday, they each planted two flowers. Every day, Maria and Celine each planted twice as many flowers as they did the day before. How many flowers did the girls plant during the week?

	Monday	Tues.	Wed.	Thur.	Fri.	Total Flowers Planted
Number of Flowers (Maria)	2	4	8	16	32	62
Number of Flowers (Celine)	2	4	8	16	32	62
Total Flowers Planted	4	8	16	32	64	124

Secondary Sample

Brody's dad told him that he could choose how he could get his allowance. With the first option, he could receive $17.00 each month. With the second option, he would get $0.57 in January, and the amount would double each month after that. Which option do you think is the better deal? Create a table showing how much he would receive each month with both options. Calculate the total with each option.

Option 1 $17.00 × 12 = $204.00

Option 2 (see chart below)

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
Amount	$0.57	$1.14	$2.28	$4.56	$9.12	$18.24	$36.48	$72.96	$145.92	$291.84	$583.60	$1,167.36

Total: $2,334.07

Creating a Table *(cont.)*

Following Patterns

Once a table is created, numbers are filled in using the information from the problem. Sometimes a pattern can be found once the numbers are inserted in a table. The pattern can be used to solve the problem.

Grades K–2 Sample

Ms. Medina's class went on a field trip to the farm. They counted the chickens in 4 different pens. In the first pen, there were 3 chickens. In the second pen, there were 4 chickens. In the third pen, there were 5 chickens. In the fourth pen, there were 6 chickens. Each chicken has 2 feet. How many total chicken feet were there in each of the pens?

Number of Chickens	3	4	5	6
Number of Feet	6	8	10	12

Grades 3–5 Sample

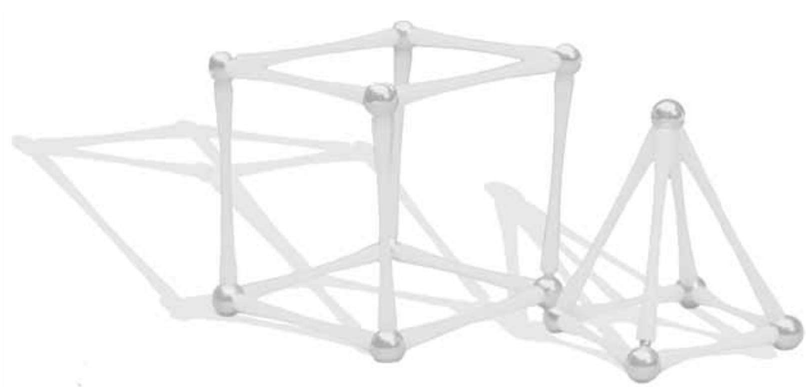
One morning the temperature was 6° celsius at 8:00 A.M. The students recorded the temperature every hour. At 9:00 A.M., the temperature was 8° celsius. At 10:00 A.M., the temperature was 12° celsius. At 11:00 A.M., the temperature was 18° celsius. If this pattern continues, what will the temperature be at 2:00 P.M.?

Time	8:00	9:00	10:00	11:00	12:00	1:00	2:00
Temperature	6°	8°	12°	18°	26°	36°	48°

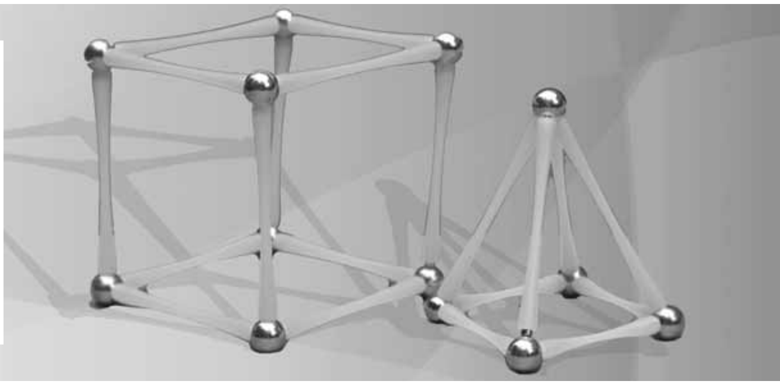
Secondary Sample

Andrew and Michael are trying to break their trampoline-jumping record from last year. The boys take turns jumping for an hour. Michael consistently makes 15% fewer jumps than Andrew does. Andrew increases the number of jumps he makes in an hour by 15 each time he jumps. During Andrew's first hour, he jumps 150 times. Based on this, how many jumps did Andrew make during his fifth hour on the trampoline?

Hour	1	2	3	4	5
Jumps	150	165	180	195	210



Looking for a Pattern



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Since mathematical patterns can be found in numbers, shapes, nature, and the world around us, looking for a pattern is one of the most frequently used problem-solving strategies. This strategy is an extension of the Creating a Table strategy and the Creating an Organized List strategy. Once information is organized, it is easier to see if a pattern exists and find the "rule."

There are several ways to check for a pattern in a problem and determine its rule:

- Determine if the numbers are increasing or decreasing by a regular sequence.
- Determine the difference between two consecutive numbers.
- Determine whether the numbers have been divided or multiplied by any given number.

Once students have determined the pattern in a problem, they should be able to continue the pattern to find a given unit within it.

There are many types of problems that can be solved using this strategy. Some of those types of problems include using spatial patterns and using tables.

Procedure

Once it is decided that looking for a pattern is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Organize the information in the problem into a table or a list. This will help students see how the information increases or decreases.
3. Look for a repeated sequence in the pattern to determine the rule for the pattern. If a repeated sequence is found, then the rule has been found.
4. If no repeated sequence exists, find the difference between the first two numbers in the pattern. Calculate the difference between other numbers in the pattern to see if it is the same. If it is the same, than the "rule" has been found.

Looking for a Pattern *(cont.)*

Procedure *(cont.)*

5. If no uniform pattern exists, look for a pattern using multiplication or division.
6. If no multiplication or division pattern exists, look for an increasing or decreasing pattern (e.g., +1, +2, +3, etc.).
7. Once the pattern is determined, reread the problem to see what information is needed from the pattern.
8. Check the work and record the solution.

Samples

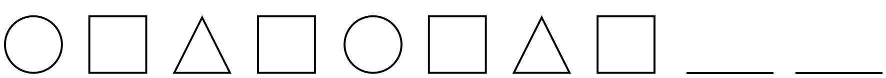
The following skills and concepts illustrate how looking for a pattern can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

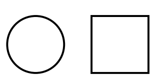
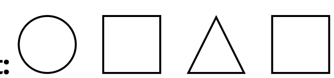
Spatial Patterns

Spatial patterns involve shapes and patterns, textures, or designs within shapes. These types of patterns are most used in elementary school, but can be appropriate for secondary-level students as functions are introduced. Spatial patterns can be repetitive or growing, depending on the ability levels of the students.

Grades K–2 Sample

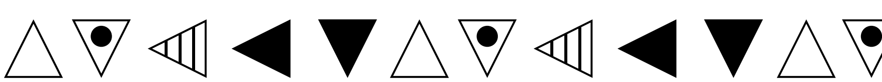
What are the next 2 shapes in this pattern? What is the pattern unit?



next shapes:  **pattern unit:** 

Grades 3–5 Sample

Continue this pattern to find the 24th shape. Create a pattern of your own that is similar to this one.



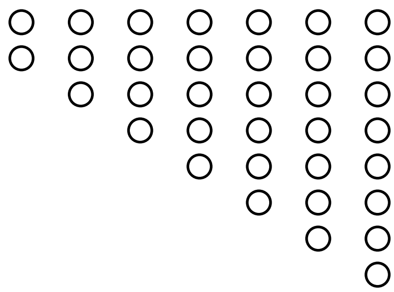
24th shape:

Looking for a Pattern *(cont.)*

Spatial Patterns *(cont.)*

Secondary Sample

Look at the pattern below. How many circles would be in the 50th unit in this pattern? What equation did you use to solve this problem?



50th unit has 51 circles

$x + 1$

Finding a Pattern in a Table

Often, students are given a table of data or information that can be used to fill in the missing pieces of that table. Students must analyze the information in the rows and columns and try different strategies (finding the difference, finding the growth using multiplication) for finding the pattern within the numbers. Once the pattern is found, the missing information can be calculated and recorded in the proper places within the table.

Grades K–2 Sample

Rory has a garden in his backyard. He picks tomatoes every day. On Monday he picked 2 tomatoes. On Tuesday, he picked 4 tomatoes. On Wednesday, he picked 6 tomatoes. If this pattern continues, how many tomatoes will he pick on Friday?

Day of the Week	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Tomatoes Picked	2	4	6	8	**10**

Grades 3–5 Sample

Each school day Ernesto saves 2 cookies from his lunch. After 10 school days, how many total cookies will he have saved?

Day	1	2	3	4	5	6	7	8	9	10
Cookies Saved	2	2	2	2	2	2	2	2	2	2
Total Cookies Saved	2	4	6	8	10	12	14	16	18	**20**

Looking for a Pattern *(cont.)*

Finding a Pattern in a Table *(cont.)*

Secondary Sample

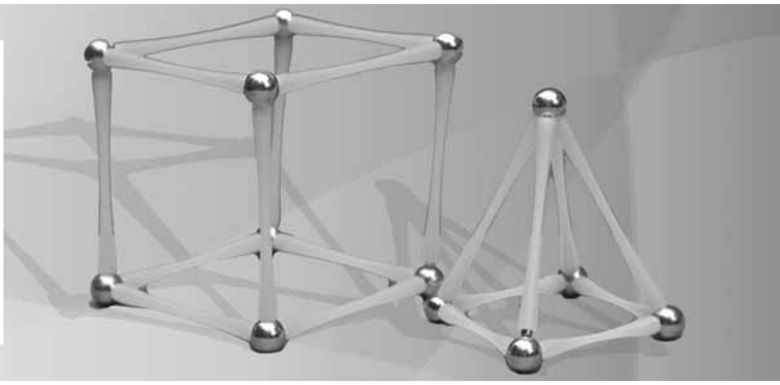
For Cho's birthday, her grandfather wants to give her money to buy some new clothes. He has agreed to give her money each day during the month of her birthday, but the amount depends on a special rule. He has decided to double the amount of money he gives her each day for 15 days, starting with $0.01 on the first day. He will only give Cho the money if she can accurately tell him how much total money she should receive from him by following this rule. How much money will Cho receive in total after the 15 days are up?

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Amount Received	$0.01	$0.02	$0.04	$0.08	$0.16	$0.32	$0.64	$1.28	$2.56	$5.12	$10.24	$20.48	$40.96	$81.92	$163.84
Total	$0.01	$0.03	$0.07	$0.15	$0.31	$0.63	$1.27	$2.55	$5.11	$10.23	$20.47	$40.95	$81.91	$163.83	**$327.67**

Guessing and Checking

Standard

- uses a variety of strategies in the problem-solving process



Background Information

Guessing and checking is a strategy that is often overlooked by teachers. So often students are told that guessing is not a good strategy. What students need to understand is that *wild* and *random* guessing is not a good strategy. Making an *educated guess* about a solution to a problem is a valuable strategy and is a good tool to use even outside of mathematics.

In this strategy, students make an educated guess as to the solution of the problem based on the information they are given. Then they check their guess against the conditions of the problem, evaluate the results, and make another guess according to the results of the previous guess. This process is repeated until the correct solution is found.

There are many types of problems that can be solved using this strategy. Some of these types of problems include using objects and using time or distance.

Procedure

Once it is decided that guessing and checking is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Note the important facts in the problem and determine the exact problem that needs to be solved.
3. Help students create a table in which to record their guesses. Tell them that a table is a good way to keep their guesses organized.
4. All students should then make an initial guess. Remind them that their guesses should be reasonable and based on the important information from the problem.
5. Record the solution to the first guess. Have students evaluate the answer. They should ask themselves if their answer correctly solves the problem.
6. If their answers do not correctly solve the problem, have them look at their original guess again. Based on their answer, they should decide whether their second guess should be higher or lower than their first guess.

Guessing and Checking *(cont.)*

Procedure *(cont.)*

7. Students should repeat steps 4–6 until the correct answer is found.
8. Check the work and record the final answer.

Samples

The following skills and concepts illustrate how guessing and checking can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Using Objects

Grades K–2 Sample

Carlos and Ami have each lost some teeth. Together, they have lost 10 teeth. Ami has lost 2 more teeth than Carlos. How many teeth has Carlos lost? How many teeth has Ami lost?

Guess	1	2	3	4	5	…
Carlos						
Ami						
Total Teeth						

Grades 3–5 Sample

Kiara, Dimitri, and Ravi have each made a photo book of their friends. Together they have 260 photos. Dimitri has 25 more photos than Ravi. Kiara has 30 more photos than Dimitri. How many photos are in each person's photo book?

Guess	1	2	3	4	5	…
Kiara						
Dimitri						
Ravi						
Total Photos						

Secondary Sample

Ling has $6.00 in dimes, nickels, and quarters in her bank. She has double the number of dimes than nickels. The number of quarters is one-fourth the number of dimes. How many of each does she have in her bank?

Guess	1	2	3	4	5	…
Dimes						
Nickels						
Quarters						
Total						

Guessing and Checking *(cont.)*

Using Time/Distance

Grades K–2 Sample

Nina, Ana, and Tess are sisters. Nina is the youngest. Ana is 2 years older than Nina. Tess is 5 years older than Ana. Their age totals 30. What are their ages?

Guess	1	2	3	4	5	…
Nina						
Ana						
Tess						
Total Age						

Grades 3–5 Sample

The Granger family is driving to their grandparents' house for the holidays. Their grandparents' house is 1,500 kilometers away, and they decide to split the drive into 4 days so that they can sightsee along the way. Each day they travel 20 kilometers more than the previous day. How many kilometers do they travel each day?

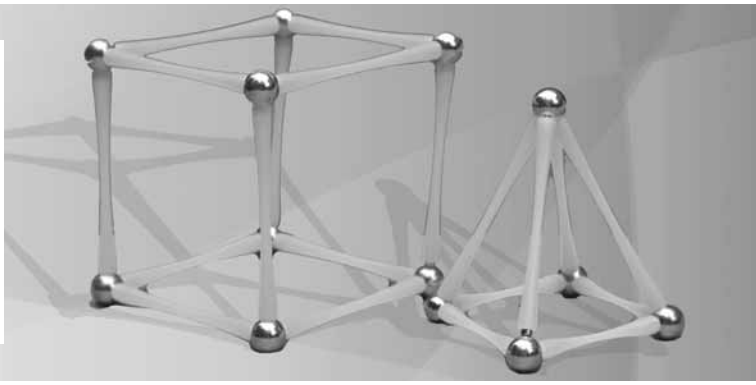
Guess	1	2	3	4	5	…
Day 1						
Day 2						
Day 3						
Day 4						
Total Distance						

Secondary Sample

Lilah, Nihal, and Kiko ran a mile in P.E. They finished with a combined time of 32.5 minutes. Lilah finished in half the time that Kiko finished. Nihal finished in two-thirds the time that Kiko finished. How long did it take each of them to run a mile?

Guess	1	2	3	4	5	…
Lilah						
Nihal						
Kiko						
Total Time						

Creating an Organized List



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Creating an organized list is a similar strategy to creating a table. Often, this strategy is used instead of a table when there is more information in the problem. This strategy is different than using a table because it allows the problem solver to arrange the information more systematically so that the answer is clear. Students should follow a sequence or procedure to make sure that all possibilities are tested or found. Creating a list ensures that no information is duplicated.

There are many types of problems that can be solved using this strategy. Some of those types of problems include sequencing and number combinations.

Procedure

Once it is decided that creating an organized list is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what information is known in the problem.
3. Decide what information should be kept the same as you work through the problem.
4. Help students create combinations and work sequentially to list the possible solutions to the problem.
5. Reread the problem and look back at the list to make sure no information is missing or repeated.
6. Record the solution.

Samples

The following skills and concepts illustrate how creating an organized list can be used with many different types of problems. Students should be comfortable with these skills to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Creating an Organized List *(cont.)*

Sequencing

Often, lists need to be completed in a specific order so that all of the combinations or pieces are written down. Sequencing items in the correct order guarantees that no information is omitted.

Grades K–2 Sample

Marisol has three picture frames. One frame is yellow, one frame is blue, and one frame is green. She wants to put them on top of the bookcase in her room. How many different ways could she put them on top of the bookcase?

Y B G	B G Y	G B Y	6 different ways
Y G B	B Y G	G Y B	

Grades 3–5 Sample

Tia has two choices for ice cream: chocolate and vanilla. She has three toppings: peanuts, fudge, and sprinkles. How many combinations of ice cream and toppings can she choose from?

chocolate—peanuts	vanilla—peanuts	
chocolate—fudge	vanilla—fudge	6 different combinations
chocolate—sprinkles	vanilla—sprinkles	

Secondary Sample

The basketball team is getting new uniforms. They can choose from three colors for the jerseys: red, blue, or silver. They can choose from two colors for the shorts: black or white. They can choose from two colors, black or white, for the numbers on the jerseys. How many combinations can be made for the new uniforms?

Jerseys	Shorts	Numbers	
red	black	black	
red	black	white	
red	white	black	
red	white	white	
blue	black	black	
blue	black	white	12 uniform combinations
blue	white	black	
blue	white	white	
silver	black	black	
silver	black	white	
silver	white	black	
silver	white	white	

Creating an Organized List *(cont.)*

Number Combinations

Sometimes problems call for students to put numbers together in specific combinations. It is also important to make sure that this is done systematically so that nothing is repeated or omitted.

Grades K–2 Sample

How many three-digit numbers can be made using the numerals 4, 8, and 2?

482	824	284
428	842	248

6 different numbers

Grades 3–5 Sample

How many addition problems can you create using the digits 7, 2, 3, and 6, with a sum between 0 and 100?

23	23	~~32~~	32	27	27	~~72~~	~~72~~	26	26	~~62~~
+ 76	+ 67	~~+ 76~~	+ 67	+ 36	+ 63	~~+ 36~~	~~+ 63~~	+ 73	+ 37	~~+ 73~~
99	90	~~108~~	99	63	90	~~108~~	~~135~~	99	63	~~135~~

62	37	37	73	~~73~~	~~36~~	36	~~63~~	63
+ 37	+ 26	+ 62	+ 26	~~+ 62~~	~~+ 72~~	+ 27	~~+ 72~~	+ 27
99	63	99	99	~~135~~	~~108~~	63	~~135~~	90

13 problems

Secondary Sample

How many problems using addition, subtraction, multiplication, or division can be created using the numbers, 3.5, $\frac{1}{4}$, 5.75, $2\frac{2}{5}$, with an answer between 3 and 10?

$3.5 + \frac{1}{4} = 3.75$
$3.5 + 5.75 = 9.25$
$3.5 + 2\frac{2}{5} = 5.9$
$3.5 - \frac{1}{4} = 3.25$
~~$3.5 - 5.75 = -2.25$~~
~~$3.5 - 2\frac{2}{5} = 1.1$~~
~~$3.5 \times \frac{1}{4} = .875$~~
~~$3.5 \times 5.75 = 20.125$~~
$3.5 \times 2\frac{2}{5} = 8.4$
~~$3.5 \div \frac{1}{4} = 14$~~
~~$\frac{3}{5} \div 5.75 = .6$~~
~~$3.5 \div 2\frac{2}{5} = 1.45$~~

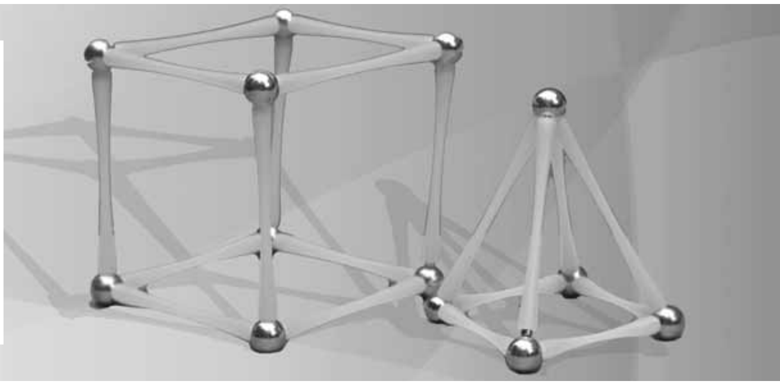
$\frac{1}{4} + 5.75 = 6$
~~$\frac{1}{4} + 2\frac{2}{5} = 2$ 13/20~~
$\frac{1}{4} - 3.5 = 3.25$
~~$\frac{1}{4} - 5.75 = -5.5$~~
~~$\frac{1}{4} - 2\frac{2}{5} = -2\frac{3}{20}$~~
~~$\frac{1}{4} \times 5.75 = 1.44$~~
~~$\frac{1}{4} \times 2\frac{2}{5} = \frac{12}{20}$~~
~~$\frac{1}{4} \div 3.5 = .07$~~
~~$\frac{1}{4} \div 5.75 = .04$~~
~~$\frac{1}{4} \div 2\frac{2}{5} = \frac{5}{48}$~~

$5.75 + 2\frac{2}{5} = 8.15$
~~$5.75 - 3.5 = 2.25$~~
$5.75 - \frac{1}{4} = 5.5$
$5.75 - 2\frac{2}{5} = 3.35$
~~$5.75 \times 2\frac{2}{5} = 13.8$~~
~~$5.75 \div 3.5 = 1.6$~~
~~$5.75 \div \frac{1}{4} = 23$~~
~~$5.75 \div 2\frac{2}{5} = 2.4$~~

~~$2\frac{2}{5} - 3.5 = -1.1$~~
~~$2\frac{2}{5} - \frac{1}{4} = 2\frac{3}{20}$~~
~~$2\frac{2}{5} - 5.75 = -3.35$~~
$2\frac{2}{5} \div 3.5 = .7$
~~$2\frac{2}{5} \div \frac{1}{4} = 9\frac{3}{5}$~~
~~$2\frac{2}{5} \div 5.75 = .4$~~

11 problems

Working Backwards



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Working backwards is a strategy to use for problems that contain linked information, where some of the information has not been provided. Usually, the missing information is at the beginning of the problem. Solving problems using this strategy requires that the problem solver start at the end of the problem and work methodically backwards until the missing information is found. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these types of problems include using the opposite operation and starting with the answer.

Procedure

Once it is decided that working backwards is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Determine what the last piece of information is in the problem. Remind students that this may be the answer in the problem.
3. Follow the steps of the problem backwards. Remind students that they may need to use the opposite (inverse) operation.
4. Check to see that the missing information in the question matches the information the students ended with.
5. Check the solution by working forward through the steps of the problem.
6. Record the solution.

Samples

The following skills and concepts illustrate how working backwards can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Working Backwards *(cont.)*

Using the Opposite Operation

Often, it is necessary to use the opposite operation when making calculations. Students need to understand how addition, subtraction, multiplication, and division are related in order to use this strategy and solve these problems correctly. Fact families are a good foundation for students before beginning this problem-solving strategy.

Grades 3–5 Sample

Taj picked some apples at the orchard during the class field trip. Sasha picked 3 more than Taj. Ernesto picked 8 less than Sasha. Jun picked 2 less than Ernesto. Ernesto picked 7 apples. How many apples did Taj pick?

Ernesto picked 7 apples.	7	
Jun picked 2 less than Ernesto.	7 – 2 = 5	The opposite operation is not needed here. Jun picked 5 apples.
Ernesto picked 8 less than Sasha.	8 + 7 = 15	The opposite operation is needed here. Sasha picked 15 apples.
Sasha picked 3 more than Taj.	15 – 3 = 12	The opposite operation is needed here. Taj picked 12 apples.

Secondary Sample

Omar has some baseball cards to trade. Amadi has 2 more than 2 times the number of baseball cards Omar has. Sean has 2 less than Amadi. Dakota has 4 less than 2 times the number of cards Sean has. Sean has 8 cards. How many cards does Omar have to trade?

Sean has 8 cards.	8	
Dakota has 4 less than 2 times the number Sean has.	(8 × 2) – 4 = 12	The opposite operation is not needed here. Dakota has 12 cards.
Sean has 2 less than Amadi.	8 + 2 = 10	The opposite operation is needed here. Amadi has 10 cards.
Amadi has 2 more than 2 times the number Omar has.	(10 – 2) ÷ 2 = 4	The opposite operation is needed here. Omar has 4 cards.

Working Backwards *(cont.)*

Starting with the Answer

When the final answer is given in a problem, students can work backwards to find the information that is missing from the beginning of the problem.

Grades 3–5 Sample

Melissa is writing a paper on the mathematician John Venn. On Monday, she wrote 123 sentences about his biographical facts. On Tuesday, she wrote 96 sentences about his contributions as a mathematician. On Wednesday, she wrote 82 sentences about his anecdotes. She has to write a total of 400 sentences to complete her report. How many more sentences does she have to write?

400 – 82 – 96 – 123 = 99 more sentences

Secondary Sample

Addie started reading a library book on Monday. The book is 110 pages. On Monday, she read 20 pages. On Tuesday, she read half the number of pages she did on Monday. On Wednesday, she read 12 pages. On Thursday, she read 4 more pages than she read on Tuesday. On Friday, she read the sum of the number of pages she read on Tuesday and Wednesday. On Saturday, she read half the sum of the number of pages she read on Thursday and Friday. Today is Sunday. How many pages does she have left to read?

Monday: 20

Tuesday: 10

Wednesday: 12

Thursday: 14

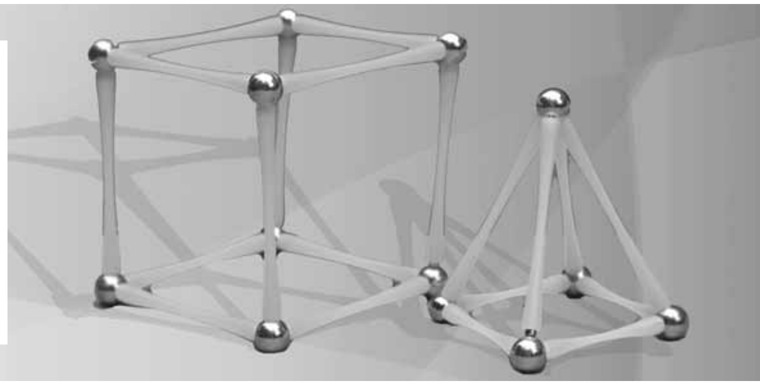
Friday: 22

Saturday: 18

Sunday: ??

110 – 20 – 10 – 12 – 14 – 22 – 18 = 14 pages to read on Sunday

Creating a Tree Diagram



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Creating a tree diagram is a very visual problem-solving strategy. It is named after the way in which the information is arranged—it looks like the branches on a tree. This strategy is used to represent relationships or to show combinations of different factors in a problem. Like other problem-solving strategies, it is designed to keep the information organized in a systematic way so that parts are not left out or repeated. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these types of problems include ordering and finding combinations.

Procedure

Once it is decided that creating a tree diagram is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide what information is known in the problem.
3. Decide what information should be kept the same as you work through the problem.
4. Help students develop the combinations, working sequentially through the problem to create the tree diagram.
5. Reread the problem and review the diagram to note any missing or repeated information.
6. Record the solution.

Samples

The following skills and concepts illustrate how creating a tree diagram can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

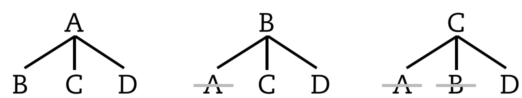
Creating a Tree Diagram *(cont.)*

Ordering

Sometimes the order of the items in a problem is key to finding the solution. Depending on the problem, there may be some possible combinations that cannot be counted as solutions. It is important to show students how to find all possible combinations first, and then cross out solutions that do not work within the parameters of the problem.

Grades 3–5 Sample

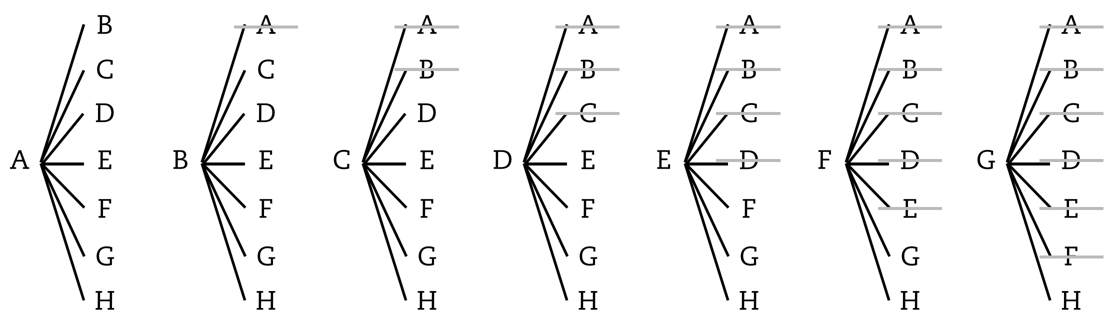
You and three of your friends are playing in a tennis tournament. Each of the players must play against each other one time. How many games will be played?



6 games played

Secondary Sample

You and seven of your friends are playing in a golf tournament. Each of the players must play against each other one time. How many games will be played?



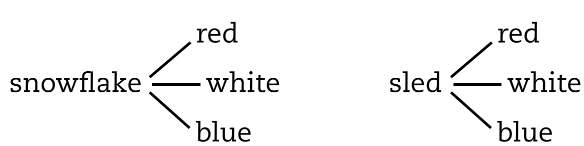
28 games played

Finding Combinations

Often the order in which the pieces of a problem are combined is not the most important factor. The solution comes when the total number of combinations is found. To do this, students must work methodically to make sure that they do not omit or duplicate combinations.

Grades 3–5 Sample

Mrs. Phillips's 4th grade class is making sugar cookies for the winter party. They will be making snowflakes and sleds with red, white, or blue icing. List all the possible combinations. If Mrs. Phillips's class wants to serve only the cookies with red and blue icing, which combination of cookies will they have for the party?



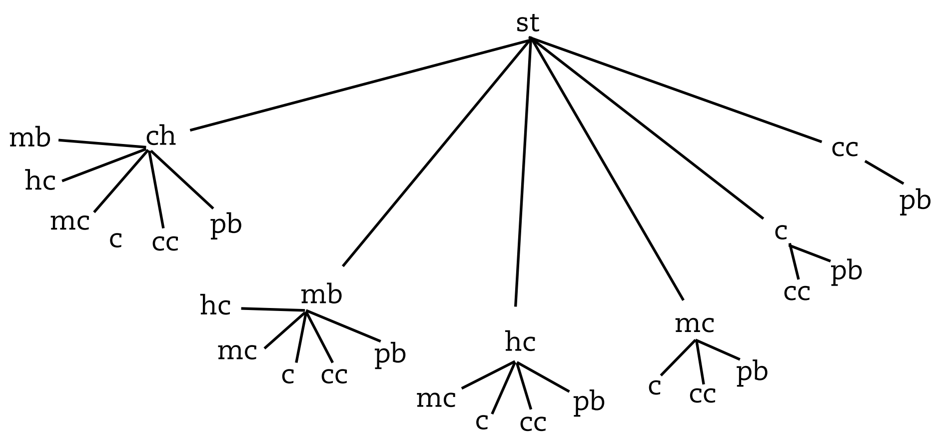
4 combinations for the party

Creating a Tree Diagram *(cont.)*

Finding Combinations *(cont.)*

Secondary Sample

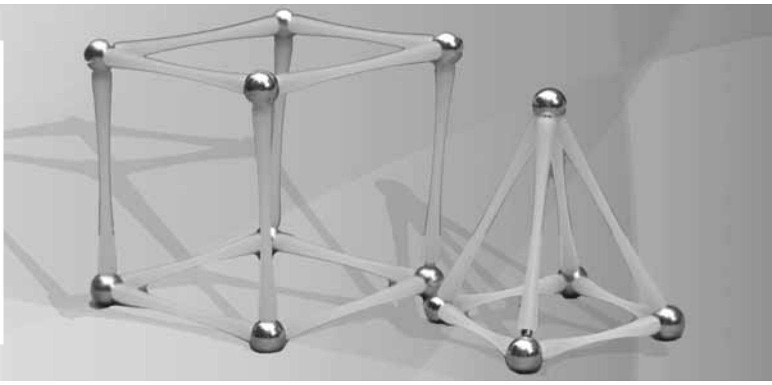
The soccer team went to an ice cream stand after its game. The flavor choices were strawberry (st), cherry (ch), mixed berry (mb), hazelnut chocolate (hc), mint chocolate (mc), coconut (c), chocolate chip (cc), and peanut butter (pb). The players each got 3 scoops of ice cream, with all 3 scoops being different flavors. List all the possible combinations if order does not matter. If Adam cannot have chocolate, how many possible combinations can he choose from?



21 different combinations

Adam can choose from 6 different combinations.

Using Simpler Numbers



Standard

- uses a variety of strategies in the problem-solving process

Background Information

Using simpler numbers is a useful strategy for solving difficult problems. In this strategy, students begin by solving the problem using an easier set of information. When the process of how to solve the problem is understood, students can solve the original problem using the scenario provided with the problem.

This problem-solving strategy can be combined with other strategies in order to understand the best solution to the more difficult problem. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these include spatial and numerical problems.

Procedure

Once it is decided that using simpler numbers is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Simplify the original problem using numbers that are easier for the students.
3. Complete the problem using the simpler numbers.
4. Help students reflect on the operations, the pattern, or the process used to solve the problem with the simpler numbers.
5. If students do not understand the general process for solving the problem, choose a different set of simpler numbers and solve the problem again. Reflect on the operations, the pattern, or the process used to solve this problem.
6. Once students can make generalizations about the process for solving the original problem, allow them to solve it using the original numbers/scenario.
7. Check the work and record the solution.

Using Simpler Numbers *(cont.)*

Samples

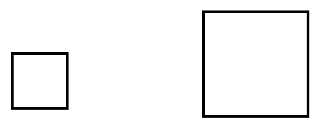
The following skills and concepts illustrate how simpler numbers can be used with many different types of problems. Students should be comfortable with them in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Spatial

Often students are asked questions about geometric designs and patterns. To solve these types of problems, students must work out the problem using a piece of the larger set or design. Then they can replicate that answer as many times as needed to solve the original problem.

Grades 3–5 Sample

What is the area and perimeter for an 8 × 8 square?



Area = 1
Perimeter = 4

Area = 4
Perimeter = 8

Secondary Sample

Your school will use triangular tables for the end-of-year school dance. Each edge of the table can seat one person. When putting tables together, each must share one side with the other. Predict how many people can sit at 50 tables.

Number of small tables	Number that can sit
1	3
2	4
3	5
4	6
5	7
6	8
7	9

Using Simpler Numbers *(cont.)*

Numerical

When problems involve difficult numbers, students often shy away from trying to solve them. Students can use the simpler numbers strategy to gain confidence in solving the problem without allowing the actual numbers to interfere with their reasoning process. Once students understand the process used for solving the problem, they can solve the original problem using the data given in that problem.

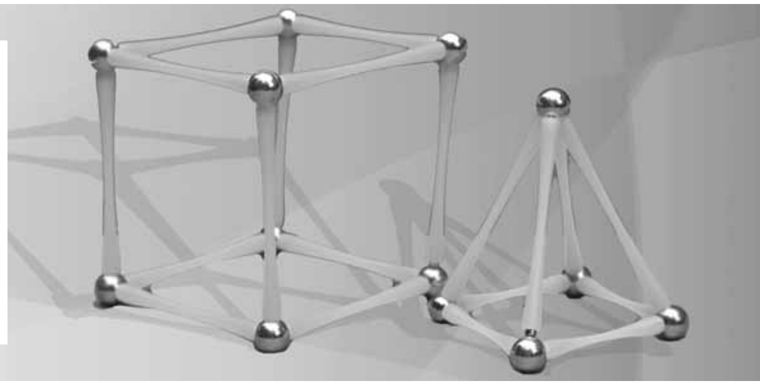
Grades 3–5 Sample

Fenway Park is home to the Boston Red Sox. As of 2005, it is the oldest American League ballpark still in operation. The park can hold 37,198 attendants. The park's sections are not numbered in numerical order. Instead, the sections are numbered 1–43, 86–97, 112, 129, 136, 148, 159, and 165. The average number of seats per section is 610. The attendants that have the digit 6 in their section number will receive an autographed baseball from the Boston Red Sox. How many sections will receive a baseball?

Secondary Sample

If it takes 6 cans of cat food to feed 5 cats for one day, how many cats can you feed for 5 days with 60 cans of cat food?

Using Logical Reasoning



Standard

- uses a variety of strategies in the problem-solving process

Background Information

On the surface, this strategy seems much like guessing and checking. However, this strategy is different because students are not making guesses to begin their problem-solving process. Instead, all of the pieces of the problem fit together like a puzzle. The logical placement of each piece of information is important in solving the problem.

Sometimes, problem solvers will have to use the information in the problem in a different order than it was presented. Inferences may also need to be made based on the information from the problem. This strategy is best used with students in grade 3 or higher.

There are many types of problems that can be solved using this strategy. Some of these types of problems include determining order and determining preferences.

Procedure

Once it is decided that using logical reasoning is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Decide on the best way to organize the information. Tell students that often a grid is needed. (It may be necessary to make grid headings or label a particular piece of the grid according to the information in the problem.)
3. Determine which information is clearly known in the problem. Record that information in the grid. Remind students that other boxes in the rows or columns on the grid may be marked based on confirmed information.
4. As information is used, have students place a line through that sentence in the problem or place a check on the first word in that sentence. This will help students keep track of the information they have recorded.
5. Examine the information that is not explicitly stated in the problem. Refer to the information on the grid when determining what inferred data should be recorded. Search each sentence for this type of data in order to complete the grid.
6. Reread the problem to make sure that the results on the grid match the information on the grid.
7. Record the final solution and answer the question from the original problem.

Using Logical Reasoning *(cont.)*

Samples

The following skills and concepts illustrate how logical reasoning can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Determining Order

Often problems will ask students to determine the order of events or objects. Placing this information in a row or grid will help students solve the problem correctly.

Grades 3–5 Sample

You are helping an engineer assemble train cars. The engineer has given you some guidelines on how to connect the cars. Using the guidelines given, connect the cars correctly.

- The types of train cars are locomotive, boxcar, hopper car, flatcar, tank car, and caboose.
- The locomotive must be first.
- The caboose must be last.
- The boxcar must be next to the caboose.
- The hopper car cannot connect to the boxcar.
- The flatcar and tank car must connect.

Secondary Sample

Kwan needs to plan his school schedule for next year. He must schedule Science from 10:00 A.M.–12:00 P.M., Study Hall at 10:00 A.M., and Gym at 1:00 P.M.

A course must be taken at the same time each day it is offered. For example, the Social Studies class can be taken from 8:00–1:00 on Tuesdays and Thursdays or from 10:00–12:00 on Tuesday and Thursdays. It cannot be taken from 8:00–10:00 on Tuesdays and from 10:00–12:00 on Thursdays.

Course	Hours Weekly	Day and Time Offered
Science	4	T/Th 10:00–12:00 or 1:00–3:00
Math	5	Daily 8:00, 9:00, 10:00, or 1:00
Study Hall	3	M/W/F 10:00 or 1:00
Language Arts	5	Daily 8:00, 10:00, 1:00, or 2:00
Lunch	5	Daily 11:00, 12:00, or 1:00
Social Studies	4	T/Th 8:00–10:00 or 10:00–12:00
Gym	3	M/W/F 8:00, 10:00, or 1:00
Computer Lab	3	M/W/F 9:00, 11:00, or 2:00
Music	2	T/Th 8:00, 10:00, 12:00, 1:00, or 2:00

Using Logical Reasoning *(cont.)*

Determining Preferences

Many logic problems discuss people's preferences for things like color, food, clothing, vehicles, and activities. These types of problems are best organized in a grid.

Grades 3–5 Sample

Antonio, Jina, Taj, and Kendra love fruit. They each have a favorite fruit: apples, oranges, strawberries, or kiwi. Use the following clues to match each with his or her favorite fruit.

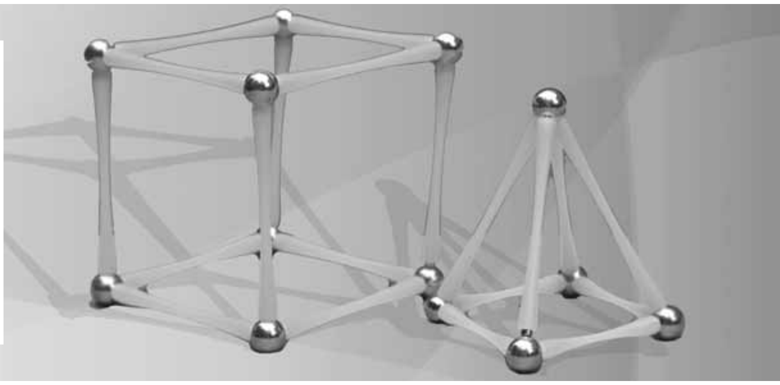
- Neither of the girls likes apples.
- Only one person's favorite fruit starts with the same letter as his or her name.
- The color of Jina's favorite fruit is also her favorite color.
- The boy with the shortest name does not like apples.
- Jina wore her favorite color for picture day—red.

Secondary Sample

Rosa, Kiara, and Ashley each have a favorite sport. Use the following clues to find out which sport—basketball, tennis, or baseball—is each girl's favorite sport.

- Rosa's favorite sport is played on a court.
- Kiara's favorite sport needs equipment to hit the ball.
- Ashley's favorite sport has five players on a team.

Analyzing and Investigating



Standard

- Uses a variety of strategies in the problem-solving process

Background Information

This problem-solving strategy is an important tool for discovering solutions to real-life problems. In this strategy, students are encouraged to investigate solutions with peers and have collaborative discussions about the solution process.

Analyzing and investigating encourages students to analyze the problem, plan a strategy for solving the problem, gather needed materials and information, and complete their investigation by solving the problem. It is especially effective when students pool their knowledge and use their unique skills to help group members solve the problem. This strategy is best used with students in grade 3 or higher.

There are many skills that students will need to acquire in order to solve problems using this strategy. Some of these skills include: estimation, mental calculations, and planning and gathering information.

Procedure

Once it is decided that analyzing and investigating is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Divide the students into small groups. Have students reread the problem.
2. In groups, allow students to plan the approach they would like to use to solve the problem.
3. Students should examine the problem for important information, discussing everyone's ideas during their planning stage.
4. Allow groups to proceed with their investigations.
5. After the investigations, discuss the process used with each group separately. If the students' solution is incorrect, help them revise previous plans and construct a different approach to solving the problem. If the solution is correct, help the group reflect on its success, asking questions to break down the thought processes they used.
6. Discuss all of the group investigations as a class.

Analyzing and Investigating *(cont.)*

Samples

The following skills and concepts illustrate how analyzing and investigating can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Estimation

Students need to realize that initial estimates can help them plan approaches and, ultimately, find solutions to problems. Students need to realize that estimates can be used to check progress of solutions and can help them see when they have made mistakes in their calculations. To do this, students must practice simple estimation activities so that they are prepared for problem-solving scenarios.

Grades 3–5 Sample

Provide students with concrete objects and have them estimate the lengths, widths, heights, etc., of the objects. Then measure the actual objects and lead discussions about the differences between their estimations and the actual measurements.

Also ask students simple calculation problems, such as the sum of 738 and 592, that require them to round and estimate to find the answer. If students can get in the habit of estimating solutions before they do the actual calculations, this will help them gauge their answers to problems later on.

Secondary Sample

Provide students with concrete objects and have them estimate the lengths, widths, heights, etc., of the objects. Then measure the actual objects and lead discussions about the differences between their estimations and the actual measurements.

Also ask students simple calculation problems, such as the average of five different numbers, that require them to round and estimate the answer. If students can get in the habit of estimating solutions before they do the actual calculations, this will help them gauge their answers to problems later on.

Analyzing and Investigating *(cont.)*

Quick Mental Computation

Quick mental computation is another method to help students hypothesize about the outcome of a problem. Using rounded numbers, doubles, multiples, or factors to estimate a solution develops a better understanding for the actual solution and steps of the problem.

Grades 3–5 Sample

Verbally provide these students with numbers to round or double without using pencils and paper or calculators. It is also a good idea to write a sequence of numbers on the board or overhead and have students mentally compute the pattern. Make sure that the numbers and patterns are easier when first practicing this skill with students. This will give them confidence and will help them become quicker as time goes on.

Secondary Sample

Verbally provide these students with numbers to round, double, factor, or find multiples of without using pencils and paper or calculators. It is also a good idea to write a sequence of numbers on the board or overhead and have students mentally compute the pattern. Make sure that the numbers and patterns are easier when first practicing this skill with students. This will give them confidence and will help them become quicker as time goes on.

Planning an Approach to Gather Information

Planning the proper approach for gathering information and for solving a problem are difficult tasks for many students and require consistent practice. Students should decide which method will work best depending on whether the task involves observations, measurements, data, surveys, or visual diagrams. They also need to choose the best way to display their results.

Grades 3–5 Sample

This strategy is very difficult for students at this age. You will need to model and complete "think-alouds" for students to get a better understanding of *how* to process and filter information. Provide students with opportunities to work in pairs with simpler problems at first. When students are comfortable with this, provide them with more detailed problems and require larger groups to work together.

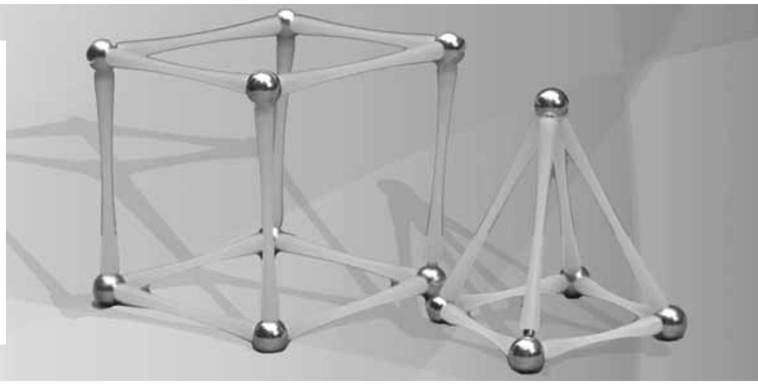
Secondary Sample

This strategy is still difficult for many students, especially if they have not spent a significant amount of time working in groups in previous grade levels. You will need to model and show samples for students to get a better understanding of *how* to process and filter information. Provide opportunities for students to work in pairs, solving simpler problems first. When student confidence increases, challenge them with more detailed problems and increase group sizes.

Solving Open-Ended Problems

Standard

- uses a variety of strategies in the problem-solving process



Background Information

This problem-solving strategy challenges students' thinking. Use this method when problems have multiple solutions. Students need practice with this type of problem because they are accustomed to problems with a single solution. They need to realize that it is acceptable in some cases to have different answers or use diversified approaches to problem solving. This is a very authentic, real-life strategy.

It is also a unique problem-solving strategy because it allows a student to work through problems at his or her ability level using the best cognitive methods already known. Words like *create*, *investigate*, *design*, and *explore* help identify this type of problem. It is a strategy best used with students in grade 3 or higher.

There are many skills that students will need to acquire in order to solve problems using this strategy. Some of these skills include the following: using labeled or numbered counters, trying different combinations, and finding as many solutions as possible.

Procedure

Once it is decided that open-ended problem solving is the best strategy to use for solving the problem, follow these steps to implement this strategy:

1. Reread the problem.
2. Have students decide on an approach they want to use to solve the problem.
3. Allow students freedom to use counters, extra sheets of paper, or other supplies to solve the problem.
4. Have students check their work and record their solutions.
5. Allow students to share their problem-solving process with the class and reflect together on the process and solution.

Samples

The following skills and concepts illustrate how open-ended problem solving can be used with many different types of problems. Students should be comfortable with these skills in order to be able to use this problem-solving strategy effectively. Read the sample problem in your grade-level range to see how this strategy can best be implemented in your classroom.

Solving Open-Ended Problems *(cont.)*

Using Labeled or Numbered Counters

Using labeled or numbered counters can help students visualize things when problems are very involved. The counters are helpful for students when trying different solutions to the problems.

Grades 3–5 Sample

You are in charge of one of the games for the fourth-grade fair. You have 6 number counters that will have the numbers 1, 2, or 3 on them. The players will pull a number counter from a bag. The number on the counter will determine the prize. If the player pulls a number 1, he or she will win a flower eraser. If the player pulls a number 2, the prize will be a blue pencil. If the player pulls a number 3, he or she will win a pen. There are 35 flower erasers, 50 blue pencils, and 10 pens. Determine which number should go on the counters. Test the game.

Secondary Sample

You need to cover a 4 by 8 foot area with colored tiles: gold, emerald, navy, and maroon. The colored tiles are 4 inches by 4 inches. Create a pattern for the area so that the emerald tile does not touch the maroon tile.

Trying Different Combinations of Numbers

Often, students must try different combinations of numbers before finding a solution in an open-ended problem. Students need to understand that they can rearrange the combinations as many times as necessary before finding a solution.

Grades 3–5 Sample

The number of newspapers Anton sold each month is listed below.

Month	Number of Newspapers
January	28
February	31
March	34
April	29

Estimate the number of newspapers he will sell by the end of the year.

Solving Open-Ended Problems *(cont.)*

Trying Different Combinations of Numbers *(cont.)*

Secondary Sample

Pablo collects baseball cards. The number of cards he collected during each of the last four months is listed below.

Month	Number of Cards
January	18
February	21
March	17
April	19

Estimate the number of cards Pablo will have collected by the end of the year.

Finding as Many Solutions as Possible

Often, problems will ask students to find the maximum number of solutions. This can be difficult if students have never practiced this skill. Students need to understand that problems can have more than one solution and work toward finding new and creative ways to solve them.

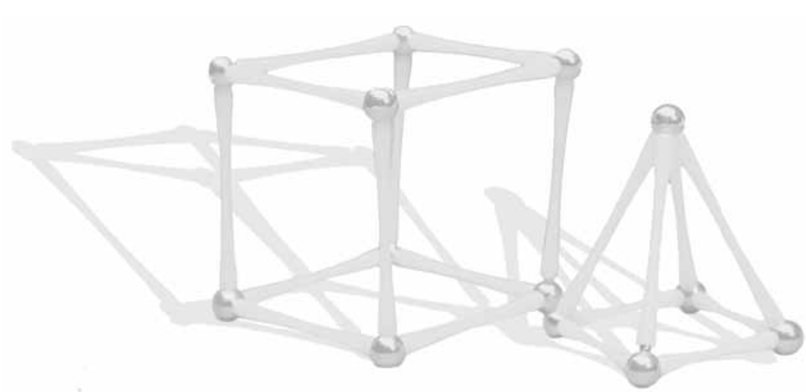
Grades 3–5 Sample

Write a multiplication problem which has a product between 120 and 350.

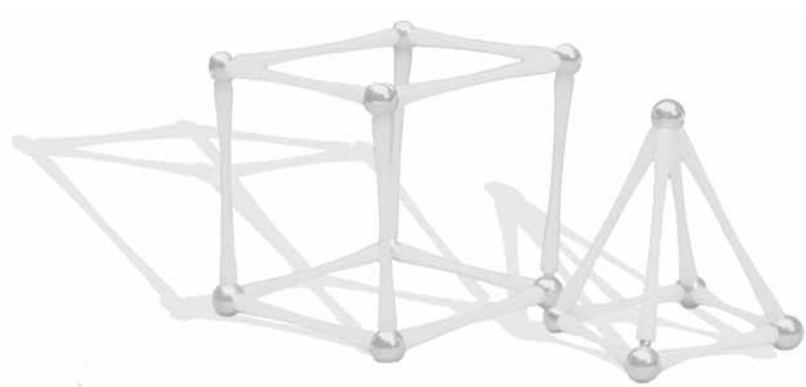
Secondary Sample

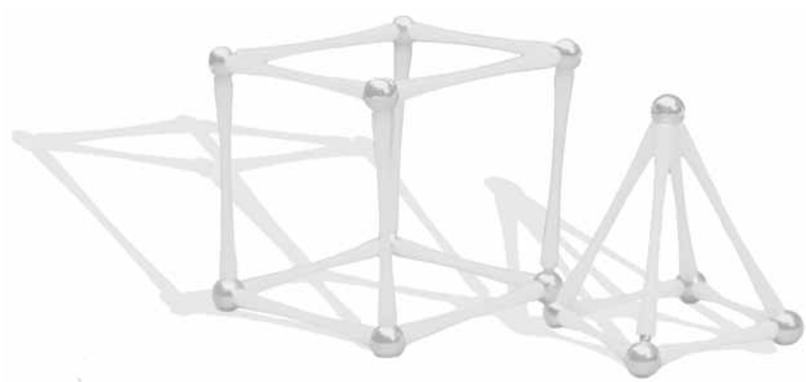
Write an addition problem which includes a mixed number and two fractions and has a sum between 3 and 5. The mixed number and fractions must have different denominators.

Notes



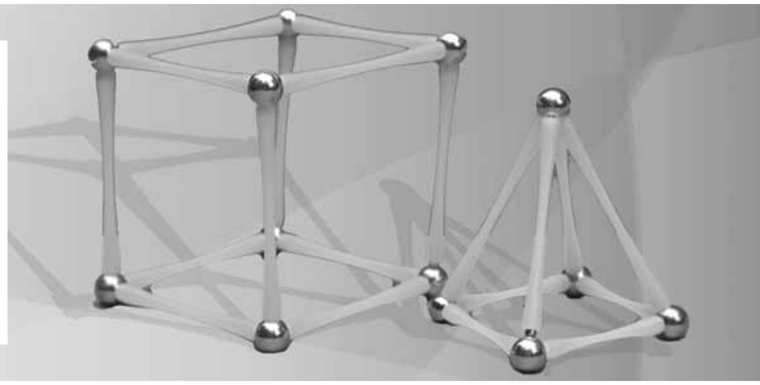
Strategies for Using Mathematical Games





Games Overview

Games help provide a medium for practice that is positive, exciting, and can generate a desire and purpose for learning.



Motivation and Mathematical Learning

Motivation is a student's willingness to give attention, time, energy, and perseverance to learning. It is the willingness to accept the challenge to understand a concept or solve a problem. Motivation is categorized as extrinsic or intrinsic. Extrinsic motivators are things such as treats, grades, and rewards. Intrinsic motivation is the desire for learning and knowledge for its own sake. As teachers, we should take both types of motivation into consideration when planning activities for children.

From an early age, most students are more extrinsically motivated than intrinsically motivated. They work for rewards. However, teachers can increase intrinsic motivation. Students are motivated when:

- they recognize concepts they have learned
- mathematical conversations relate to real-world problems
- tasks and assignments are meaningful and they feel successful

Attitude is also an important part of motivation. Students are motivated to learn when they feel good about a subject and their ability to do well in the subject. There is also a positive correlation between attitude and achievement in mathematics. When students are motivated, they attend to instruction, strive for meaning, and persevere when difficulties arise (Cathcart et al. 2000).

Teachers' attitudes toward mathematics are also influential in forming students' attitudes. The teacher needs to be positive and show enthusiasm for, and interest in, mathematics. (Cathcart et al. 2000) When teachers are positive and enthusiastic, the students are more likely to get on board and reflect the same attitudes.

Motivation with Games

Games are a good source of motivation. They are a fun way for students to develop, maintain, and reinforce mastery of basic facts. Games eliminate the tedium of most mathematical drills. They can be used in whole-group, small-group, and individual settings.

In classrooms where competitive games may pose a problem, rules can always be modified so that harmony may rule. However, fair and friendly competition can generate many positive outcomes such as challenge, independence, excitement, and determination. Modeling good sportsmanship is also important. Students need to be shown the proper way to react when they win or lose. They need to be explicitly shown how boasting, teasing, blaming, and anger are never appropriate responses no matter whether one wins or loses.

Games Overview *(cont.)*

Motivation with Games *(cont.)*

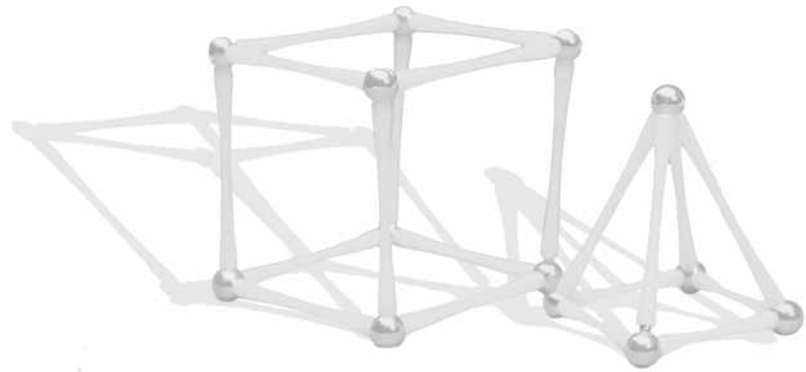
It is important that the teacher provides a positive learning environment where students are under minimal stress, emphasizes meaning and understanding rather than rote memorization, relates real-world concepts, and works in well-organized groups. Games help provide a medium for practice that is positive, exciting, and can generate a desire and purpose for learning.

Games Are for All Learners

Play is one of the first developmental skills that students acquire. It is the brain's link from the inner world to reality and the foundation of creativity (Jensen 2001). So often, people think of play as a meaningless method of entertainment for students; however, when mathematical content is incorporated into play, it provides a context for the students and can actually develop their reasoning skills and mathematical understandings (Olson 2007).

Games come in many forms. They can be as simple as verbal questioning for points and as complex as using technological equipment to engage in interactive play. Because games are so versatile, many different types of learning styles can be addressed.

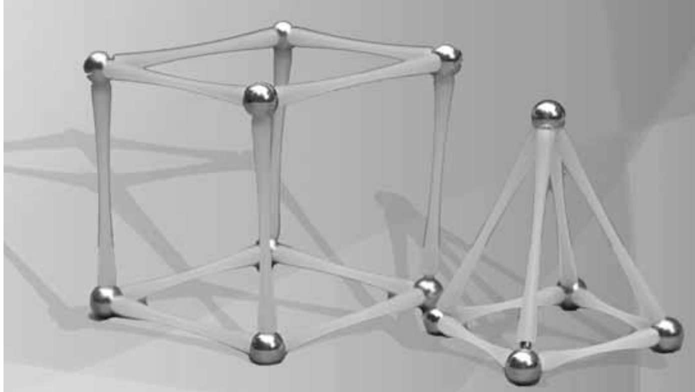
Research from Howard Gardner (2006) shows that students learn information in many different ways. Teachers need to present information in those various ways so that all students can understand the concepts in the way that is best for them. Games can address many of the learning modalities Gardner presents, such as bodily-kinesthetic, visual-spatial, verbal-linguistic, and interpersonal. By playing games, students are given opportunities to reinforce concepts in meaningful ways.



Concept-Based Games

Materials

- game boards (1 for each group of 4–6 students) (Templates are provided on the Teacher Resource CD: *games1.pdf*, *games2.pdf*, and *games3.pdf*)
- *Concept-Based Games Rules* (page 253; page253.pdf)
- *Game Cards Template* (page 254; page254.doc)
- *Games Recording Sheet* (page 262; page262.pdf)
- dice or spinners
- game markers to use as playing pieces



Background

Concept-based games use content that follows the specific concept being taught in class. To play, students move around a game board using cards with information such as questions, diagrams, visuals, or equations that pertain to the information they are learning in class. Students in kindergarten through second grade should be given a set of 15–18 cards to use during play. Students in grades 3–5 should be given a set of 18–28 cards. Secondary students should be given a set of 28–32 cards. At a student's turn, he or she draws a card from the deck and reads it to the rest of the players. Everyone in the group answers the question, and then the group discusses the answer and comes to consensus. Those players who answer the question correctly get to roll a die and move the displayed number of spaces. Those players who answer the question incorrectly stay where they are on the board and wait for the next question to be read to the group. After everyone moves, the next player draws a card, and the sequence continues until one player has crossed the finish line on the game board.

To assess students' progress and on-task behavior, students must show their work and record their answers on the *Games Recording Sheet* (page 262). Then students can submit their paper as evidence of their work throughout the game.

Procedure

1. Divide the students into groups. Depending on how you would like to differentiate this game, you can choose to homogeneously or heterogeneously group the students.
2. Distribute the following supplies to each group: 1 game board, 1 die or spinner, 1 set of concept-based game cards, and 1 game marker per student in the group.
3. Instruct students to lay the game cards face down in the middle of the board. All students should be able to reach the cards.

Concept-Based Games *(cont.)*

Procedure *(cont.)*

4. In order to start the game, give students a directive, such as the tallest player goes first and then play passes to the left, or the player whose birthday is closest to today goes first and then play passes to the right.
5. Explain the rules of the game to the students (page 253).
6. Allow students to play for a set amount of time. Students may play more than one round if the allotted time permits. When time expires, instruct two students from each group to gather the materials and put them away.

   Extension Idea: You can also combine cards from several concepts and use them as a cumulative review of the concepts covered over a specific amount of time.

Differentiation

Above-Level Learners

Before playing, allow students time to create their own game cards that cover the concepts being taught in class. Explain to students that they want a balance of medium-level and challenging-level questions. Students can also be instructed to go around the game board twice before they can win.

Below-Level Learners

Allow students to play the game in pairs. They may quietly confer with their partners while answering the questions. Remind them that they want to speak softly so that other pairs do not copy their answers.

English Language Learners

Provide students with additional visual aids for them to use when answering the questions. These visual aids will differ depending on the mathematical content being studied in the game. For example, if addition and subtraction are part of the game, students can be given hundred boards. If place value is covered in the game, students could be given number cards and place-value charts.

Concept Suggestions

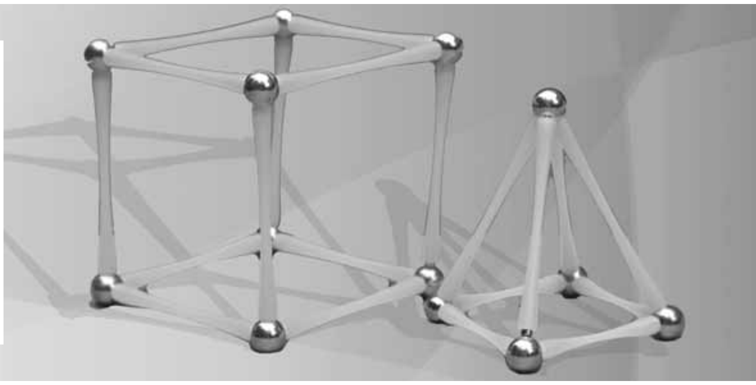
Many mathematical concepts can be reinforced through the use of the game. Some ideas are provided below:

Grades K–2
- data analysis
- shapes
- addition
- subtraction
- time

Grades 3–5
- fractions, decimals, and percents
- time
- data analysis
- multiplication and division
- word problems

Secondary
- central tendencies
- ratios and proportions
- word problems
- solving linear equations
- calculating measurement using formulas

Who Has? Games



Materials

- *Who Has? Games Rules* (page 255; page255.pdf)
- *Who Has? Template* (page 256; page256.doc)

Background

The game *Who Has?* can be used to reinforce, practice, or assess concepts and vocabulary associated with the content being taught in class. To prepare for the game, a set of cards is created, at least one for each person in class, including the teacher. Each card has a content-related question on it, as well as the answer to a different card's question. The teacher begins the game by asking aloud the question on his or her card. The students look at the answers on their cards, and the student who has the correct answer reads it aloud and then asks the question on his or her card to the rest of the class. The student whose card contains the answer to that question reads it and then asks his or her question aloud to the class. This question-and-answer format continues until the teacher reads aloud the answer on his or her card.

The cards can be used repeatedly because there is very little chance of a student getting the same card a second time the game is played. Sample *Who Has?* game cards are provided on the Teacher Resource CD (*whohas.doc*).

Procedure

1. Give each student a card. If there are more cards than students, give them more than one card until all cards are distributed.
2. Explain the rules of the game with the students (page 255).
3. Remind students to listen carefully to the questions being read so that they can hear when it is their turn.
4. Begin the game by reading the question on your card.
5. Whoever has the answer to the question will then read his or her card. "I have ________. Who has __________?"
6. Continue playing until you read the answer on your card.

Extension Idea: You could also make fewer cards and implement this game in a small-group setting instead of with the whole class.

Who Has? Games *(cont.)*

Differentiation

Above-Level Learners

Distribute two cards to these students before beginning the game. This will enable them to listen for more than one answer. You can also allow these students to develop the game cards as new concepts are studied throughout the year. Make sure that the students have played the game several times and understand the cyclical nature of the questions before allowing them to create cards on their own.

Below-Level Learners

Allow these students to play the game in pairs. This will enable them to discuss each question before deciding if it is their turn to answer the question. Before the game begins, allow pairs to discuss the content of their cards with you if they are unsure of the concepts being presented.

English Language Learners

Allow these students to play the game in pairs. This will enable them to discuss each question before deciding if it's their turn to answer the question. Before the game begins, allow them to make an illustration of the content on their cards so that they have a reminder of the concepts covered on their cards.

Concept Suggestions

Many mathematical concepts can be reinforced through the use of the game. Some ideas are provided below:

Grades K–2

- numbers
- money
- months/calendar
- more/less
- addition

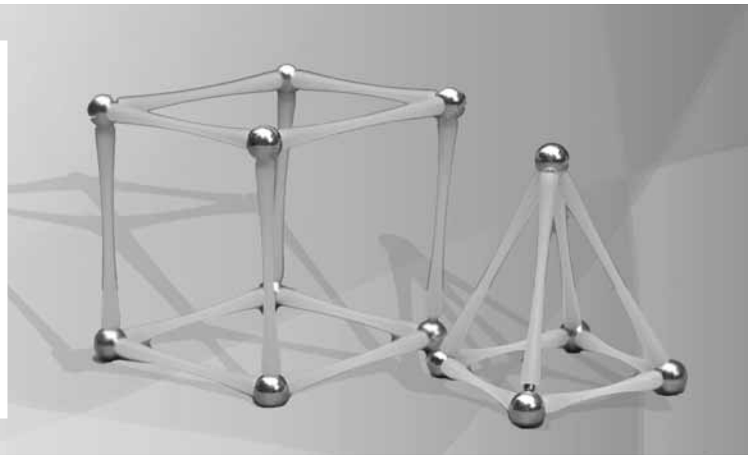
Grades 3–5

- 2-D shapes
- 3-D shapes
- translations
- multiplication
- subtraction

Secondary

- fractions
- division
- measurement conversions
- division
- squares/square roots

Category Quiz Games



Materials

- Grid with categories and number values (can be poster or displayed on the board)
- small bag or container (1 for each team)
- small strips of paper with each student's name on them
- *Category Quiz Games Rules* (page 257; page257.pdf)

Background

Category quiz games can use content that follows the specific concept being taught in class or can use content that has been taught over a long period of time as a review. To play, students are divided into teams. Then they are shown a grid with categories across the top row, such as *Multiplication*, *Division*, *Basic Fractions*, and *Miscellaneous*. The remaining squares on the grid have point values in each row and are color coded red, blue, green, and yellow. Each team is given two colors that they can choose from on the grid. At its turn, a team chooses a category and a point value from one of its colors and is then asked the question that corresponds to that square on the grid. As a team, students work to answer the question. If they answer correctly, they receive the points. If they answer incorrectly, the other teams in the room are allowed to respond. It is important to come up with a system for "ringing in" when teams want to answer at this point in the game.

To create the game grid, choose five different categories that correspond with the concept(s) being reviewed. Then assign the following point values to each of the rows: row 1 = 100 points, row 2 = 200 points, row 3 = 300 points, row 4 = 400 points, and row 5 = 500 points. Then assign colors to each of the squares in random order, keeping the numbers as equal as possible. Next assign questions to each of the squares. Red squares should be questions suitable for only above-level students. Blue squares should be questions suitable for above-level and on-level students. Green squares should be questions suitable for on-level and below-level students, and yellow squares should be questions suitable for only below-level students.

Procedure

1. Prior to playing the game, write each student's name on a small strip of paper.
2. Divide the students into teams based on their ability levels. Make sure that English language learners are placed in their proper ability groups, not all in a group by themselves. Give each team a bag or container. Place the correct paper strips into the bags/containers so that each team has all of its team members' names in its bag/container.
3. Display the game grid to the students and read the categories aloud.
4. Explain the rules of the game to students (*Category Quiz Games Rules* page 257) and allow them to ask any questions they may have about the game or the categories.

Category Quiz Games *(cont.)*

Procedure *(cont.)*

5. Have teams pick numbers between 1 and 10 to decide which team goes first. Instruct that team to pick a category and a point value, keeping in mind to choose only from the assigned colors.
6. Read the corresponding question to the students and allow them one minute to answer the question. Everyone on the team needs to understand the answer to the question.
7. At the end of one minute, choose one name from the team's container and allow only that student to answer the question.
8. If the student answers correctly, the team receives the points. If the student answers the question incorrectly, allow the other teams to answer the question.
9. Replace the student's name in the bag/container. Whichever team answered the question correctly has control of the grid and gets to choose the next category and point value. Continue playing like this until all of the questions have been asked. Whichever team has the most points at the end of the game wins.

Differentiation

Above/On-Level Learners

Above-level students should pick from both red and blue squares only. On-level students should pick from blue and green squares only.

Below-Level Learners

Below-level students should pick from green and yellow squares only.

English Language Learners

These students should be placed in their correct ability group (above, on, below) for this activity. In order to aid them with their language, allow them to have small 3x5 cards with vocabulary and diagrams that will scaffold their language for the concepts being reviewed in the game.

Concept Suggestions

Many mathematical concepts can be reinforced through the use of the game. Some ideas are provided below:

Grades K–2
- addition
- subtraction
- time
- 2-D shapes
- place value

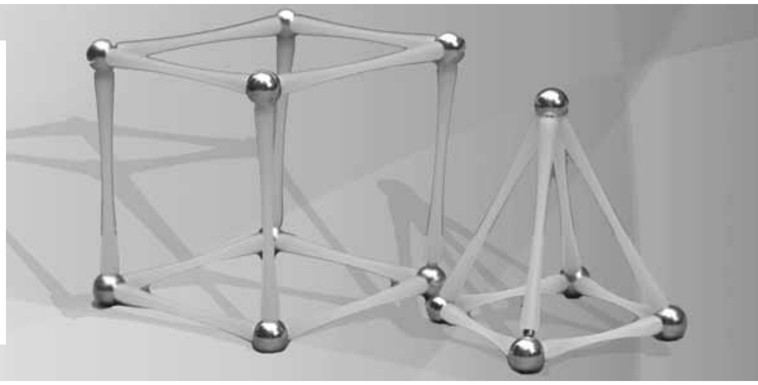
Grades 3–5
- multiplication
- division
- symmetry
- fractions
- 3-D shapes

Secondary
- data analysis
- polynomials
- ratios
- Pythagorean theorem
- surface area/volume

MATHO Games

Materials

- *MATHO Games Rules* (page 258; page258.pdf)
- *MATHO Boards* (page 259; page259.pdf)
- *MATHO Problem Cards* (page 260; page260.doc)
- *Games Recording Sheet* (page 262; page262.pdf)
- small objects to use as game markers



Background

MATHO is a game that is played like Bingo. Problem cards are created for this game based on the concepts being taught in class. Each problem card has a letter in the corner that corresponds to the letters printed on the MATHO boards. To play, students receive a pile of game markers and a MATHO board that has 25 squares on it, each with a different letter printed in the spaces. The center space on the card is a free space. A problem card is then displayed to the students either on the overhead or on a document projector. Every student is required to solve the problem on his or her *Games Recording Sheet*. Then the problem is discussed as a class and the answer is shared. Students who answered the problem correctly and whose cards have the corresponding problem letter on them can cover the spaces. Those who answered the question incorrectly or do not have the letter on their boards do not cover any spaces. The first player to accurately cover five spaces in a row wins.

To assess students' understanding of the concept being used in the game, collect the *Games Recording Sheets* and check their work.

Procedure

1. Use the *MATHO Problem Cards* template to create a set of problem cards that correspond to the concept being taught in class. Run the problem cards on transparency film or paper depending on whether you will use an overhead projector or document projector for the game.
2. Distribute one MATHO board, one *Games Recording Sheet*, and a handful of game markers to each student.
3. Explain the rules of the game to the students (page 258).
4. Display a problem to the students and allow them time to solve it. Then, discuss the answer as a class.
5. Point out the letter in the corner of the problem card. Students who answered the problem correctly and who have that letter on their boards can cover the spaces with game markers. Students who did not answer the problem correctly or do not have that letter on their boards do not cover any spaces.
6. Continue displaying problems until a student accurately covers five spaces in a row. The spaces can be covered down, across, or diagonally.

MATHO Games *(cont.)*

Differentiation

Above-Level Learners

Allow students to play this game in a small group. The students can take turns displaying the problem cards to the group. Everyone in the group answers the question, and then the group discusses the answer and comes to consensus.

Below-Level Learners

Allow students to play this game in a small, teacher-directed group. That way, the problems can be worked with teacher guidance and more support can be provided to students who answer problems incorrectly.

English Language Learners

If playing this game in a large group, have students play in pairs. Allow them to discuss procedures while solving the problems. Students can also play this game in a small group so that the problems can be worked with teacher guidance and difficult vocabulary can be discussed.

Concept Suggestions

Many mathematical concepts can be reinforced through the use of the MATHO game. Some ideas are provided below:

Grades K–2

- addition
- subtraction
- time
- 2-D shapes
- place value

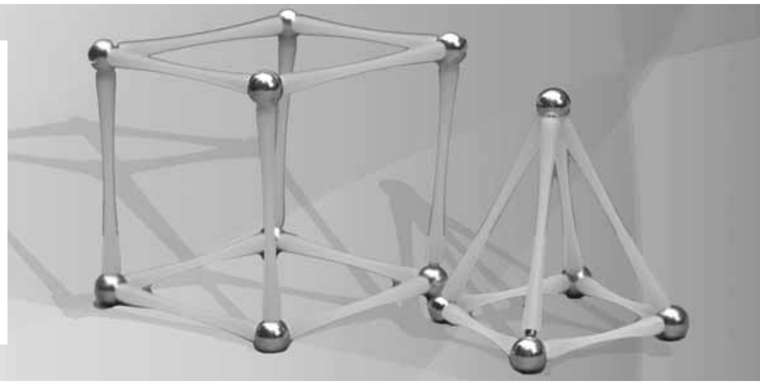
Grades 3–5

- multiplication
- division
- decimals
- fractions
- rounding

Secondary

- equivalent forms of numbers
- adding polynomials
- conversions
- Pythagorean theorem
- surface area/volume

Matching Games



Materials

- *Game Cards Template* (page 254; page254.doc)
- *Matching Games Rules* (page 261; page261.pdf)
- *Games Recording Sheet* (page 262; page262.pdf)

Background

Matching games can be created to follow any concept being taught in class. To play, students need a set of cards that have problems or questions on them and a set of cards that have the corresponding answers. The cards should be mixed ahead of time so that students do not know which problems go with which answers. Students in kindergarten through second grade should be given a set of 14–18 total cards to use during play. Students in grades 3–5 should be given a set of 18–28 cards. Secondary students should be given a set of 28–34 cards. The cards are placed facedown in equal rows in the middle of all of the players. At a student's turn, he or she turns over two cards. If either of the two cards are problem cards, everyone in the group solves them, discusses the answer, and comes to consensus. If a matching problem and answer card have been turned, the student gets to keep the match. If both of the cards are answer cards or the problem and answer cards do not match, they are turned back over and play moves to the next student. Students will use their work to evaluate whether they have turned over matching problem and answer cards. Problems do not need to be solved more than once. When all of the matches have been found, the student who has collected the most matches wins.

To assess students' progress and on-task behavior, collect the *Games Recording Sheets*.

Procedure

1. Divide the students into groups of 3–4 students. Depending on how you would like to differentiate this game, you can choose to homogeneously or heterogeneously group the students. Each group needs a set of cards and enough *Games Recording Sheets* to give one to each student.
2. Instruct the groups to lay the cards facedown in equal rows.
3. In order to start the game, give students a directive, such as the shortest player goes first and then play passes to the left, or the youngest player goes first and then play passes to the right.
4. Remind students of the rules of the game (page 261).
5. Allow students to play for a set amount of time.
6. When time expires, have students count their total matches and determine who the winner is. If two or more students are tied for the most matches, they are all winners.

Matching Games *(cont.)*

Procedures *(cont.)*

7. Instruct one student from each group to clean up and gather the materials and put them away.
8. Have the students turn in their *Games Recording Sheets*.

Differentiation

Above-Level Learners

Before playing, allow these students time to create some or all of the cards they will use for this game. They should create questions that cover the concepts being taught in class. Then they can play according to the procedures provided.

Below-Level Learners

Allow groups to play with fewer cards. You can also have the students separate the cards into two piles, answers and problems, before laying them out. Then have students make two separate face-down sets: one set has just problem cards and the other set has just answer cards. When students turn cards, they turn one from each set.

English Language Learners

Before playing the game, review the problem cards as a group. Without solving the problems, discuss the procedures, any necessary vocabulary, and any visuals they might want to reference. This will help the students understand what they will need to do in order to play the game. Then allow the students to play the game following the procedures provided above.

Concept Suggestions

Many mathematical concepts can be reinforced through the use of the game. Some ideas are provided below:

Grades K–2
- fact families
- addition
- subtraction
- missing addends
- time

Grades 3–5
- inequalities
- area
- symmetry
- data analysis
- fractions

Secondary
- slope
- graphing lines
- order of operations
- ratios and proportions
- data analysis

Concept-Based Games Rules

Object of the Game

- Be the first player to go around the game board.

Setting up the Game

- Place the game board in the middle of all of the players.
- Shuffle the cards. Place them facedown on the board. Make sure that everyone can reach the pile.
- Place your *Games Recording Sheet* near you.
- Each player places a game marker on **Start**.

How to Play the Game

- Your teacher tells you who will start. Then, play goes around to the left.
- For each turn, draw the top card from the deck.
- Everyone solves the problem on their *Games Recording Sheets*.
- Compare your answers. Decide who solved the problem correctly.
- If you solved the problem correctly, roll the die and move forward that many spaces. If you did not solve the problem correctly, stay where you are.
- Place the used card in a discard pile. Once you run out of cards, shuffle the discard pile. Then, use the cards again.

How to Win the Game

- The first player to travel around the whole game board is the winner.

Game Cards Template

Directions: Write questions, true or false statements, diagrams, etc., on the game cards below. Create a set of cards for each game board.

Who Has? Games Rules

Object of the Game

- To correctly identify the answer on your card.

Setting up the Game

- Place the card you receive from your teacher on the desk in front of you.
- Read your card.
- Think about the answer on your card. What question does it answer?

How to Play the Game

- Your teacher will read the first question aloud to the class. The person whose card has the answer to that question reads his or her question next.
- Listen to the question being read.
- If your card has the answer, read it aloud to the class.
- If your card does not have the answer, wait for the next question to be read.
- Once you read your card, turn it over on your desk. Listen to the other questions and answers.

How to End the Game

- The game ends when the teacher reads the answer on his or her card.

Who Has? Template

Directions: Fill in the blanks on the cards below to match the mathematical concepts you are teaching in class.

I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?
I have ______________. Who has ____________?	I have ______________. Who has ____________?	I have ______________. Who has ____________?

Category Quiz Games Rules

Object of the Game

- Be the team with the most points when all of the questions have been asked.

Setting up the Game

- Look at the categories on the game grid. Think about what you know in each of those categories.
- Find the squares that correspond with the colors your team is assigned.

How to Play the Game

- Your teacher tells you who will start.
- For each turn, your team must choose a category and a point value from the squares that correspond to its assigned color values.
- Listen to the question read aloud by the teacher. Discuss the answer as a group and make sure that each person can answer the question. Your team gets one minute to answer the question.
- The teacher randomly picks a name from your team, and that student answers the question.
- If the answer is correct, your team gets the points for that square. If the answer is incorrect, the other teams get a chance to answer the question.
- Whichever team answers the question correctly gets control of the grid and chooses a new question.
- The game is finished when all of the questions on the grid have been read.

How to Win the Game

- The team with the most points after all of the questions have been read wins.

MATHO Games Rules

Object of the Game

- Be the first player to cover 5 spaces in a row on your MATHO Board.

Setting up the Game

- Place your MATHO Board in front of you.
- Place your pile of game markers and your *Games Recording Sheet* near you.
- Place one game marker on the **Free** space in the middle of your board.

How to Play the Game

- Look at the problem card that is displayed by the teacher. Think about how to solve the problem.
- Write the problem on your *Games Recording Sheet*.
- Solve the problem.
- If you answer the problem correctly, look at the letter in the corner of the problem card. If you have that letter on your MATHO Board, cover it with a game marker. If you do not have that letter on your MATHO Board, wait until the next problem is shown.
- If you did not solve the problem correctly, wait until the next problem is shown.

How to Win the Game

- The first player to cover 5 spaces in a row on his or her MATHO Board is the winner.

MATHO Boards

Directions: Cut the MATHO boards apart on the thick lines below. There are four MATHO boards on this page. For durability, consider laminating this page before cutting apart the boards.

M	A	T	H	O	M	A	T	H	O
Q	V	O	H	G	A	R	O	E	G
W	I	P	J	F	X	U	P	L	H
X	K	FREE	L	E	V	D	FREE	C	S
S	M	N	B	Y	K	J	T	M	Q
R	T	U	C	D	F	W	N	B	Z

M	A	T	H	O	M	A	T	H	O
F	K	M	A	I	B	S	O	C	M
W	L	E	J	Q	X	R	L	A	F
G	Y	FREE	H	B	Q	K	FREE	W	N
X	V	Z	R	P	D	H	P	Y	I
U	T	N	O	S	S	J	T	V	G

MATHO Problem Cards

Directions: Write a problem in each of the squares below. The letter in the corner corresponds to the letter found on the MATHO Boards.

A	**B**	**C**
D	**E**	**F**
G	**H**	**I**
J	**K**	**L**
M	**N**	**O**
P	**Q**	**R**
S	**T**	**U**
V	**W**	**X**
Y	**Z**	

Matching Games Rules

Object of the Game

- Be the player with the most matches at the end of the game.

Setting up the Game

- Shuffle the cards.
- Place the game cards face down in equal rows. Make sure that everyone can reach the cards.

How to Play the Game

- Your teacher tells you who will start. Then, play goes around to the right.
- For each turn, flip over two cards. You are trying to match problems with answers. Make sure that everyone can see the cards when you have turned them over.
- If the first card you turn over is a problem card, stop and let everyone solve the problem on their *Games Recording Sheets*. Compare your answers. Decide what the correct answer is. Then flip your second card to see if you can find the answer.
    - If you find a match, you keep the cards and then your turn is over.
    - If you do not find a match, your turn is over and play goes to the next person.
- If the first card you turn over is an answer card, try to find the correct problem card.
    - If you find a match, you keep the cards and then your turn is over.
    - If you do not find a match, your turn is over and play goes to the next person.

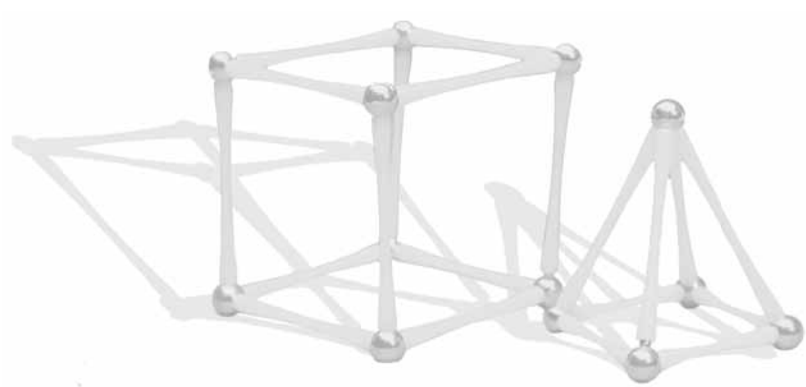
How to Win the Game

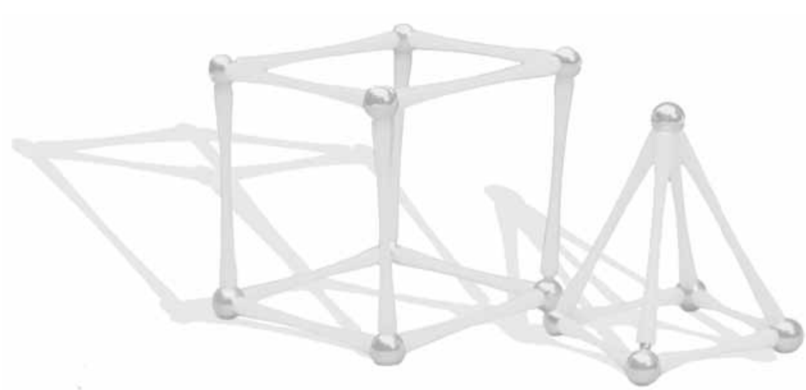
- The player with the most matches wins.

Games Recording Sheet

Directions: Record the problems you solve during your game in the space below.

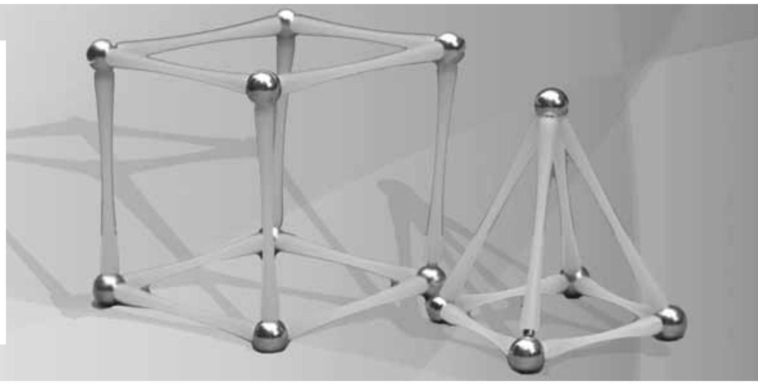
Strategies for Assessing Mathematical Thinking





Assessment Overview

When students understand the expectations and criteria for success they realize that evaluation is part of the learning process and do not feel threatened by it.



Assessment is the means by which we determine what students know and can do. It tells teachers, students, parents, and policymakers something about what students have learned: the mathematical terms they recognize and can use, the procedures they can carry out, the kind of mathematical thinking they do, the concepts they understand, and the problems they can formulate and solve (Kilpatrick, Swafford, and Findell 2001).

It is clear that assessment is a necessary part of education today. All of education is held to high standards, and yearly achievement improvements are of the utmost importance. Assessment for learning provides feedback to improve teaching and learning in the classroom. Although both assessment opportunities have merit, many teachers focus on measuring what students have learned at the end of a unit of instruction and fail to allow opportunities for feedback and revision during instruction (Carr 2008).

In general, there are two types of assessment: summative and formative. According to the Association for Supervision and Curriculum Development (1996), summative assessment is defined as "a culminating assessment, which gives information on students' mastery of content" and formative assessment is defined as "assessment which provides feedback to the teacher for the purpose of improving instruction."

As previously mentioned, summative assessments are usually formal assessments that are given at the end of a unit of study or as a culminating test for the year. These assessments are often paper-and-pencil type tests with multiple choice, matching, free response, essay, or open-ended questions. They are specifically graded, and although they can be used to drive future instruction, they are often used for the purpose of evaluating a student's overall progress.

Formative assessments can be formal or informal. They are primarily used to provide the teacher with feedback on student understanding and progress during a unit of study. Based on the results, the teacher decides if instruction needs to be slowed or quickened to best meet the students' needs. This type of assessment should not be viewed as extra work. "Formative assessment is part of good teaching" and should provide a seamlessness between instruction and assessment (McIntosh 1997).

Assessment Overview *(cont.)*

The following chart is adapted from Richard Pergente's book (1994), *Charting Your Course: How to Prepare to Teach More Efficiently*.

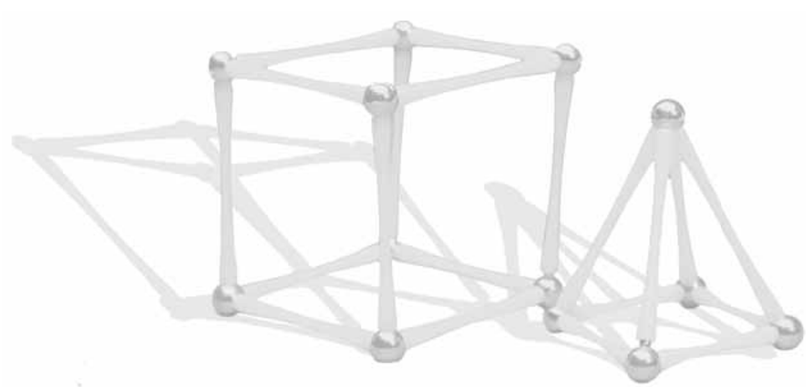
	Summative Assessment	Formative Assessment
Time	at the conclusion of an activity or unit of study	during an activity or unit of study
Goal	to make a decision or provide a grade	to improve learning and modify instruction
Feedback	final decision	return to material and instruction

In general, mathematics standards can be divided into three main categories:

- mathematical content
- mathematical processes, such as reasoning, communicating, problem solving, and making connections
- mathematical disposition, such as a student's attitude, confidence, persistence, and cooperation with other students

These standards must be assessed in order for teachers to know whether students are making adequate progress at their grade level. It is also important to assess content in those three areas frequently so that instruction can be modified and enhanced based on the results.

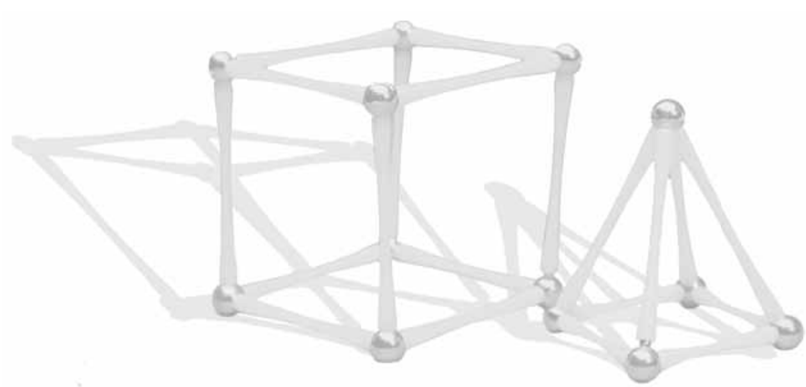
It is important to discuss assessment with students. When students understand the expectations and the criteria for success, they realize that evaluation is part of the learning process and do not feel threatened by it (Carr 2008). This type of discussion also allows students to metacognitively reflect on their own learning styles, their strengths, and their weaknesses, therefore becoming more aware of how they learn and what they should do to become better students.



Assessment Overview *(cont.)*

The chart below illustrates types of assessments that can be used in each of the categories listed on the previous page. Many of these assessments will be further explored in this section.

Category	Suggested Assessments
Mathematical Content	• written test/quiz • observation • self evaluation • performance task • graphic organizer • rubric
Mathematical Processes	• interview • self evaluation • journal • portfolio • rubric • graphic organizer
Mathematical Dispositions	• observation • self evaluation • journal • peer evaluation



Interviews

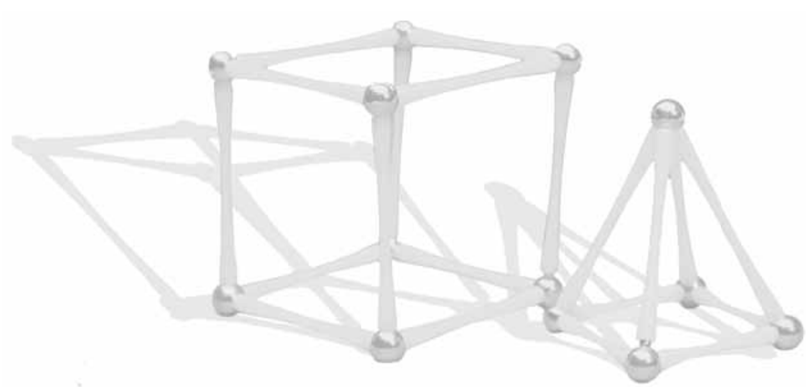
Interviews are an important strategy to use when assessing students' mathematical thinking. They can be completed formally or informally, depending on the purpose of the interview. By engaging in dialog with students, you are able to ask them what they know and then respond with follow-up questions to more deeply understand their thinking. Interviewing also allows you to get an idea of students' ability to make connections and communicate their thinking processes (McIntosh 1997).

Additionally, interviews can help you identify exact student needs and misconceptions. Instruction can then be modified to reteach those precise areas in which the need exists. For above-level students, interviews can be a method of discourse where they are able to explain to you their heightened understanding of the concepts being discussed.

The following are suggestions when using interviews in your classroom:

- Keep records of the interviews you conduct (e.g., rubrics, notes, or audio recordings).
- Allow plenty of wait time so that students can give thoughtful responses.
- Allow students to change or add to their responses if they do not feel that they adequately explained themselves on their first try.
- If students are assigned a task or activity to complete during the interview, ask them to describe what they are doing and why.
- Not every student needs to be interviewed about each concept, but every student should be interviewed at several points during the year.

The following pages show interview rubrics or questions. Full-size, blank templates of these documents can be found on the Teacher Resource CD in the *Assessment* folder. See page 318 for more details.



Interviews *(cont.)*

Sample Formal Interview

This interview was conducted after students were taught how to make up stories based on number sentences. They were also provided with counters to use as concrete representations of the story.

Student Name: *Hector*

Teacher Question	Student Answer	Teacher Notes
What story can you make up based on the number sentence written on this card? 9 + 4 = 13	My sister and I went to the store. I bought 9 packs of bubble gum and she bought 4 packs. We bought 13 packs altogether.	Hector correctly described a scenario that used the number sentence.
Add to your story using the number sentence written on this card: 13 – 6 = 7	We went to another store and bought 6 more packs of bubble gum. Now there are 7 packs of bubble gum.	Hector does not appear to understand what the symbol for subtraction means.
Incorrect understanding. **STOP**		

Adapted from the Alaska Department of Education and Early Development (1996).

Interviews *(cont.)*

Sample Project Interview Questions

This type of interview could be given after students have completed some type of project or mathematical task. The project could have been completed in pairs or groups, but the interview should be conducted individually so that you understand what each student learned or understood during the process.

Student Name:

1. What did you do first?

Student Response:

2. Why did you do that first?

Student Response:

3. Describe your solution.

Student Response:

4. What did you do next?

Student Response:

5. What changes did you make based on your observations?

Student Response:

6. Describe any patterns that you saw.

Student Response:

Adapted from the Alaska Department of Education and Early Development (1996).

Interviews *(cont.)*

Sample Problem-Solving Interview Questions

This interview should be conducted while students are working on problem solving. While you are conducting the interview, students' lack of understanding will be immediately apparent if they have difficulty describing the problem or are completing incorrect work. At this point, you can see who needs extra scaffolding or simpler problems. Record notes about the student's actions and responses in the Student Responses column of the chart. Then record a score to indicate the student's level of understanding. A score of 1 indicates a low level of understanding. A score of 3 indicates a high level of understanding.

1. Describe the problem in your own words. What are the most important details?
2. Solve the problem and tell me what you are doing as you work.
3. There may be other strategies you could use to solve this problem. Please describe at least one other strategy.
4. Describe a real-world problem that is similar to your original problem.

Student Responses	1	2	3
The student was able to describe the problem and identify the most important details.			
The student solved the problem correctly, and adequately described the steps he or she was following.			
The student was able to describe other strategies that could be used to solve the problem.			
The student was able to relate the original problem to other types of real-world problems.			

Observation

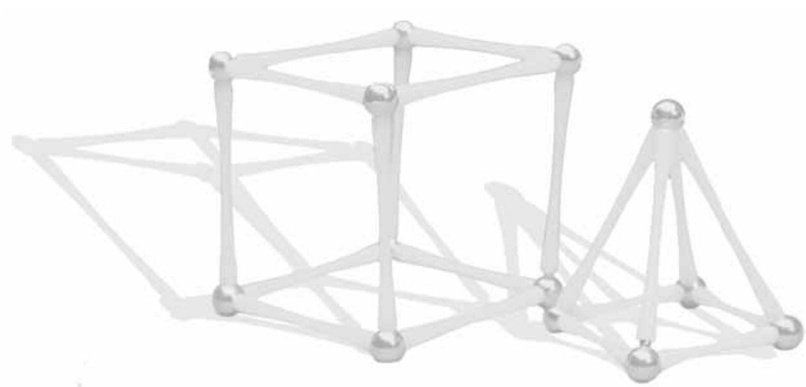
Observation is commonly used as an informal assessment strategy. Observations can be made and recorded at many different points during instruction of a concept. Once recorded, observations make for a true picture of the progression of student understanding on a concept. They can be helpful to use when assessing behaviors such as concentration, attitude toward problem solving, perseverance, accuracy using manipulatives, and ability to work with others on mathematical tasks.

Additionally, observations can take the form of notes, checklists, video recordings, or simple rubrics. When observational data is compiled, the data can be used to guide future instruction and supplement academic grades when reporting overall mathematics grades.

The following are suggestions when conducting observations in your classroom:

- Keep observational data organized according to student name.
- Record the date and concept/skill being observed each time observations are conducted.
- Do not call attention to your actions as you record observations. Students may become nervous and not perform to their best abilities if they feel they are being watched for grading purposes.
- Record all behaviors or actions, even if they do not align with what you originally intended to observe. You may gain a different insight into students' mathematical thinking.
- Record student behaviors using video-recording equipment. Later they can be used to show yearly growth or viewed by parents to better understand their child's mathematical understanding in the classroom.

The following pages show observation checklists or charts. Full-size, blank templates of these documents can be found on the Teacher Resource CD in the *Assessment* folder. See page 318 for more details.



Observation *(cont.)*

Sample Observation Checklist

This checklist could be used to observe a student's understanding of creating a graph from a set of data. Place a check mark in the "Yes" column if the student demonstrates accurate understanding. Place a check mark in the "No" column if the student does not demonstrate accurate understanding.

Student Name: *Veronique* **Date:** *10/15/2008* **Criteria:**	**Yes**	**No**	**Observation Notes**
Carefully reads the directions.			
Communicates clearly with group members.			
Accurately creates a bar graph.			
Labels the *x*- and *y*-axes appropriately.			
Labels the graph with appropriate title.			
Accurately transfers data from table to bar graph.			
Works well with the other members of the group.			
Interprets results carefully and accurately.			

Sample Observation Note Card

This note card could be used to evaluate students' understanding of specific mathematical tasks or concepts. Once the observations are complete, the cards can be filed away in a recipe box behind dividers with the students' names on them. This note card records an observation of a student performing an angle measurement using a protractor.

Student Name: *Amal* **Date:** *3/21/2008*

Task/Concept: *Demonstrates an understanding of measuring angles using a protractor.*

Observations:

Placed the protractor at the vertex of the angle. Read the protractor backwards and recorded angle measure of 150° instead of 30°. Need to follow up with students about reflecting on answer and not relying on tool to give correct measurement all the time. Also, student may have a limited understanding of obtuse and acute angles.

Adapted from the Alaska Department of Education and Early Development (1996).

Observation *(cont.)*

Sample Observation Chart

This chart could be used to evaluate students' understanding of general mathematical tasks. To organize these charts, create a notebook with a tab for each student in your class. Place blank charts behind each student's tab and record observations as necessary throughout the class period.

Student Name: Greta

Date	Task	Observations	Misconceptions	Reteaching Strategies
9/06	classifying triangles and quadrilaterals	• Created 3 piles: triangles, quadrilaterals, and others. • Counted sides of shapes before placing them in piles, but doesn't seem to use that information as a means of classifying the shapes. • Only placed regular triangles and quadrilaterals in the correct piles. • Placed irregular triangles and quadrilaterals in the "others" pile.	• Doesn't understand that any 3-sided closed shape is a triangle. Greta is incorrectly placing triangles in the "others" pile. • Doesn't understand that any 4-sided closed shape is a quadrilateral. Greta is incorrectly placing rectangles, trapezoids, and rhombi in the "others" pile.	• Review properties of triangles using visuals and concrete materials that show irregular triangles. • Review properties of quadrilaterals using visuals and concrete materials that show irregular quadrilaterals. • Need to help her understand connection between number of sides and type of shape.
9/15	classifying triangles and quadrilaterals	• Created 3 piles: triangles, quadrilaterals, and others. • Counted sides of shapes before placing them in piles. • Accurately placed all triangles in correct pile. • Accurately placed squares, rectangles, and rhombi in correct pile. • Trapezoids are still placed in the "others" pile.	• Doesn't fully understand that any 4-sided closed shape is a quadrilateral. Greta doesn't see how trapezoids can fit in the category of quadilraterals.	• Have a one-on-one discussion with her about what she knows about quadrilaterals. Discuss specific properties of trapezoids and design a test to conduct together to see if they fit in the category of quadrilaterals.

Performance Tasks

Performance tasks are learning activities where a student is asked to complete a task or project to demonstrate his or her understanding of a particular skill or set of skills. The student is evaluated according to specific criteria. This type of authentic assessment allows the teacher to directly observe and question students about their procedure, their understanding, and their ability to communicate their ideas. In one study, Marzano (1994) found that over two-thirds of the teachers polled said performance tasks provided them with better assessment data than did traditional evaluation techniques such as tests and quizzes.

Performance tasks can be brief and measure one specific understanding. For example, a teacher may provide a student with several blocks, have the student create a pattern, and tell about his or her thought process. Or, a student may be given fraction bars, asked to generate a proportion, and tell when in real life that proportion could be relevant. Often, these types of tasks are performed in front of the teacher while he or she takes notes and makes observations. The teacher can ask follow-up questions right away to gauge student understanding or to clarify incorrect procedures.

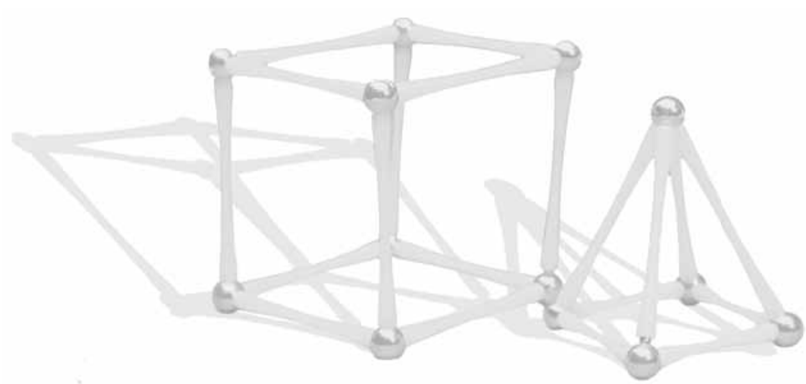
Performance tasks can also be complex and measure a variety of skills. For example, a student may be asked to write about a mathematical problem he or she sees in real life, create a presentation about the problem and the solution found, and then make the presentation to the class. Or, a student may be asked to create a budget and itinerary for a trip using particular guidelines.

It is important to share the grading procedures with students prior to conducting the performance tasks. Depending on the task, it may be necessary to give students copies of the assessment form (rubric, checklist, etc.) or simply discuss the grading criteria.

Here are some general areas where using performance tasks are appropriate:

- making generalizations
- describing patterns or data
- describing solutions
- discussing real-life mathematical problems
- explaining understanding of a particular concept or skill
- creating models of problems or concepts
- using manipulatives to demonstrate understanding

The following pages show performance rubrics and checklists. Full-size, blank templates of these documents can be found on the Teacher Resource CD in the *Assessment* folder. See page 318 for more details.



Performance Tasks *(cont.)*

Performance Task Rubric Sample

In this performance task, the students were asked to create a pattern using pattern blocks and explain the process and thinking behind their work.

Directions: Read each of the criteria listed below. Choose a score that best matches your performance. You and your teacher will score your performance. A score of 1 = poor, 2 = good, and 3 = excellent.

Criteria	1	2	3	Student Score	Teacher Score
The student creates a pattern using the shapes provided.	The student is unable to create a pattern with the shapes provided.	The student creates a pattern with the shapes provided, but makes 1–2 mistakes.	The student accurately creates a pattern with the shapes provided.		
The student describes the pattern he or she created.	The student is unable to describe the pattern created or did not create a pattern.	The student somewhat describes the pattern created.	The student accurately describes the pattern created.		
The student explains how he or she created the patterns and his or her thought process.	The student is unable to explain how he or she created the pattern and his or her thought process.	The student somewhat explains how he or she created the pattern and his or her thought process.	The student accurately explains how he or she created the pattern and his or her thought process.		

Performance Tasks *(cont.)*

Performance Task Checklist Sample

In this task, students were asked to create an itinerary and budget for a trip anywhere in North America.

Directions: Read the items listed below. Check the item off the list when you have completed it during your project.

Budget and Itinerary Checklist	Student	Teacher
1. I created an itinerary for my trip.		
2. My Itinerary includes:		
• arrival date and time		
• 5 events/activities to do on the trip		
• return date and time		
3. My itinerary is neat and organized.		
4. I created a budget for my trip.		
5. My budget includes:		
• travel cost		
• cost of each event/activity		
• lodging cost		
• food cost		
• daily spending money		
6. The plans do not exceed the original budget.		
7. My budget is neat and organized.		
8. I presented my itinerary and budget to the class in a clear and well-organized manner.		

Self- and Peer-Evaluations

Using self- and peer-evaluations after instruction is a valuable way to see how students and their peers view their performances. "When students use self-evaluation, they review and evaluate their own performance by comparing it against some standard. In the process, students must step back and reflect on what and how they learn" (Carr 2008). It also allows them to assess their work habits, attitudes, and thinking processes.

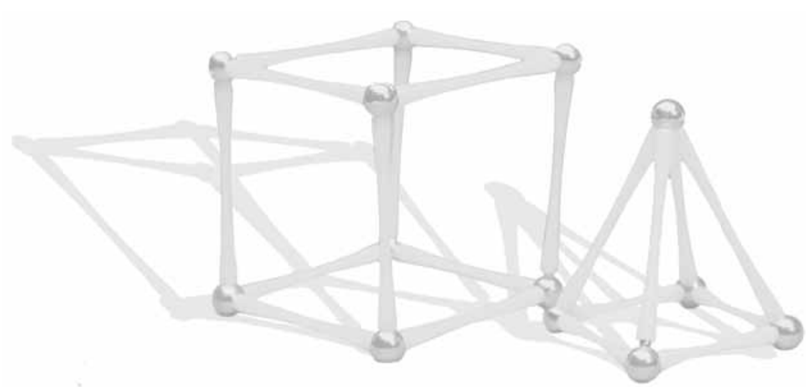
By utilizing self- and peer-evaluations during instruction, teachers allow students to collaborate about their understanding and strengthen their abilities. Students need a supportive learning environment in which they feel comfortable taking risks in order for self- and peer-evaluation to work effectively (Carr 2008). Teachers can provide this type of environment by:

- discussing the criteria for evaluation openly.
- modeling self-evaluation through think-alouds.
- providing samples of completed evaluations for students to view.
- using misconceptions/mistakes as learning opportunities instead of failures.
- using constructive feedback when informally discussing student performance.
- allowing students to complete private self-evaluations before completing those that are collected for teacher viewing.

Self- and peer-evaluations can be used as formative or summative assessments, depending on the type of information teachers wish to gather and how the information will be used. If the evaluation is used as a formative assessment, instruction can be modified to meet the student's needs, and misconceptions can then be corrected. If the evaluations are used as a summative assessment, final grades could be recorded and used for overall evaluation of a student's understanding.

Self- and peer-evaluations can be completed in many different ways, depending on the type of information that needs to be gathered. The most common evaluation types are rubrics, checklists, and rating scales.

The following pages show self- and peer-evaluation recording sheets. Full-size, blank templates of these documents can be found on the Teacher Resource CD in the *Assessment* folder. See page 318 for more details.



Self- and Peer-Evaluations *(cont.)*

Self- and Peer-Evaluation Rubric

This evaluation rubric is used for problem solving. For younger students, use smiley faces instead of numbers.

Student Name: ________________________________

Peer Name: __________________________________

Directions: Read each of the criteria listed below. Choose a score that best matches your understanding. You and a peer will score your performance. A score of 1 = poor, 2 = good, and 3 = excellent.

Criteria	1	2	3	Self	Peer
Understands the problem	The student does not understand the problem.	The student understands part of the problem.	The student understands the problem.		
Discusses the solution process with others	The student does not discuss the solution process with others.	The student can only discuss part of the solution process with others.	The student can clearly discuss the solution process with others.		
Works with others to solve the problem	The student does not work with others to solve the problem.	The student works with others to solve part of the problem.	The student works with others to solve the problem completely.		
Shows the solution clearly	The student cannot show the solution clearly.	The student can show part of the solution clearly.	The student shows all of the solution clearly.		

Self- and Peer-Evaluations *(cont.)*

Self- and Peer-Evaluation Checklist

The checklist below can be used for problem solving. Place clip art or examples next to each statement to aid English language learners and struggling readers.

Directions: Read each item below. Check it off the list as you complete each item.

Problem-Solving Checklist

- ❏ I read the problem.
- ❏ I underlined the important words in the problem.
- ❏ I underlined the numbers I need in the problem.
- ❏ I threw out the information I didn't need to solve the problem.
- ❏ I chose a strategy to use to solve the problem.
- ❏ I showed the work I did to complete the problem.
- ❏ I wrote my answer clearly.
- ❏ I checked my solution.
- ❏ I explained my solution to another student.

Self- and Peer-Evaluations *(cont.)*

Self-Evaluation Rating Scale

This rating scale is used for problem solving.

Directions: Read each statement below. Decide where you belong on the scale and shade in the bubble. A score of 1 shows the least understanding and a score of 5 shows the most understanding.

①	②	③	④	⑤
I do not understand the problem.				I understand the problem.
I cannot identify the important details in the problem.				I can identify the important details in the problem.
I do not know a strategy I can use to solve the problem.				I know a strategy I can use to solve the problem.
I do not know how to start.				I know where to start and can explain my steps.
I do not know how to check my solution.				I know how to check my solution.
This was a hard problem.				This was an easy problem.

Adapted from the Alaska Department of Education and Early Development (1996).

Graphic Organizers

> "Graphic organizers make powerful tools that are easy to implement and allow you to thoroughly examine student thinking and learning. Graphic organizers also let you assess ongoing learning that is conceptual, hierarchical, cyclical, and sequential" (Struble 2007).

Graphic organizers have many uses in the mathematics classroom. They can be used to organize information about a specific topic, they can be used to aid students in taking notes, they can be used to visually represent a problem or sequence, and they can be used for assessment both before and after a concept has been introduced.

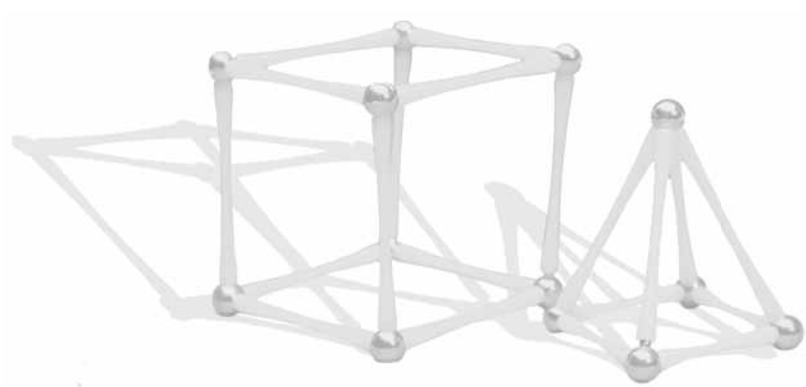
Prior to teaching a lesson, teachers can assess students' prior knowledge of a particular concept by using a graphic organizer called a KWL chart. This graphic organizer can be completed by students individually or as a class. Teachers can then use this information to tailor instruction to the needs of the students and the understanding they already have.

After teaching a concept, students can provide a "visual representation of knowledge" and understanding through a graphic organizer that fits the concept (Bromley, Irwin-DeVitis, and Modlo 1999). The following lists some graphic organizers that work well with mathematics instruction:

- flow chart
- Venn diagram
- triple Venn diagram
- circle map (Hyerle 2004)
- concept web
- double bubble map (Hyerle 2004)

Once students have visually represented their understanding by completing a graphic organizer, it is important to provide them with feedback. This feedback should be visual by writing it on the organizer and then discussing it with the student. This process will help students correct misconceptions, deepen their understanding of the concept, and make connections to prior knowledge. Feedback can also be provided to students who do not have misconceptions about the concept. You can further their understanding of the concept by exposing them to new related content or by making a new connection between concepts that other students may not be ready for.

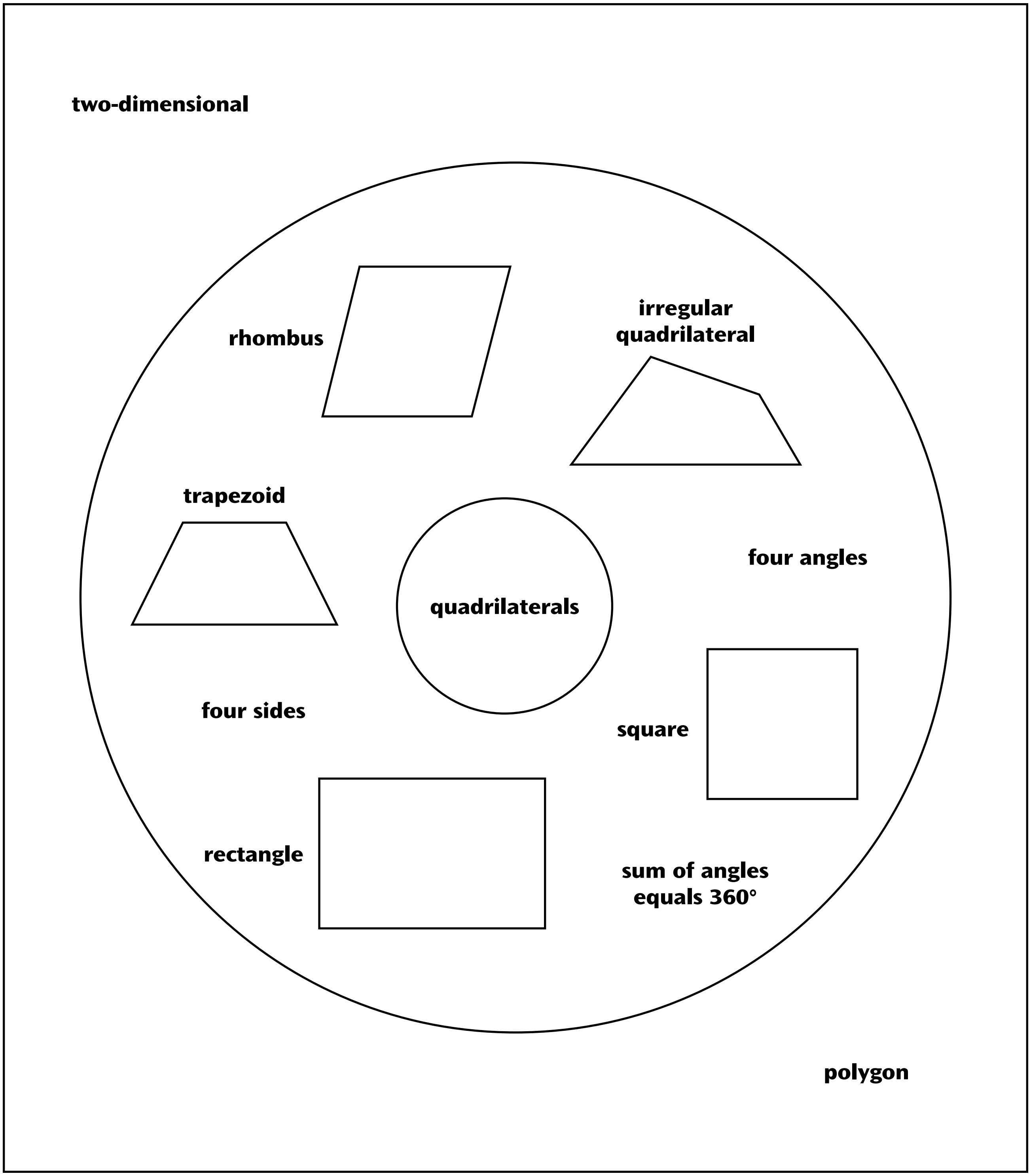
The following pages show sample graphic organizers.



Graphic Organizers *(cont.)*

Circle Map Sample

This circle map demonstrates a student's understanding of the concepts of quadrilaterals. Circle maps can be used to explain details about many mathematical concepts.



Adapted from David Hyerle (2004).

Graphic Organizers *(cont.)*

Flow Chart Sample

This flow chart shows a student's understanding of the procedures for finding the average from a set of data. Many procedures can be explained using this graphic organizer.

Directions: Write the steps in the boxes below to explain the procedure for finding the average of a set of numbers.

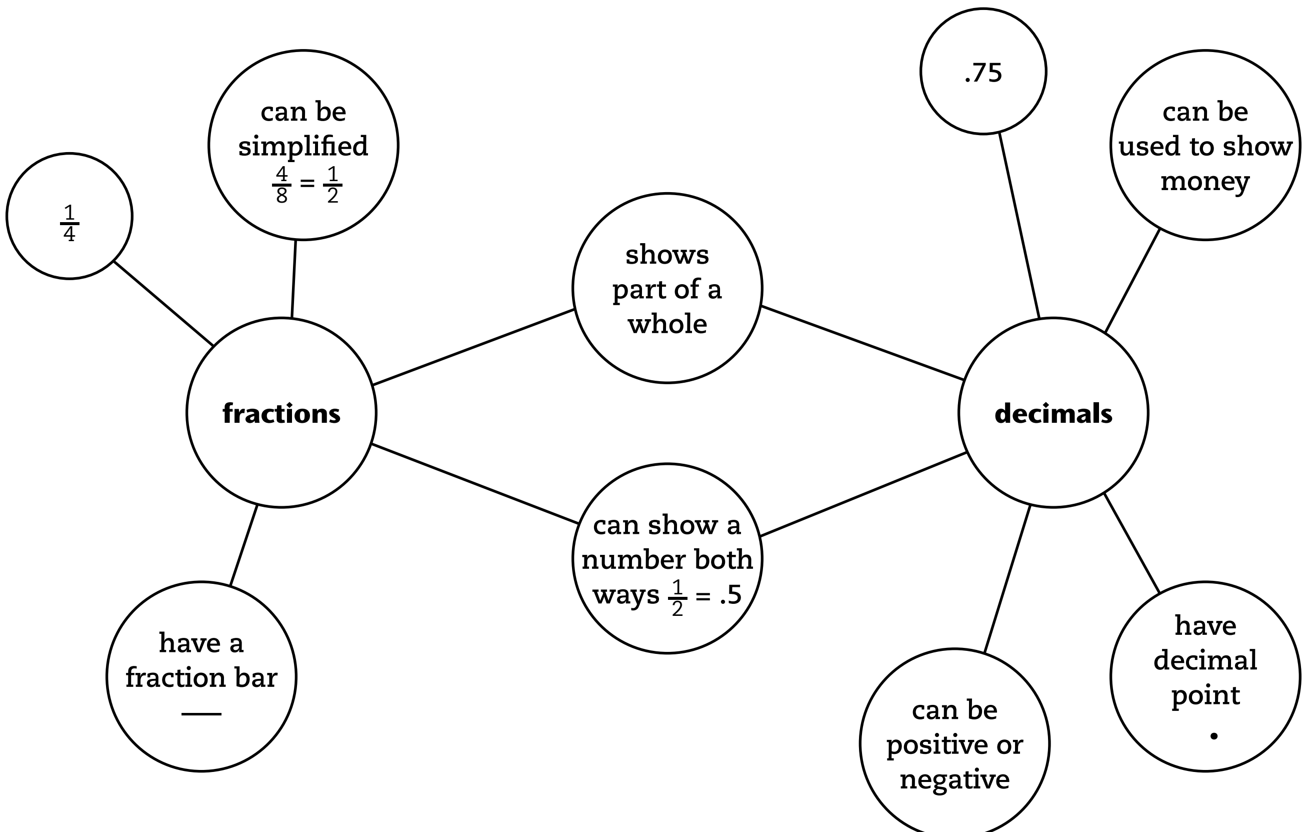
To find the average of a set of numbers:

First, add the numbers in the set to find the total. 20 24 24 23 26 18 21 19 28 + 27 230	**Then, count the numbers to find out how many are in the set.** 20, 24, 24, 23, 26, 18, 21, 19, 28, 27 10 numbers in the set.	**Last, divide the total by the amount of numbers in the set. This number is the average.** $230 \div 10 = 23$ average = 23

Graphic Organizers *(cont.)*

Double Bubble Map Sample

Double-bubble maps are used for comparing and contrasting. This map compares fractions and decimals.



Adapted from David Hyerle (2004).

Mathematics Journals

It is important for students to understand that stories and essays are not the only things that can be written. Mathematics can and should also be written. Mathematics journals are a valuable way to organize thoughts, communicate ideas, and make connections. And, because of the content that can be recorded, these communications are also a valuable assessment tool for teachers.

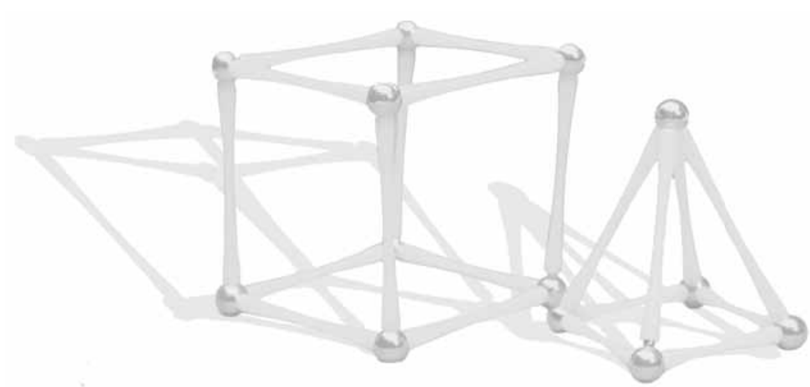
Using journals in the classroom can:

- provide students with an opportunity to reframe newly acquired knowledge into their own words or sketches.
- permit students to review, reiterate, and deepen their understanding of a mathematical concept.
- ensure that students will order their thoughts about a concept or procedure, thus allowing a way to offer their thoughts about thinking and develop their metacognitive skills.
- provide teachers an opportunity to view the various ways the students construct an understanding of a mathematical concept or process (adapted from Cook 1995).

When assessing the content of a student's journal entry, it is important to keep in mind the following suggestions:

- Allow students to respond to journal prompts or questions with written explanations, charts, diagrams, symbols, and drawings. All of these response techniques will provide valuable information.
- The journal response is gauging mathematical understanding, not writing mechanics. Do not grade sentence structure, grammar, etc.
- Students will need various amounts of time to complete the journal entries. Consider allowing them time in class and time at home to complete their thoughts.
- Respond to students' entries by writing back to them in their journals. This will be valuable dialog for both of you and can serve as a record of student progress.
- Allow students to give feedback to one another on their entries.
- The journal response can take the place of other assignments. If journal responses are always an extra assignment, students will not always provide their best work.

The following pages show sample journal responses and prompts.



Mathematics Journals *(cont.)*

Journal Response Sample

This type of journal response allows for teachers and students to have open dialog about a concept. Students receive personal feedback and can correct mistakes in a meaningful way. Many concepts can be explored in this fashion.

Directions: Copy the chart below into your journal. Show your work in the column labeled Work. You may use written computations, diagrams, or drawings to show your work. Write any questions or comments you have in the My Comments column. I will respond to your questions and give you more feedback in the Teacher Comments column.

Problem	Work	My Comments	Teacher Comments
15 ÷ 4 =			
27 ÷ 8 =			

Mathematics Journals *(cont.)*

Journal Prompts and Question Samples

The prompts and questions below can be used with students for mathematics journals.

- What did you learn today in class?
- How can you apply the strategy you learned today in your own life?
- How will the activity we did in class today help you learn things in other classes?
- Write step-by-step directions explaining the procedure we learned in class today. Include visuals and diagrams when appropriate.
- Draw a picture of the concept we discussed today.
- Write a letter to a friend explaining what you learned today.
- Describe the concept we learned in class today for a friend who was absent.
- Write a story for younger students that explains what we learned today.
- Describe a real-life situation where you would need to use the information we learned today.
- Create a poem that discusses the concept we learned today.
- Write a word problem that uses addition. (Teacher Note: You can substitute any other skill you study in your class for addition.)
- Reflect on how you learn mathematics best. Write a letter to me so that I know how you like to learn in class.
- Write notes for a friend who has fallen behind in class. Use diagrams and visuals to explain the concepts.

Rubrics

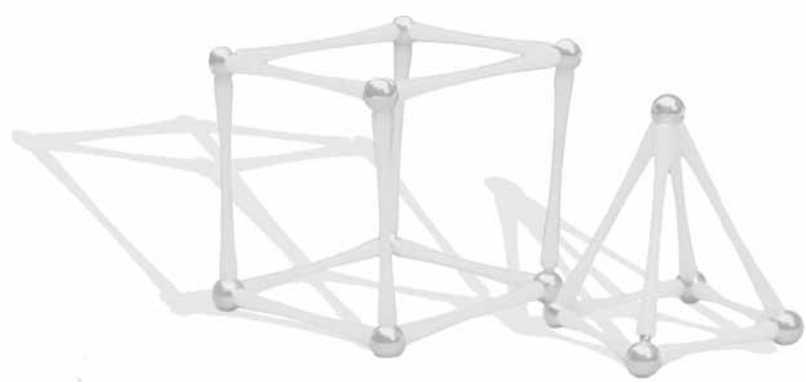
"A rubric is a scoring tool that lists the criteria for a piece of work, or 'what counts'…it also articulates gradations of quality for each criterion, from excellent to poor" (Goodrich 1996). It consists of a grid that outlines the criteria of the project or performance being assessed in the left-most column and the gradations of quality that can exist in the next columns (usually three or four gradations from poor to excellent). The final column(s) are used for scoring. Depending on the project, there can be a column for a self-score, a peer score, and a teacher score.

"Rubrics can improve student performance, as well as monitor it, by making teachers' expectations clear and by showing students how to meet these expectations" (Goodrich 1996). Rubrics also provide students with feedback on their strengths as well as how to improve their performance.

To create a rubric:

1. Think about the end result the students should produce/perform.
2. List criteria of quality work. This is the goal of the project or task and what all students should strive to accomplish.
3. Think about gradations of quality in the work. Describe the best and worst quality and then fill in the middle, based on the list created in the previous step. However, avoid using negative language.
4. Share the rubric with students before beginning the project or task.

The following pages show sample rubrics. Full-size, blank templates of these documents can be found on the Teacher Resource CD in the *Assessment* folder. See page 318 for more details.



Rubrics *(cont.)*

Rubric Sample 1

This generic rubric can be used to evaluate many mathematical tasks, activities, and projects.

Two-Point Rubric

Criteria	1	2	3	Score
The student explains the process or procedure accurately.	The student cannot explain the process or procedure.	The student explains the process or procedure with minimal errors.	The student accurately explains the process or procedure.	
The student demonstrates the process or procedure accurately.	The student cannot demonstrate the process or procedure.	The student demonstrates the process or procedure with minimal errors.	The student accurately demonstrates the process or procedure.	
The solution is correct.	The solution is incorrect.	The solution is partially correct.	The solution is completely correct.	

Rubric adapted from FCAT 2004 Sample Test Materials (Florida Department of Education 2004).

Rubrics *(cont.)*

Rubric Sample 2

This generic rubric can be used to evaluate many mathematical tasks, activities, and projects.

Four-Point Rubric

Points	Criteria
4	• The solution is correct and the student has demonstrated a thorough understanding of the concept or procedure. • The task has been fully completed, using sound mathematical methods. • The response may contain minor flaws, but it is thorough and the understanding is evident.
3	• The solution demonstrates an understanding of mathematical concepts or procedures involved in the task. • The response is mostly correct but contains small errors in execution of mathematical procedures or demonstration of conceptual understanding.
2	• The response is only partially correct. • The solution may be correct, but the response demonstrates only a partial understanding of underlying mathematical concepts or procedures. • Errors show misunderstanding of parts of the task or faulty conclusions.
1	• The response shows limited understanding of mathematical concepts and procedures. • The response is incomplete, and the parts of the problem that have been solved and/or explained contain serious flaws or incomplete conclusions.
0	• The solution is completely incorrect. • The response is incomprehensible and/or demonstrates no understanding of the concept.

Rubric adapted from FCAT 2004 Sample Test Materials (Florida Department of Education 2004).

Portfolios

A portfolio is a collection of student work that can be used to determine a student's understanding, progress, and accomplishments over a period of time. The collection of work is chosen by both student and teacher and can comprise homework, in-class assignments, projects, problem-solving activities, journal entries, or other pieces of work that exemplify the student's understanding. "This not only helps to develop self-assessment skills, but also promotes self-esteem as students decide which entries best indicate their abilities" (Micklo 1997). Since portfolios encompass so many different pieces of work, a student is also able to choose items that demonstrate his or her learning styles.

Portfolios provide teachers with information about students' progress, thought processes, achievements, and needs. By collecting information over a period of time, it will provide teachers with a more accurate picture of students' understanding than a single written test would. Portfolios also facilitate communication about mathematical understanding and progress among students, teachers, and, sometimes, parents.

Portfolios should be used to assess student's general progress and overall mathematical understanding as the year progresses. However, it is important to remember that each piece that is chosen for a portfolio should be assessed for its specific purpose prior to its inclusion in the portfolio. As a whole, a portfolio should measure concepts that are not easily assessed on a written test or on single assignments, such as:

- a student's ability to incorporate mathematical language and symbols into his or her writing
- a student's ability to use pictures and diagrams to help communicate mathematically
- a student's ability to problem solve (understand the problem, choose strategies, and investigate and verify the results)
- a student's ability to demonstrate mathematical reasoning (use facts, properties, and relationships; draw conclusions; justify the answers)
- a student's ability to make mathematical connections (link concepts and procedures, describe relationships among different mathematical topics, and relate mathematics to his or her daily life) (adapted from Micklo 1997)

Once students choose a piece of work to include in the portfolio, they can also complete a reflective personal assessment of the piece. Completing statements such as *I chose this piece for my portfolio because…*, *If I could improve one part of this piece I would…*, or *While working on this piece I learned…* will give teachers further insight into their students' understanding and encourage students to continually reflect on their performance. This type of reflection is called metacognition and is an important skill for students to possess (Desoete 2008).

The following page shows a sample portfolio evaluation. A full-size, blank template of this document can be found on the Teacher Resource CD in the *Assessment* folder. See page 318 for more details.

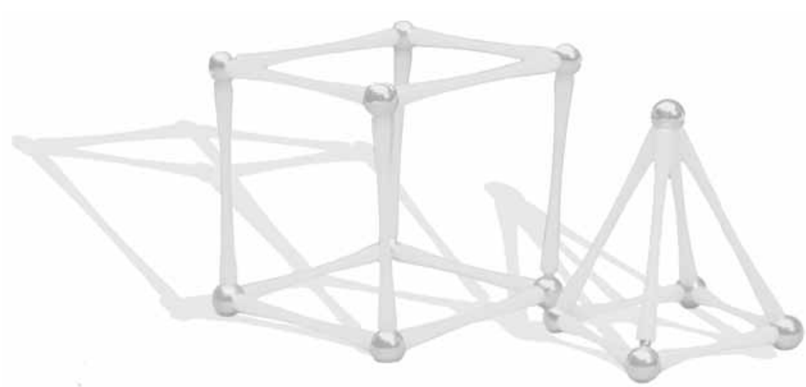
Portfolios *(cont.)*

Portfolio Assessment Sample

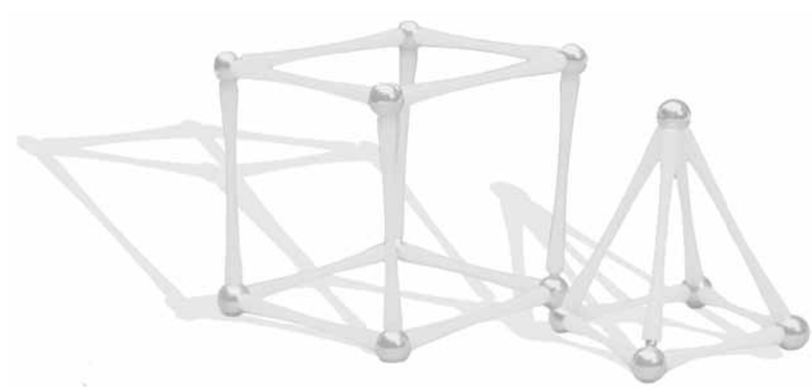
Here are suggestions for assessing a student's portfolio:

Score	Criterion
Excellent	• A wide variety of work was included. • Student takes responsibility in choosing samples. • Work chosen clearly displays the student's learning style. • The number of samples exceeds the minimum requirements. • Samples show strong evidence of student's mathematical language, reasoning, and problem-solving skills.
Good	• A variety of work was included. • Student takes some responsibility in choosing samples. • Work chosen somewhat displays the student's learning style. • The number of samples meets the minimum requirements. • Samples show some evidence of student's mathematical language, reasoning, and problem-solving skills.
Fair	• Some variety of work was included. • Student takes little responsibility in choosing samples. • Work chosen loosely displays the student's learning style. • The number of samples is slightly below the minimum requirements. • Samples show little evidence of student's mathematical language, reasoning, and problem-solving skills.
Poor	• Little variety of work was included. • Student takes no responsibility in choosing samples. • Work chosen doesn't display the student's learning style. • The number of samples is far below the minimum requirements. • Samples show no evidence of student's mathematical language, reasoning, and problem-solving skills.

Notes



Appendices



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Mathematics-Related Children's Literature List

Number Sense		
Title	**Author**	**Interest Level**
The 500 Hats of Bartholomew Cubbins	Seuss	PK–3
The 512 Ants on Sullivan Street	Losi	PK–3
A Million Chameleons	Young	PK–3
A Remainder of One	Pinczes	K–6
Anno's Hat Tricks	Nozaki	PK–3
Anno's Magic Seeds	Nozaki	PK–3
Anno's Mysterious Multiplying Jar	Nozaki	4–6
Arithme-Tickle: An Even Number of Odd Riddle-Rhymes	Lewis	PK–3
At the Edge of the Woods	Cotten	K–3
Bats on Parade	Appelt	PK–3
Ben Franklin and the Magic Squares	Murphy	PK–3
Big Numbers	Packard	PK–3
Divide & Ride	Murphy	PK–3
Each Orange Had 8 Slices	Giganti, Jr.	PK–3
Eating Fractions	McMillan	PK–3
Fraction Fun	Adler	3–6
Gator Pie	Mathews	3–6
George Washington's Teeth	Chandra & Comora	PK–3
A Grain of Rice	Pittman	4–8
The Great Math Tattle Battle	Bowen	2–5
Homecoming	Voigt	6–8
How Many, How Much	Silverstein	6–8
The Icky Bug Counting Book	Pallotta	PK–3
The King's Chessboard	Birch	4–6
The King's Commissioners	Friedman	2–5
The Man Who Counted: A Collection of Mathematical Adventures	Tahan	6–8
Math Curse	Scieszka	6–8
Math for All Seasons	Tang	2–5
Minnie's Diner	Dodds	3–5
The $1.00 Word Riddle Book	Burns	4–6
The Patchwork Quilt	Flournoy	6–8
Piece = Part = Portion: Fractions = Decimals = Percents	Gifford	3–6
Spunky Monkeys on Parade	Murphy	K–3
Uno's Garden	Base	1–8
Zero: Is It Something? Is It Nothing?	Zaslavsky	PK–3

Mathematics-Related Children's Literature List *(cont.)*

Algebra and Functions		
Title	**Author**	**Interest Level**
Among the Odds and Evens: A Tale of Adventure	Turner	4–8
A House for Hermit Crab	Carle	PK–3
Brown Bear, Brown Bear, What Do You See?	Martin, Jr.	PK–3
The Button Box	Reid	PK–3
The Dinosaur Data Book	Lambert	4–8
Do the Math: Secrets, Lies and Algebra	Lichtman	6–8
Each Orange Had 8 Slices	Giganti, Jr.	K–2
Exactly the Opposite	Hoban	PK–3
The Grapes of Math	Tang	4–6
Great Moments in Baseball History	Christopher & Stout	4–6
The Great Turkey Walk	Karr	3–6
How Many Snails?	Giganti Jr.	1–8
I Spy Two Eyes: Numbers in Art	Micklethwait	3–5
If You Hopped Like a Frog	Schwartz	3–6
In My World	Ehlert	PK–3
The King's Chessboard	Birch	4–6
Knots on a Counting Rope	Martin, Archambault, & Rand	PK–3
Math Doesn't Suck: How to Survive Middle School Math With-Out Losing Your Mind or Breaking a Nail	McKellar	6–8
More M & M Chocolate Candies Math	McGrath	4–6
My Father's Boat	Garland	K–5
Over in the Meadow	Keats	PK–3
Pattern (Math Counts)	Pluckrose	PK–3
The Rajah's Rice: A Mathematical Folktale from India	Barry	6–8
Saturday Night at the Dinosaur Stomp	Shields	6–8
Shoes from Grandpa	Fox	PK–3
Six Dinner Sid	Moore	K–2
Ten Black Dots	Crews	PK–3
Tiger Math: Learning to Graph from a Baby Tiger	Nagda & Bickel	4–6
Tools	Morris	PK–3
What's Faster Than a Speeding Cheetah?	Wells	K–3

Mathematics-Related Children's Literature List *(cont.)*

Geometry and Measurement		
Title	**Author**	**Interest Level**
The Adventures of Penrose The Mathematical Cat	Pappas	3–8
Alexander, Who Used to be Rich Last Sunday	Viorst	PK–3
Bats Around the Clock	Appelt	PK–3
Bunny Money	Wells	PK–3
Cubes, Cones, Cylinders, and Spheres	Hoban	PK–3
8 O'Cluck	Creighton	PK–3
Flat Stanley	Brown	PK–3
The Foot Book	Seuss	PK–3
Fun Ideas for Getting A-Round In Math CIRCLES	Ross	6–8
The Grouchy Ladybug	Carle	PK–3
The Greedy Triangle	Burns	1–6
How Big Is a Foot?	Myller	PK–3
How Tall, How Short, How Far Away	Adler	4–6
Inch by Inch	Lionni	PK–3
I Read Symbols	Hoban	PK–3
Is a Blue Whale the Biggest Thing There Is?	Wells	1–8
Jim and the Beanstalk	Briggs	PK–3
Just A Minute	Slater & Burns	PK–3
The Long and Short of It	Nathan & McCourt	K–3
Measuring Penny	Leedy	2–6
Mirette on the High Wire	McCully	PK–3
Money	Cribb	PK–3
My Very First Book of Shapes	Carle	PK–3
"One Inch Tall" from Where the Sidewalk Ends	Silverstein	6–8
Ovals	Burke	3–5
Pigs Will be Pigs	Axelrod	3–5
The Shape of Me and Other Stuff	Seuss	PK–3
The Silly Story of Goldie Locks And the Three Squares	MacCarone	K–2
Sir Cumference and the First Round Table	Neuschwander	6–8
Sir Cumference and the Dragon of Pi	Neuschwander	6–8
Sweet Clara and the Freedom Quilt	Hopkinson	6–8
Telling Time With Big Mama Cat	Harper	PK–3

Mathematics-Related Children's Literature List *(cont.)*

Data Analysis, Probability, and Statistics		
Title	**Author**	**Interest Level**
"A Closet Full of Shoes" from Falling Up	Silverstein	K–4
"A Rice Sandwich" from A House on Mango Street	Cisneros	4–8
All Mixed Up!	Gukova	4–7
A Million Fish…More or Less	McKissack	4–8
Bad Luck Brad	Herman	PK–3
Bart's Amazing Charts	Ochiltree	PK–3
Chrysanthemum	Henkes	PK–3
Cloudy with a Chance of Meatballs	Barrett	PK–3
Country Fair	Gibbons	PK–3
Daley B	Blake	PK–3
The Day Gogo Went to Vote	Sisulu	PK–3
Do You Wanna Bet?	Cushman	4–8
Equal Shmequal	Kroll	6–8
Freight Train	Crews	K–2
Grandfather Tang's Story	Tompert	3–5
Guess Who My Favorite Person Is?	Baylor	All
In the Next Three Seconds	Morgan	PK–3
Jessie Bear, What Will You Wear?	Carlstrom	PK–3
Lemonade For Sale	Murphy	PK–3
The Little Red Hen	retold by Williams	PK–3
The Mouse Who Owned the Sun	Derby	PK–3
The Mysterious Tadpole	Kellogg	PK–3
The Napping House	Wood	PK–3
Pigs at Odds	Axelrod	PK–3
Poppa's New Pants	Medearis	PK–3
The Principal's New Clothes	Calmenson	6–8
Probably Pistachio	Murphy	PK–3
Red is Best	Stineson	PK–3
Round Trip	Jonas	PK–3
Sea Squares	Hulme	3–5
10 for Dinner	Bogart	4–6
Tiger Math	Nagda & Bickel	4–6
The Usborne Book of Facts and Lists	Bresler & Gibson	4–6
Who's Got Spots?	Aber	PK–3
Why Mosquitoes Buzz In People's Ears	retold by Aardema	K–2

Mathematics-Related Children's Literature List (cont.)

Mathematical Reasoning		
Title	**Author**	**Interest Level**
Africa Counts	Zaslavsky	6–8
Afterwards: Folk & Fairy Tales with Mathematical Ever Afters	Kaye	1–6
Arithmetickle	Ashley	4–8
The Art and Craft of Problem Solving	Zeitz	6–8
Chasing Vermeer	Balliet	4–8
Comic Strip Math	Greenberg	4–8
Cool Math	Maganzini	4–8
Counting Caterpillars and Other Math Poems	Franco	PK–3
Dave's Down-To-Earth Rock Shop	Murphy	3–6
Day With No Math	Kaye	4–6
Esio Trot	Dahl	3–8
Every-Day-of-the-School-Year Math Poems	Miller & Lee	3–6
How the Other Half Thinks: Adventures in Mathematical Reasoning	Stein	6–8
The I Hate Mathematics Book	Allison & Burns	4–8
No Fair!	Holtzman & Burns	PK–3
Frog Math	Kopp	PK–3
Math Games & Activities from Around the World	Zaslavsky	4–6
Math for Smarty Pants	Allison & Burns	4–6
Math Rashes: And Other Classroom Tales	Evans	4–6
Math Trek: Adventures in the Math Zone	Peterson & Henderson	4–6
Midnight Math: Twelve Terrific Math Games	Ledwon	PK–3
Mother Goose Math	Schecter	PK–3
Murderous Maths	Poskitt	4–6
My First Number Book	Heinst	PK–3
Purple, Green, and Yellow	Munsch	PK–3
Sideways Arithmetic from Wayside School	Sachar	4–6
Skittles Riddles Math	McGrath	PK–3
Testing Miss Malarkey	Finchler	3–8
25 Mini-Math Mysteries	Johnson	3–6

Grade-Level Vocabulary Lists

Grades K–2 Suggested Vocabulary

Number and Operations

about
add
addition
all
amount
between
cardinal number
closer to
combine
compare
count
difference
digit
double
dozen
equal
equal sign
even number
fact
fewer
fraction
greater than
half
hundreds place
identify
label
least
less
less than
million
likely
list
minus
more
most
near
negative
number
number fact
number line
numeral
odd number
ones place
order
ordinal number
part
property
relationship
represent
rule
similar
solve
strategy
subtract
subtraction
sum
symbol
ten thousands place
tens place
thousands place
total
unequal
unit
value
whole number
zero

Algebra

decreasing pattern
increasing pattern
numeric pattern
pattern
pattern unit
pattern extension
shape pattern

Geometry

above
behind
below
between
circle
corner
edge
face
horizontal
in front
inside
left
near
outside
parallel
rectangle
rhombus
right
side
square
trapezoid
triangle
under
vertical
vertex (vertices)

Measurement

A.M.
afternoon
area
autumn
calendar
Celsius
cent
centimeter
cents
change
clock
coin
cost
counting
cup
customary system
day
degrees
dime
distance
dollar
dollar bill
dollar sign
elapsed time
estimate

Grade-Level Vocabulary Lists *(cont.)*

Grades K–2 Suggested Vocabulary *(cont.)*

coin
Fahrenheit
fall
foot
gallon
gram
half-dollar
half-gallon
half hour
height
hour
inch
inches
kilogram
kiloliter
kilometer
length
liter
location
measure
measuring cup
meter
metric system
metric ton
midnight
mile
milligram
milliliter
millimeter
minute
money
month
morning
nickel
night
noon
ounce
P.M.
penny
pint
pound
quart
quarter
Roman numerals
ruler
season
second
silver dollar
size
spring
stopwatch
summer
temperature
thermometer
time
time interval
ton
value
volume
week
weight
width
winter
yard
yardstick
year

Data Analysis and Probability

bar graph
chance
chart
circle graph
column
data
graph
group
grouping
key
least
less than
likely
more than
most
outcome
pictograph
predict
prediction
table
tallies
tally chart

Problem Solving

diagram
draw
list
manipulative
model
outcome
problem
problem solving
solve

Grade-Level Vocabulary Lists *(cont.)*

Grades 3–5 Suggested Vocabulary

Number and Operation
addend
addition algorithm
approximate
associative property
average
common denominator
common factor
common fractions
common multiple
commutative property
composite number
computation
convert
decimal
denominator
distributive property
divide
dividend
divisibility
division
divisor
equation
equivalent
equivalent fractions
estimate
even number
expanded notation
factor
fraction
function
greatest common factor
improper fraction
inequality
integer
least common multiple
mean
median
mixed number
mode
multiple
multiplication
multiply
negative number
number lines
number sentence
numerator
odd number
operation
order of operations
percent
place value
prime number
product
quotient
range
remainder
reversing order of operations
rounding
rounding numbers
subtraction algorithm
value

Algebra
equation
equivalent representation
expression
geometric pattern
growing pattern
horizontal axis
linear pattern
open sentence
repeating pattern
shrinking pattern
symbol
variable
vertical axis

Geometry
acute
acute angle
angle
area
characteristic
circumference
classes of triangles
congruent
corresponding angle
corresponding side
cube
cylinder
edge
equivalent
face
flip
hexagon
intersecting
isosceles triangle
obtuse angle
octagon
parallel
parallelogram
perpendicular
polygon
prism
pyramid
reflect
reflection
regular
rhombus
right angle
right triangle
rotation
scale
similarity
slide
sphere
spiral
symmetry
three-dimensional
transformation
translation
trapezoid
two-dimensional
turn
vertex (vertices)

Grade-Level Vocabulary Lists *(cont.)*

Grades 3–5 Suggested Vocabulary *(cont.)*

Measurement
afford
allowance
angle
area
budget
capacity
centimeter
circumference
convert
costs
cubic centimeter
cubic inch
cubic unit
customary system
earnings
estimation
expenses
gram
grids
income
kilogram
linear dimensions
mass
measurement
meter
metric system
perimeter
profit
protractor
square foot
square inch
square unit
standard unit
surface area
ton
value
volume

Data Analysis and Probability
analyze
bar graph
chart
certain
certainty
circle graph
compare
comparison
conclusion
data
data cluster
data collection method
display
examine
graph
grid
improbability
label
likelihood
line graph
key
number line
organize
pictograph
possible
probability
sample
sample size
tallies
tally chart
unlikely

Problem Solving
chart
counter example
error
diagram
draw
invalid
list
model
outcome
problem
reasoning
relevant information
results
solution
solve
trial
validity
verification

Grade-Level Vocabulary Lists *(cont.)*

Secondary Suggested Vocabulary

Number and Operations
base
binary system
composite number
constant difference
constant rate of change
constant ratio
cube number
cube root
decrease
equality
equivalent
exponent
exponential
factor
multiple
integer
interval
negative integer
percent
positive integer
prime
prime factor
proportion
quantity
rate
ratio
rational numbers
relative magnitude
Roman numerals
root
scientific notation
square number
square root

Algebra
algebraic expression
algebraic representation
exponential pattern
function
graph
linear equation
linear pattern
maximum value
minimum value
nonlinear equation
nonlinear function
ordered pair
pattern
placeholder
rectangular coordinates
slope
slope-intercept form
table
value
variable
Venn diagram
x-intercept
y-intercept

Geometry
accuracy
angle bisector
array
characteristic
circumference
closed figure
compass
complementary angle
congruent
convert
coordinate geometry
coordinate plane
corresponding
cylinder
degree
diagonal
dilation
edge
endpoint
growth rates
hexagon
intersecting lines
irregular polygon
line segment
model
origin
pentagon
planar cross sections
plane
plane figure
plot
polygon
polyhedron
prism
property
proportion
protractor
Pythagorean theorem
quadrilateral
radius
ray
reflection
rotation
rotational symmetry
scale
scale drawings
segment
semicircle
solid figure
supplementary angle
symmetry
tetrahedron
three-dimensional figure
transformation
translation
vertex
vertices

Measurement
abbreviate
area
capacity
centigram
centiliter
circle formula
circumference
cubic foot
cubic meter

Grade-Level Vocabulary Lists *(cont.)*

Secondary Suggested Vocabulary *(cont.)*

cubic unit
decimeter
diameter
Fahrenheit
fluid ounce
formula
gallon
gram
inch
linear dimensions
metric system
metric ton
mile
millimeter
ounce
perimeter
pint
rate
square centimeter
square yard
surface area
thermometer
volume

Data Analysis and Probability

area model
bar grap
biased sample
box-and-whisker plot
central tendency
clusters
compare
conclusion
data
determine
dispersion
distribution
double bar graph
experimental probability
extremes
fair chance
frequency
frequency distribution
gaps
input/output table
line of best fit
mean
median
mode
multi-line graph
outliers
plot
probability
random samples
range
sample space
sampling error
scatter plot
set
simulation
stem-and-leaf plot
table
theoretical probability
tree diagram
values

Problem Solving

analyze
apply
argument
chart
conclusion
conjecture
counter example
diagram
draw
error
infer
invalid
interpret
investigate
list
outcome
prediction
reasoning
results
solution
summarize
synthesize
validity
verification

Answer Key

***Counting By 2s and 5s* (page 85)**

Each box should contain the correct number of circles as shown below each rectangle.

1. 10
2. 16; 18
3. 14; 16; 18
4. 20; 25; 30
5. 25; 30
6. 15; 20; 25
7. 30; 35; 40

***Using Arrays* (page 88)**

1. Answers will vary. Possible arrays include: 1 × 18; 18 × 1; 2 × 9; 9 × 2; 3 × 6; 6 × 3
2. Answers will vary. Possible arrays include: 1 × 24; 24 × 1; 2 × 12; 12 × 2; 3 × 8; 8 × 3; 4 × 6; 6 × 4
3. Answers will vary. Possible arrays include: 1 × 20; 20 × 1; 2 × 10; 10 × 2; 5 × 4; 4 × 5

***Cups and Chips* (page 91)**

1. $x = 3\frac{2}{8}$ or $3\frac{1}{4}$
2. $x = 5\frac{2}{3}$
3. $x = -2\frac{1}{4}$
4. $x = -9$
5. $x = -7$
6. $x = 4$
7. $x = 3\frac{4}{5}$
8. $x = 10\,\frac{1}{2}$
9. $x = -3\frac{1}{7}$

***Our Pocket Graph* (page 94)**

Graphs will vary. Check student work.

***Areas and Perimeters* (page 97)**

1. A = 32 ft.^{2}; P = 24 ft.; Diagrams will vary.
2. A = 24 ft.^{2}; P = 28 ft.; Diagrams will vary.
3. A = 45 ft.^{2}; P = 28 ft.; Diagrams will vary.
4. A = 44 ft.^{2}; P = 30 ft.; Diagrams will vary.
5. length = 10 ft.; Diagrams will vary.
6. width = 5 in.; Diagrams will vary.
7. 18 feet long; Diagrams will vary.

***Cube Probability* (page 100)**

1. yellow = 15%; red = 30%; blue = 5%; orange = 10%; green = 15%; brown = 25%
2. Answers will vary. Check student work.
3. Answers will vary. Check student work.
4. Answers will vary. Check student work.
5. Answers will vary. Check student work.
6. Answers will vary but should include that experimental probability more closely aligns with theoretical probability the more trials that are performed.

***Addition Fun* (page 103)**

1. 44
2. 78
3. 45
4. 91
5. 89
6. 78
7. 75
8. 86
9. 81

***Daring Division* (page 106)**

1. 126
2. 42
3. 106
4. 214
5. 131
6. 92
7. 33
8. 62
9. 151
10. 41
11. 155
12. 162

Problems illustrated on the place-value mat will vary.

***Combining Like Terms* (page 110)**

1. $5x^2 + 4x - 1$
2. $-x^2 + 5x + 4$
3. $-3x^2 - x - 2$

Answer Key *(cont.)*

Combining Like Terms *(cont.)*

4. $9x^2 - 6x - 4$
5. $10x^2 - 2x - 4$
6. $3x^2 + 12x - 3$
7. $6x^2 + 6x + 5$
8. $6x^2 - 4x - 1$
9. $4x^2 + x + 10$

Pattern Block Combinations **(page 113)**

Answers will vary. Check student work.

Transformations **(page 118)**

Answers will vary. Check student work.

Rotational Symmetry **(page 121)**

Answers will vary. Check student work.

Addition Practice **(page 138)**

1. 57

	TENS	ONES	
15 =	10	5	
42 =	40	2	
	50 +	7	= 57

2. 79

	TENS	ONES	
26 =	20	6	
53 =	50	3	
	70 +	9	= 79

3. 76

	TENS	ONES	
45 =	40	5	
31 =	30	1	
	70 +	6	= 76

4. 94

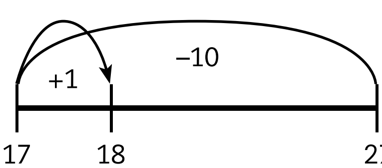
	TENS	ONES	
56 =	50	6	
38 =	30	8	
	80 +	14	= 94

5. 86

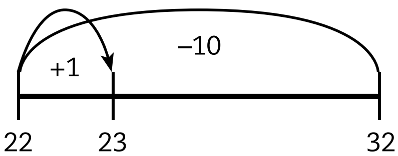
	TENS	ONES	
29 =	20	9	
47 =	40	7	
	60 +	16	= 76

Practicing Subtraction **(page 141)**

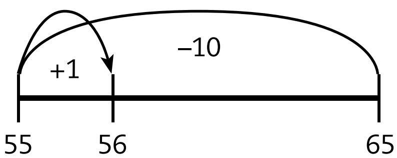
1. 18



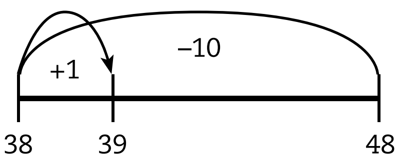
2. 23



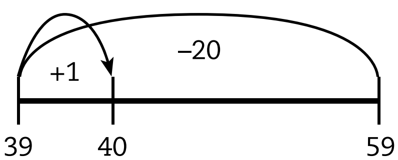
3. 56



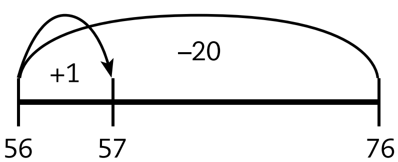
4. 39



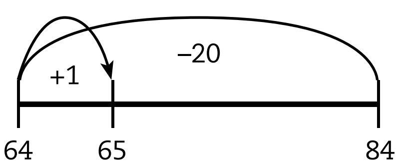
5. 40



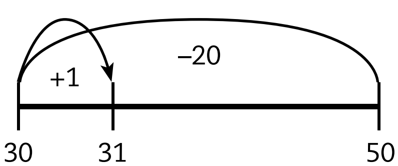
6. 57



7. 65



8. 31



Challenge
Responses will vary.

Multiplication Grids **(page 144)**

1. 126

	20	1	
6	120	6	= 126

2. 100

	20	5	
4	80	20	= 100

3. 62

	30	1	
2	60	2	= 62

Answer Key *(cont.)*

Multiplication Grids *(cont.)*

4. 126

	40	2	
3	120	6	= 126

5. 185

	30	7	
5	150	35	= 185

6. 144

	40	8	
3	120	24	= 144

7. 312

	50	2	
6	300	12	= 312

8. 136

	60	8	
2	120	16	= 136

9. 171

	50	7	
3	150	21	= 171

10. 425

	80	5	
5	400	25	= 425

11. 120

	20	4	
5	100	20	= 120

12. 128

	30	2	
4	120	8	= 128

13. 288

	40	8	
6	240	48	= 288

Challenge
Venn diagrams will vary.

Division Using Multiples (page 147)

1. 22 R5
2. 18 R3
3. 19 R2
4. 47 R4
5. 84
6. 21
7. 32 R5
8. 42 R3
9. 56
10. 48 R6
11. 47 R2
12. Problems and answers will vary.

Working with Fractions (page 150)

1. $\frac{9}{12}$
2. $\frac{6}{8}$
3. $\frac{11}{12}$
4. $\frac{8}{8}$ or 1
5. $\frac{7}{9}$
6. $\frac{7}{10}$
7. $\frac{6}{7}$
8. $\frac{8}{11}$
9. No, $\frac{1}{3} + \frac{1}{3}$ does not equal $\frac{2}{6}$. Explanations will vary but should include that the denominator does not change when adding fractions.
10. $\frac{4}{8} + \frac{3}{8}$; $\frac{7}{8}$
11. $\frac{2}{8} + \frac{2}{8} + \frac{2}{8}$; $\frac{6}{8}$
12. $\frac{6}{15} + \frac{7}{15}$; $\frac{13}{15}$

Missing Numbers (page 153)

1. 3
2. 8
3. 10
4. 9
5. 10
6. 9
7. 11
8. 12

Challenge
Answers will vary.

Simplifying by Combining (page 156)

1. $3x^2 - 5x + 2$
2. $2x^2 + 5x + 10$
3. $-3x^2$ and $8x^2$ are like terms because they both share only the variable x^2; $4xy$ and $8xy$ are like terms because they both share only the variable xy; $-7x^2y$ and $-6x^2y$ are like terms because they both share only the variable x^2y
4. Answers will vary, but should include determining like terms and adding them.
5. Answers will vary, but should include determining like terms and adding them.

Answer Key *(cont.)*

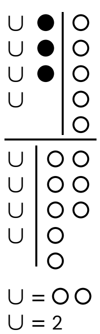
Simplifying by Combining *(cont.)*

6. Answers will vary, but should include that combining like terms means to add together numbers with the same variables.
7. $9a + 5b$
8. $-m + 10n + 4$
9. $-3a - 4b - 6$
10. $3a + 5ab + 11c$
11. $7b + 2a + 3$
12. $-10a + 10b - 6$
13. $23a$
14. Drawings and responses will vary.
15. $17x^2 - 6x + 11$
16. $23a + 19b + 6ab$

Practicing Two-Step Equations **(page 159)**

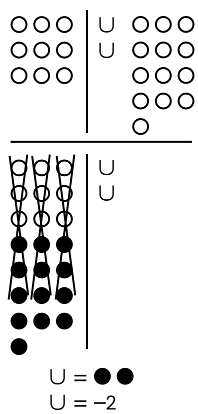
1. $x = 2$

$4x - 3 = 5$



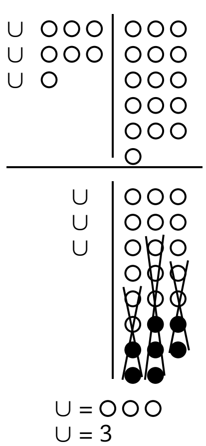
2. $x = -2$

$9 = 2x + 13$



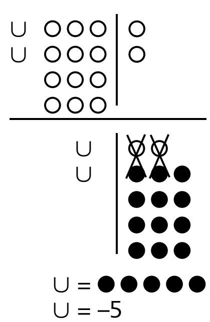
3. $x = 3$

$3x + 7 = 16$



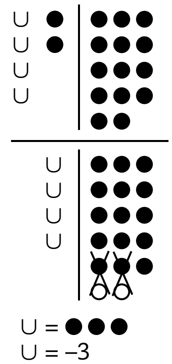
4. $x = -5$

$2x + 12 = 2$



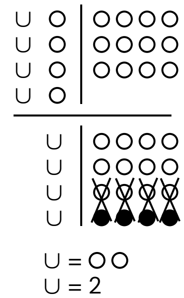
5. $x = -3$

$4x - 2 = -14$



6. $x = 2$

$4x + 4 = 12$



7.
$$\begin{array}{l} 4x - 5 = -37 \\ \quad\underline{+5} \quad \underline{+5} \\ \dfrac{4x}{4} = \dfrac{-32}{4} \\ x = -8 \end{array}$$

8.
$$\begin{array}{l} -9 = 3x + 18 \\ \underline{-18} \quad \underline{-18} \\ \dfrac{-27}{3} = \dfrac{3x}{3} \\ -9 = x \end{array}$$

9.
$$\begin{array}{l} 12x + 23 = 119 \\ \quad\underline{-23} \quad \underline{-23} \\ \dfrac{12x}{12} = \dfrac{96}{12} \\ x = 8 \end{array}$$

10.
$$\begin{array}{l} 10x - 7 = -37 \\ \quad\underline{+7} \quad \underline{+7} \\ \dfrac{10x}{10} = \dfrac{-30}{10} \\ x = -3 \end{array}$$

11.
$$\begin{array}{l} 21x + 8 = -76 \\ \quad\underline{-8} \quad \underline{-8} \\ \dfrac{21x}{21} = \dfrac{-84}{21} \\ x = -4 \end{array}$$

12.
$$\begin{array}{l} 2.1x - 4 = 8.6 \\ \quad\underline{+4} \quad \underline{+4} \\ \dfrac{2.1x}{2.1} = \dfrac{12.6}{2.1} \\ x = 6 \end{array}$$

13.
$$\begin{array}{l} 2(x + 3) = 8 \\ 2x + 6 = 8 \\ \quad\underline{-6} \quad \underline{-6} \\ \dfrac{2x}{2} = \dfrac{2}{2} \\ x = 1 \end{array}$$

14.
$$\begin{array}{l} 2.7(x - 3.4) = 16.2 \\ 2.7x - 9.18 = 16.2 \\ \quad\underline{+9.18} \quad \underline{+9.18} \\ \dfrac{2.7x}{2.7} = \dfrac{25.38}{2.7} \\ x = 9.4 \end{array}$$

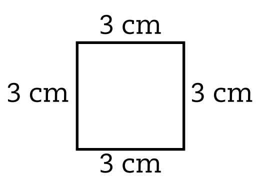
Answer Key *(cont.)*

Working with Area **(page 162)**

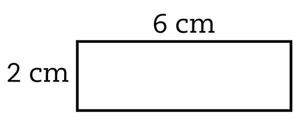
1. 6 square units
2. 18 square units
3. 48 m^2
4. 36 $mi.^2$
5. 40 $ft.^2$
6. $208.00
7. 2 units
8. 7 units
9. 5 units
10. 11 units
11. Responses will vary.
12. 3,600 $ft.^2$

Working with Perimeter **(page 165)**

1. 12 m
2. 76 cm
3. 20 m
4. 8 units
5. 16 units
6. 12 units
7. 34 units
8. 12 cm



9. 16 cm



10. Designs will vary. Check student work.

Working with Volume **(page 168)**

1. 12 cm^3
2. 40 cm^3
3. 45 cm^3
4. 30 cm^3
5. 24 cm^3
6. 42 cm^3
7. 240 cm^3
8. 240 cm^3
9. 2 cm
10. 2 cm
11. 8 cm
12. 20 cm
13. No, that box only holds 2,400 cubic centimeters.
14. 30 cm

Looking at Triangles and Quadrilaterals from All Sides and Angles **(page 171)**

1. Sketches will vary. Check student work.
2. Sketches will vary. Check student work.
3. Sketches will vary. Check student work.
4. Sketches will vary. Check student work.
5. Sketches will vary. Check student work.
6. Sketches will vary. Check student work.
7. True; Explanations will vary but should include that squares have two sets of congruent sides and four 90° angles.
8. True; Explanations will vary but should include that a right triangle can have three different angle measures.
9. False; Explanations will vary but should include that an acute triangle must have three acute angles.
10. True; Explanations will vary but should include that all three angles in the example triangle are acute.
11. 5 cm
12. 7 cm
13. 90°
14. 160°
15. 90°
16. 145°

Classroom Measures **(page 174)**

Items measured and listed in chart will vary.

1. Check student work.
2. Check student work.
3. Check student work.
4. Check student work.

Challenge
Answer will vary.

Answer Key *(cont.)*

***Conversions: All About Changing the Names* (page 177)**

1. 36.8 ounces
2. 4.5 miles
3. 54,400 ounces
4. .2007 km
5. 2,700 mm
6. .00024 km
7. 3.2 decades
8. 240,000 centimeters
9. 2.5 cups
10. It is better to buy 4 pounds for $3.87 because 60 ounces is only 3.75 pounds.
11. 5 ft. 1 in.
12. 9.5 inches
13. Problems will vary. Check student work.
14. Responses will vary. Check student work.

***Tendencies of Beans* (page 183)**

mean = 5

mode = 2

median = 6

Contents of Teacher Resource CD

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52	Root Word Map	page052.pdf
63	Vocabulary Flip Book	page063.pdf

Manipulatives

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85	Counting by 2s and 5s	page085.pdf
88	Using Arrays	page088.pdf
91	Cups and Chips	page091.pdf
94	Our Pocket Graph	page094.pdf
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Teaching Procedures

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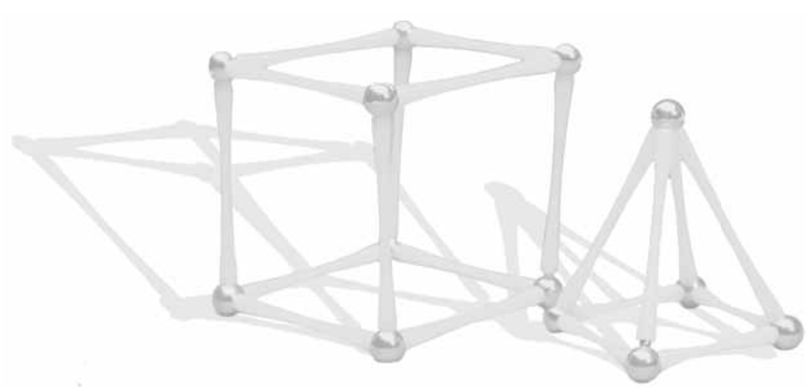

Notes