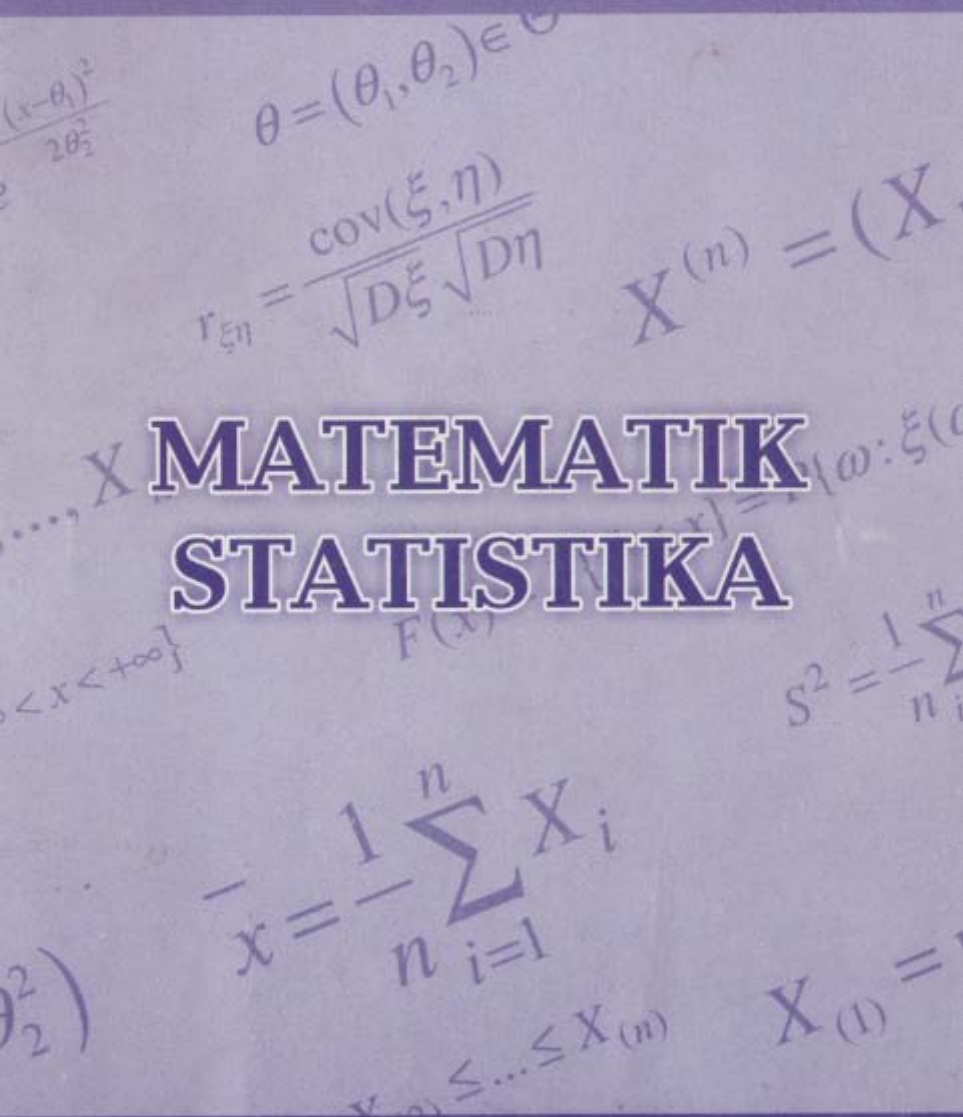


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MATEMATIK STATISTIKA



O'ZBEKISTON RESPUBLIKASI
OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI
MIRZO ULUG'BEK NOMIDAGI
O'ZBEKISTON MILLIY UNIVERSITETI

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MATEMATIK STATISTIKA
*(parametrlarni baholash va gipotezalarni
tekshirish misollarda)*

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Qo'llanma matematik statistika fani chuqur o'rganiladigan universitetlarning talabalari uchun mo'ljallangan bo'lib, u bakalavriatning "Matematika", "Amaliy matematika va informatika", "Informatika va axborot texnologiyalari", "Axborot havfsizligi" va "Mexanika" yo'nalishlari Davlat Ta'lim Standartlariga mos keladi. Ushbu qo'llanmadan "Ehtimollar nazariyasi va matematik statistika" mutaxassisligi magistrantlari, katta ilmiy xodim-izlanuvchilar ham foydalanishlari mumkin. Qo'llanmada matematik statistika bo'yicha asosiy tushunchalar qisqacha keltirilgan bo'lib, 300 dan ziyod misol va masalalar berilgan, hamda ulardan 20 dan ortig'i batafsil yechimi bilan ko'rsatilgan.

Taqrizchilar:

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SO'Z BOSHI

Matematik statistika fani matematikaning tasodifiy tajribalar natijalarini qayta ishlashning matematik usullariga bag'ishlangan bo'limidir. Biz tasodifiy tajriba deganda natijasini oldindan aytib bo'lmaydigan, ya'ni tasodifiyatga bog'liq bo'lgan harakatlar majmuasini tushunamiz. Ma'lumki, ehtimollar nazariyasi tasodifiy jarayonlarni matematik modelini o'rganadi. Agar bu model to'g'ri tanlangan bo'lsa, biz u yoki bu tasodifiy hodisa ehtimolini hisoblay olamiz va ehtimollarning turg'unlik xossasiga asoslangan holda bu hodisalarning ro'y berish sanog'ini taqriban ayta olamiz. Matematik statistika alohida soha sifatida o'rganilsada, uning asosiy usullari ehtimollar nazariyasi doirasida qolaveradi. Buning sababi shundaki, matematik statistika masalalari o'ziga xos hususiyatlarga ega bo'lib, ma'lum ma'noda ehtimollar nazariyasi masalalariga teskaridir. Agar biz ehtimollar nazariyasida tasodifiy jarayonlar modeli ma'lum deb hisoblangan holda shu jarayonlar haqida ma'lum xulosalar qilsak, matematik statistikada esa aksincha qandaydir tasodifiy hodisalarning sonli xarakterga ega bo'lgan statistik ma'lumotlariga asoslangan holda tegishli ehtimollik modelini tanlaymiz. Demak, matematik statistika tasodifiy tajribalarni o'rganish nuqtai nazaridan qaralganda ehtimollar nazariyasini teskari masalalari bilan shug'ullanar ekan.

Ushbu qo'llanmada matematik statistikani o'rganish uchun kerakli barcha tushunchalar keltirilgan bo'lib, unda mustaqil yechish uchun misollar keltirilgan. Ushbu qo'llanma universitetlarning bakalavriatura yo'nalishi talabalari bilan bir qatorda "Ehtimollar nazariyasi va matematik statistika" mutaxassisligi magistrantlari, katta ilmiy xodim-izlanuvchilar uchun ham foydalidir. Bu qo'llanmadan mustaqil ta'lim uchun ham foydalansa bo'ladi. Mualliflar qo'llanmani nashrga tayyorlashdagi yordami uchun N.F.Usmonovaga o'z minnatdorchiliklarini bildiradilar.

1-§. TASODIFIY MIQDORLAR VA ULARNING SONLI KARAKTERISTIKALARI

Agar Ω elementar hodisalar fazosida aniqlangan ξ sonli funksiya har bir ω elementar hodisaga $\xi(\omega)$ sonni mos qo'ysa, ya'ni $\xi = \xi(\omega)$, $\omega \in \Omega$ bo'lsa, u tasodifiy miqdor deyiladi. Demak, tajriba natijasida u yoki bu qiymatni qabul qilishi oldindan ma'lum bo'lmagan miqdor tasodifiy miqdor deb atalar ekan. Agar t.m. ko'pi bilan sanoqli qiymatlar qabul qilsa, bunday t.m. diskret tipdagi t.m. deyiladi. Agar t.m. qabul qiladigan qiymatlari biror oraliqdan iborat bo'lsa, u holda uzluksiz tipdagi t.m. deyiladi.

Demak, diskret t.m. bir-biridan farqli alohida qiymatlarni, uzluksiz t.m. esa biror oraliqdagi ixtiyoriy qiymatlarni qabul qilar ekan.

$F(x)$ funksiya ξ t.m.ning taqsimot funksiyasi $\forall x \in R$ son uchun quyidagicha aniqlanadi: $F(x) = P\{\xi < x\} = P\{\omega : \xi(\omega) < x\}$.

Taqsimot funksiyasi quyidagi xossalarga ega:

1. $F(x)$ chegaralangan: $0 \leq F(x) \leq 1$.
2. $F(x)$ kamaymaydigan funksiya: agar $x_1 < x_2$ bo'lsa, u holda $F(x_1) \leq F(x_2)$.
3. $F(-\infty) = \lim_{x \rightarrow -\infty} F(x) = 0$, $F(+\infty) = \lim_{x \rightarrow +\infty} F(x) = 1$.
4. $F(x)$ funksiya chapdan uzluksiz: $\lim_{x \rightarrow x_0-0} F(x) = F(x_0)$.
5. ξ diskret t.m. t.f. $F(x)$ ning sakrash nuqtalari to'plami ko'pi bilan sanoqlikdir.

ξ t.m. uzluksiz deyiladi, agar uning t.f. ixtiyoriy nuqtada uzluksiz bo'lsa. Uzluksiz t.m. *zichlik funksiyasi* yordamida beriladi.

Uzluksiz t.m. *zichlik funksiyasi* deb, shu t.m. taqsimot funksiyasidan olingan birinchi tartibli hosilaga aytiladi. Zichlik funksiyasini $f(x)$ orqali belgilaymiz:

$$f(x) = F'(x). \quad (1)$$

Zichlik funksiyasi quyidagi xossalarga ega:

1. $f(x)$ funksiya manfiy emas: $f(x) \geq 0$.
2. ξ uzluksiz t.m.ning $[a, b]$ oraliqqa tegishli qiymatni qabul qilishi ehtimolligi zichlik funksiyaning a dan b gacha olingan aniq integraliga

teng:

$$P\{a \leq \xi \leq b\} = \int_a^b f(x) dx. \quad (2)$$

3. Uzluksiz t.m. taqsimot funksiyasi zichlik funksiya orqali quyidagicha ifodalanadi:

$$F(x) = \int_{-\infty}^x f(t) dt. \quad (3)$$

4. Zichlik funksiyasidan $-\infty$ dan $+\infty$ gacha olingan integrali birga tengdir:

$$\int_{-\infty}^{+\infty} f(x) dx = 1.$$

ξ diskret t.m.ning taqsimot qonuni berilgan bo'lsin: $\{p_i = P\{\xi = x_i\}, i = 1, 2, \dots, n, \dots\}$.

ξ diskret t.m.ning matematik kutilmasi $\sum_{i=1}^{\infty} x_i p_i$ qator orqali hisoblanadi va $M\xi$ orqali belgilanadi:

$$M\xi = \sum_{i=1}^{\infty} x_i p_i. \quad (4)$$

ξ uzluksiz t.m.ning matematik kutilmasi

$$M\xi = \int_{-\infty}^{+\infty} x f(x) dx \quad (5)$$

integral orqali hisoblanadi.

ξ t.m.ning dispersiyasi $M(\xi - M\xi)^2$ ifoda bilan aniqlanadi va $D\xi$ orqali belgilanadi. T.m. dispersiyasini hisoblash uchun quyidagi formula qulaydir: $D\xi = M\xi^2 - (M\xi)^2$.

Ikki o'lchovlik tasodifiy miqdorlar. Faraz qilaylik, (Ω, \mathcal{A}, P) ehtimollik fazosida aniqlangan ξ, η t.m.lar berilgan bo'lsin. (ξ, η) vektorga tasodifiy vektor yoki *2-o'lchovli t.m.* deyiladi. Ikki o'lchovli t.m.

har bir elementar hodisa ω ga 2 ta ξ va η t.m.larning qabul qiladigan qiymatlarini mos qo'yadi. (ξ, η) ikki o'lchovli diskret t.m. taqsimot qonunini

$$p_{ij} = P\{\xi = x_i, \eta = y_j\}; \quad i = \overline{1, n}, \quad j = \overline{1, m} \quad (6)$$

sistema yordamida yoki quyidagi jadval ko'rinishda berish mumkin:

ξ	η				$P\{\xi = x_i\} \quad i = \overline{1, n}$
	y_1	y_2	...	y_m	
x_1	p_{11}	p_{12}		p_{1m}	$\sum_{j=1}^m p_{1j}$
x_2	p_{21}	p_{22}		p_{2m}	$\sum_{j=1}^m p_{2j}$
					\vdots
x_n	p_{n1}	p_{n2}		p_{nm}	$\sum_{j=1}^m p_{nj}$
$P\{\eta = y_j\}$					
$j = \overline{1, m}$	$\sum_{i=1}^n p_{i1}$	$\sum_{i=1}^n p_{i2}$		$\sum_{i=1}^n p_{im}$	1

bu yerda barcha p_{ij} ehtimolliklar yig'indisi birga teng (chunki birgalikda bo'lmagan $\{\xi = x_i, \eta = y_j\} \quad i = \overline{1, n}, \quad j = \overline{1, m}$ hodisalar to'la gruppni tashkil etadi: $\sum_{i=1}^n \sum_{j=1}^m p_{ij} = 1$).

Agar (ξ, η) ikki o'lchovli diskret t.m.ning birgalikdagi taqsimot qonuni berilgan bo'lsa, har bir komponentaning alohida (marginal) taqsimot qonunlarini topish mumkin. Har bir $i = \overline{1, n}$ uchun $\{\xi = x_i, \eta = y_1\}, \{\xi = x_i, \eta = y_2\}, \dots, \{\xi = x_i, \eta = y_m\}$ hodisalar birgalikda bo'lmagani sababli: $p_{x_i} = P\{\xi = x_i\} = p_{i1} + p_{i2} + \dots + p_{im}$. Demak, (ξ, η) tasodifiy vektorning marginal taqsimotlari $p_{x_i} = P\{\xi = x_i\} = \sum_{j=1}^m p_{ij}$, $i = \overline{1, n}$, $q_{y_j} = P\{\eta = y_j\} = \sum_{i=1}^n p_{ij}$ $j = \overline{1, m}$.

$F(x, y) = P\{\xi < x, \eta < y\}$ ikki o'lchovli funksiya (ξ, η) tasodifiy vektorning t.f. yoki ξ, η t.m.larning birgalikdagi taqsimot funksiyasi deyiladi.

Ikki o'lchovli $F(x, y)$ t.f.ning asosiy xossalarini keltiramiz:

1. $\forall x, y: 0 \leq F(x, y) \leq 1$, ya'ni taqsimot funksiya chegaralangan.

2. $F(x, y)$ funksiya har qaysi argumenti bo'yicha kamayuvchi emas va chapdan uzluksiz.

3. $F(x, y)$ funksiyaning kamida bir argumenti $-\infty$ ga teng bo'lsa (limit ma'nosida), u holda $F(x, y)$ funksiya nolga teng: $F(x, -\infty) = F(-\infty, y) = F(-\infty, -\infty) = 0$.

4. Agar $F(x, y)$ funksiyaning bitta argumenti $+\infty$ bo'lsa (limit ma'nosida), u holda

$$F(x, +\infty) = F_{\xi}(x); F(+\infty, y) = F_{\eta}(y). \quad (7)$$

5. Agar ikkala argumenti $+\infty$ bo'lsa (limit ma'nosida), u holda $F(+\infty, +\infty) = 1$.

Agar ikki o'lchovlik (ξ, η) t.m. t.f. $F(x, y)$:

1. uzluksiz;
2. har bir argumenti bo'yicha differensiallanuvchi;
3. $F''_{xy}(x, y)$ ikkinchi tartibli aralash hosila mavjud bo'lsa, u holda (ξ, η) uzluksiz t.m. deyiladi.

Ikki o'lchovlik (ξ, η) t.m.ning zichlik funksiyasi

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y} = F''_{xy}(x, y) \quad (8)$$

-ikkinchi tartibli xususiy va aralash hosila orqali aniqlanadi.

$f(x, y)$ zichlik funksiyasi quyidagi xossalarga ega:

1. $f(x, y) \geq 0$;
2. $P\{(\xi, \eta) \in D\} = \iint_D f(x, y) dx dy$; (9)

3. $F(x, y) = \int_{-\infty}^x \int_{-\infty}^y f(u, v) du dv$; (10)

4. $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = 1$;

5. ξ va η t.m.larning marginal zichlik funksiyalarini quyidagi formulalar yordamida topish mumkin:

$$\int_{-\infty}^{+\infty} f(x, y) dy = f_{\xi}(x); \quad \int_{-\infty}^{+\infty} f(x, y) dx = f_{\eta}(y). \quad (11)$$

Agar $\forall x, y \in R$ uchun $\{\xi < x\}$ va $\{\eta < y\}$ hodisalar bog'liqsiz bo'lsa, ξ va η t.m.lar *bog'liqsiz* deyiladi. Endi t.m.lar bog'liqsizligining zarur va yetarli shartini keltiramiz.

Teorema. ξ va η t.m.lar bog'liqsiz bo'lishi uchun barcha x, y lar uchun

$$F(x, y) = F_{\xi}(x)F_{\eta}(y) \quad (12)$$

tenglik bajarilishi zarur va yetarlidir.

ξ va η uzluksiz t.m.lar bog'liqsiz bo'lishi uchun $\forall x, y$ lar uchun $f(x, y) = f_{\xi}(x) f_{\eta}(y)$ tenglikning bajarilishi zarur va yetarlidir;

ξ va η diskret t.m.lar bog'liqsiz bo'lishi uchun esa, ixtiyoriy $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$ larda $P\{\xi = x_i, \eta = y_j\} = P\{\xi = x_i\} P\{\eta = y_j\}$ tenglikning bajarilishi zarur va yetarlidir ($n, m \leq \infty$).

Diskret ξ va η t.m.larning *matematik kutilmalari* birgalikdagi taqsimoti orqali

$$M\xi = m_x = \sum_{i=1}^n \sum_{j=1}^m x_i p_{ij}, \quad M\eta = m_y = \sum_{i=1}^n \sum_{j=1}^m y_j p_{ij} \quad (13)$$

formulalar, uzluksiz t.m.larning *matematik kutilmalari* esa

$$M\xi = m_x = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x f(x, y) dx dy, \quad M\eta = m_y = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} y f(x, y) dx dy \quad (14)$$

integrallar orqali hisoblanadi.

ξ va η t.m.larning *kovariatsiyasi*

$$K_{\xi\eta} = cov(\xi, \eta) = M((\xi - m_x)(\eta - m_y)) \quad (15)$$

matematik kutilma bilan aniqlanadi. Agar (ξ, η) t.m. diskret bo'lsa, ularning kovariatsiyasi $K_{\xi\eta} = \sum_{i=1}^n \sum_{j=1}^m (x_i - m_x)(y_j - m_y)p_{ij}$, agar ular uzluksiz bo'lsa, $K_{\xi\eta} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - m_x)(y - m_y)f(x, y)dx dy$ integral orqali hisoblanadi.

Kovariatsiyani quyidagicha hisoblash ham mumkin:

$$K_{\xi\eta} = cov(\xi, \eta) = M\xi\eta - M\xi M\eta. \tag{16}$$

ξ va η t.m.larning *korrelatsiya koeffitsienti*

$$r_{\xi\eta} = \frac{cov(\xi, \eta)}{\sqrt{D\xi}\sqrt{D\eta}} \tag{17}$$

formula bilan aniqlanadi.

Korrelatsiya koeffitsiyentining xossalari:

1. $|r_{\xi\eta}| \leq 1$, ya'ni $-1 \leq r_{\xi\eta} \leq 1$;
2. Agar $\xi \perp \eta$ bo'lsa, u holda $r_{\xi\eta} = 0$;
3. Agar $|r_{\xi\eta}| = 1$ bo'lsa, u holda ξ va η t.m.lar chiziqli funksional bog'liq va aksincha.

Bir argumentning funksiyalari. Agar ξ t.m.ning har bir qiymatiga biror qoida bo'yicha mos ravishda η t.m.ning bitta qiymati mos qo'yilsa, u holda η ni ξ tasodifiy argumentning funksiyasi deyiladi va $\eta = \varphi(\xi)$ kabi yoziladi.

ξ diskret t.m. x_1, x_2, \dots, x_n qiymatlarini mos p_1, p_2, \dots, p_n ehtimolliklar bilan qabul qilsin: $p_i = P\{\xi = x_i\}$, $i = 1, 2, \dots, n$. Ravshanki, $\eta = \varphi(\xi)$ t.m. ham diskret t.m. bo'ladi va uning qabul qiladigan qiymatlari $y_1 = \varphi(x_1), y_2 = \varphi(x_2), \dots, y_n = \varphi(x_n)$, mos ehtimolliklari esa p_1, p_2, \dots, p_n bo'ladi. Demak, $p_i = P\{\eta = y_i\} = P\{\eta = \varphi(x_i)\}$, $i = 1, 2, \dots, n$. Shuni ta'kidlash lozimki, ξ t.m.ning har xil qiymatlariga mos η t.m.ning bir xil qiymatlari mos kelishi mumkin. Bunday holdalarda qaytarilayotgan qiymatlarning ehtimolliklarini qo'shish kerak bo'ladi.

$\eta = \varphi(\xi)$ t.m.ning matematik kutilmasi va dispersiyasi

quyidagicha aniqlanadi:

$$M\eta = \sum_{i=1}^n \varphi(x_i) p_i, \quad D\eta = \sum_{i=1}^n (\varphi(x_i) - M\eta)^2 p_i.$$

Zichlik funksiyasi $f(x)$ bo'lgan ξ uzluksiz t.m. va η t.m. ξ t.m.ning funksiyasi $\eta = \varphi(\xi)$ bo'lsin. η t.m.ning taqsimotini aniqlaymiz. $\eta = \varphi(\xi)$ funksiya ξ t.m.ning barcha qiymatlarida uzluksiz, (a, b) intervalda qat'iy o'suvchi va differensiallanuvchi bo'lsin, u holda $y = \varphi(x)$ funksiyaga teskari $x = \psi(y)$ funksiya mavjud. η t.m.ning taqsimot funksiyasi $G(y) = P\{\eta < y\}$ formula orqali aniqlanadi. $\{\eta < y\}$ hodisa $\{\xi < \psi(y)\}$ hodisaga ekvivalent. Yuqoridagilarni e'tiborga olsak,

$$\begin{aligned} G(y) &= P(\eta < y) = P(\xi > \psi(y)) = 1 - P(\xi < \psi(y)) = \\ &= 1 - F_{\xi}(\psi(y)) = 1 - \int_a^{\psi(y)} f(x) dx. \end{aligned} \quad (18)$$

(18) ni y bo'yicha differensiallaymiz va η t.m.ning zichlik funksiyasini topamiz: $g(y) = \frac{dG(y)}{dy} = f(\psi(y)) \frac{d}{dy}(\psi(y)) = f(\psi(y))\psi'(y)$.

Demak, $g(y) = f(\psi(y))\psi'(y)$. Agar $y = \varphi(x)$ funksiya (a, b) intervalda qat'iy kamayuvchi bo'lsa, u holda $\{\eta < y\}$ hodisa $\{\xi < \psi(y)\}$ hodisaga ekvivalent. Shuning uchun,

$$G(y) = \int_{\psi(y)}^b f(x) dx = - \int_b^{\psi(y)} f(x) dx.$$

Bu yerdan, $g(y) = -f(\psi(y))\psi'(y)$. Zichlik funksiya manfiy bo'linasligini hisobga olib, bu formulalarni umumlashtirish mumkin: $g(y) = f(\psi(y)) \left| \psi'(y) \right|$.

Agar ξ zichlik funksiyasi $f(x)$ bo'lgan uzluksiz t.m. bo'lsa, u holda $\eta = \varphi(\xi)$ t.m.ning sonli xarakteristikalarini hisoblash uchun

η t.m.ning taqsimotini qo'llash shart emas:

$$M\eta = M(\varphi(\xi)) = \int_{-\infty}^{+\infty} \varphi(x)f(x)dx,$$

$$D\eta = D(\varphi(\xi)) = \int_{-\infty}^{+\infty} (\varphi(x) - M\eta)^2 f(x)dx. \quad (19)$$

Ikki argumentning funksiyalari. Agar ξ va η t.m.lar qabul qiladigan qiymatlarining har bir juftligiga biror qoidaga ko'ra ζ t.m. mos qo'yilsa, u holda ζ t.m. ikki ξ va η tasodifiy argumentning funksiyasi deyiladi va $\zeta = \varphi(\xi, \eta)$ kabi belgilanadi. Agar ξ va η t.m.lar bog'liqsiz bo'lsa, u holda $\zeta = \xi + \eta$ t.m. zichlik funksiyasi

$$f_{\zeta}(z) = f_{\xi+\eta}(z) = \int_{-\infty}^{+\infty} f_{\xi}(x)f_{\eta}(z-x)dx,$$

yoki

$$f_{\zeta}(z) = f_{\xi+\eta}(z) = \int_{-\infty}^{+\infty} f_{\xi}(z-y)f_{\eta}(y)dy \quad (20)$$

formulalar bilan hisoblanadi. Bog'liqsiz t.m.lar yig'indisining taqsimoti shu t.m.lar taqsimotlarining *kompozitsiyasi* deyiladi. ζ t.m.ning zichlik funksiyasi $f_{\xi+\eta} = f_{\xi} * f_{\eta}$ ko'rinishda yoziladi, bu yerda * - kompozitsiya belgisi. Umumiy holda, $\xi \perp \eta$ bo'lsa, kompozitsiyani t.f.lar orqali yozish mumkin:

$$F_{\zeta}(z) = F_{\xi+\eta}(z) = \int_{-\infty}^{+\infty} F_{\eta}(z-x)dF_{\xi}(x) = \int_{-\infty}^{+\infty} F_{\xi}(z-y)dF_{\eta}(y).$$

1-misol. ξ t.m.ning $f(x)$ zichlik funksiyasi berilgan bo'lsin:

$$f(x) = \begin{cases} 0, & x < 1, \\ C/x^A, & x \geq 1. \end{cases} \quad \text{Quyidagilarni hisoblang: a) } C; \text{ b) } F(x);$$

c) $M\xi$; d) $P(\xi > M\xi)$;

▷ a) C sonini zichlikning $\int_{-\infty}^{\infty} f(x)dx = 1$ kossasidan foydalanib hisoblaymiz: $1 = \int_{-\infty}^{\infty} f(x)dx = C \int_1^{\infty} x^{-4}dx = \frac{C}{3}$; $C = 3$.

b) Endi t.f. ni hisoblaymiz: $x \in [1, +\infty)$ $F(x) = \int_1^x \frac{3}{t^4} dt = -t^{-3} \Big|_1^x = 1 - \frac{1}{x^3}$, $x \in (-\infty, 1)$: $F(x) = 0$.

$$\text{Demak, } F(x) = \begin{cases} 1 - \frac{1}{x^3}, & x \geq 1, \\ 0, & x < 1. \end{cases}$$

$$\text{c) } M\xi = \int_{-\infty}^{+\infty} x f(x)dx = \int_1^{+\infty} x \frac{3}{x^4} dx = -3 \frac{x^{-2}}{2} \Big|_1^{+\infty} = \frac{3}{2};$$

$$\text{d) } P(\xi > M\xi) = P(\xi > \frac{3}{2}) = P(\frac{3}{2} < \xi < +\infty) = F(+\infty) - F(\frac{3}{2}) = 1 - \left(1 - \frac{1}{(\frac{3}{2})^3}\right) = \left(\frac{2}{3}\right)^3. <$$

I. ξ t.m.ning $f(x)$ zichlik funksiyasi berilgan bo'lsin:

$$1. f(x) = \begin{cases} Cx, & x \in [0, 1], \\ C, & x \in (1, 2], \\ 0, & \text{aks holda.} \end{cases}$$

$$2. f(x) = \begin{cases} C(1 - x/3), & x \in [0, 3], \\ 0, & x \notin [0, 3]. \end{cases}$$

$$3. f(x) = \begin{cases} 0, & x \leq 0, \\ C/(x+1)^4, & x > 0. \end{cases}$$

$$4. f(x) = \begin{cases} 0, & x < 1, \\ Ce^{1-x}, & x \geq 1. \end{cases}$$

$$5. f(x) = \begin{cases} C/\sqrt{1-x^2}, & x \in [-1, 1], \\ 0, & x \notin [-1, 1]. \end{cases}$$

$$6. f(x) = \begin{cases} C\sqrt{1-x}, & x \in [0, 1], \\ 0, & x \notin [0, 1]. \end{cases}$$

$$7. f(x) = \begin{cases} C/(1+x^2), & x \in [0, \sqrt{3}], \\ 0, & x \notin [0, \sqrt{3}]. \end{cases}$$

$$8. f(x) = \begin{cases} 0, & x \leq 0, \\ Cxe^{-0.5x}, & x > 0. \end{cases}$$

$$9. f(x) = \begin{cases} C/x, & x \in [1/e, e], \\ 0, & x \notin [1/e, e]. \end{cases}$$

$$10. f(x) = \begin{cases} 0, & x < 0, \\ C/(x+1)^5, & x \geq 0. \end{cases}$$

$$11. f(x) = \begin{cases} C(1 - 0.5|x|), & x \in [-2, 2], \\ 0, & x \notin [-2, 2]. \end{cases}$$

$$12. f(x) = \begin{cases} 0, & x \notin (0, \pi/2) \\ C \sin x, & x \in (0, \pi/2) \end{cases}$$

$$13. f(x) = \begin{cases} C \cos x, & x \in [0, \frac{\pi}{2}], \\ 0, & x \notin [0, \frac{\pi}{2}]. \end{cases}$$

$$14. f(x) = \begin{cases} C(|x| + \frac{1}{4}), & x \in [-1, 1], \\ 0, & x \notin [-1, 1]. \end{cases}$$

$$15. f(x) = \begin{cases} 0, & x \leq 0, \\ Cxe^{-x}, & x > 0. \end{cases}$$

$$16. f(x) = \begin{cases} C \ln x, & x \in [1, e], \\ 0, & x \notin [1, e]. \end{cases}$$

$$17. f(x) = \begin{cases} 2x/3, & x \in [0, 1], \\ C(3-x), & x \in (1, 3], \\ 0, & \text{aks holda.} \end{cases}$$

$$18. f(x) = \begin{cases} C(1 - |x|), & x \in [-1, 1], \\ 0, & x \notin [-1, 1]. \end{cases}$$

$$19. f(x) = \begin{cases} C\sqrt[3]{1-x}, & x \in [0, 1], \\ 0, & x \notin [0, 1]. \end{cases}$$

$$20. f(x) = \begin{cases} (x+1)/2, & x \in [-1, 0], \\ (C-x)/2C, & x \in (0, C], \\ 0, & x \notin [-1, C]. \end{cases}$$

Quyidagilarni hisoblang: a) C ; b) $F(x)$; c) $M\xi$; d) $D\xi$; e) $P(\xi > M\xi)$.

II. Bir argumentning funksiyalari

2-misol. ξ t.m. α parametrli ko'rsatkichli taqsimotga ega: $P(\xi < x) = 1 - e^{-\alpha x}$, $x \geq 0$. $\eta_1 = \sqrt{\xi}$ t.m.ning zichlik funksiyasini toping.

▷ Avval η_1 ning taqsimot funksiyasini topamiz: agar $x \geq 0$ bo'lsa, $F_{\eta_1}(x) = P(\sqrt{\xi} < x) = P(\xi < x^2) = F_{\xi}(x^2) = 1 - e^{-\alpha x^2}$; $x < 0$ uchun $F_{\eta_1}(x) = 0$.

Demak, $F_{\eta_1}(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-\alpha x^2}, & x \geq 0, \end{cases}$ t.f.ning zichlik

funksiyasi esa $f_{\eta_1}(x) = (F_{\eta_1}(x))' = \begin{cases} 0, & x < 0, \\ 2\alpha x e^{-\alpha x^2}, & x \geq 0, \end{cases}$ ekan.

◁

3-misol. ξ t.m. $[0, 1]$ oraliqda tekis taqsimlangan. Quyidagi t.m.ning zichlik funksiyasini toping: $\eta_2 = -\ln(1 - \xi)$.

▷ Avval η_2 ning taqsimot funksiyasini topamiz: agar $x > 0$ bo'lsa, $F_{\eta_2}(x) = P(\eta_2 < x) = P(-\ln(1 - \xi) < x) = P(0 \leq \xi \leq 1 - e^{-x}) = F_{\xi}(1 - e^{-x})$. Bundan $f_{\eta_2}(x)$ ni hisoblaymiz. $x > 0$ da: $f_{\eta_2}(x) = F_{\xi}'(1 - e^{-x}) = e^{-x} f_{\xi}(1 - e^{-x}) = e^{-x}$ va $x \leq 0$ da: $f_{\eta_2}(x) = 0$.

◁

Misollar

1. $\xi \sim R(-4, 4)$ bo'lsin, u holda $\eta_1 = \frac{\xi+4}{8}$ va $\eta_2 = \frac{1-\xi}{2}$ t.m.larning zichlik funksiyalarini toping.

2. $\xi \sim E(\alpha)$ bo'lsin, u holda $\eta_1 = \ln(\xi + 1)$ va $\eta_2 = 1/(\xi + 1)$ t.m.larning zichlik funksiyalarini toping.

3. $\xi \sim N(a, \sigma^2)$ bo'lsin, u holda $\eta_1 = 2\xi + 1$ va $\eta_2 = 1 - \xi$ t.m.larning zichlik funksiyalarini toping.

$$4. \xi \text{ t.m.ning t.f. } F_{\xi}(x) = \begin{cases} 0, & x \leq 0, \\ x, & 0 < x < 1, \text{ bo'lsin. } \eta_1 = \xi^2 + 1 \\ 1, & x \geq 1 \end{cases}$$

va $\eta_2 = 1/(1 + \xi)$ t.m.larning zichlik funksiyalarini toping.

5. $\xi \sim R(-1, 1)$ bo'lsin, u holda $\eta_1 = |\xi|$ va $\eta_2 = \frac{|1-\xi|}{2}$ t.m.larning zichlik funksiyalarini toping.

6. $\xi \sim N(a, \sigma^2)$ bo'lsin, u holda $\eta_1 = 5\xi - 1$ va $\eta_2 = -\xi + 1$ t.m.larning zichlik funksiyalarini toping.

7. $\xi \sim E(\alpha)$ bo'lsin, u holda $\eta_1 = \xi^2$ va $\eta_2 = e^{-\xi}$ t.m.larning zichlik funksiyalarini toping.

8. ξ t.m.ning zichlik funksiyasi $f_{\xi}(x) = \frac{1}{\pi} \frac{1}{1+x^2}$ bo'lsin. $\eta_1 = \xi^2$ va $\eta_2 = 1/(1 + \xi^2)$ t.m.larning zichlik funksiyalarini toping.

9. $\xi \sim N(0, 1)$ bo'lsin, u holda $\eta_1 = \xi^2$ va $\eta_2 = \frac{\xi+1}{2}$ t.m.larning zichlik funksiyalarini toping.

10. $\xi \sim R(0, 1)$ bo'lsin, u holda $\eta_1 = -\ln(1 - \xi)$ va $\eta_2 = tg(\pi(\xi - \frac{1}{2}))$ t.m.larning zichlik funksiyalarini toping.

11. $\xi \sim E(\alpha)$ bo'lsin, u holda $\eta_1 = \xi^2 + 1$ va $\eta_2 = \xi/(\xi + 1)$ t.m.larning zichlik funksiyalarini toping.

12. ξ t.m.ning zichlik funksiyasi $f_{\xi}(x) = \frac{1}{\pi} \frac{1}{1+x^2}$ bo'lsin. $\eta_1 = \frac{2\xi}{1-\xi^2}$ va $\eta_2 = \frac{1}{\xi}$ t.m.larning zichlik funksiyalarini toping.

13. $\xi \sim N(5, \sigma^2)$ bo'lsin, u holda $\eta_1 = 3\xi - 1$ va $\eta_2 = -\xi + 2a$ t.m.larning zichlik funksiyalarini toping.

$$14. \xi \text{ t.m.ning t.f. } F_{\xi}(x) = \begin{cases} 0, & x \leq 0, \\ x^3, & 0 < x < 1, \text{ bo'lsin. } \eta_1 = \xi^2 + 1 \\ 1, & x \geq 1 \end{cases}$$

va $\eta_2 = 1/\xi$ t.m.larning zichlik funksiyalarini toping.

15. $\xi \sim R(-2, 2)$ bo'lsin, u holda $\eta_1 = \frac{|\xi|+4}{8}$ va $\eta_2 = \frac{2-\xi}{2}$ t.m.larning zichlik funksiyalarini toping.

16. $\xi \sim E(\alpha)$ bo'lsin, u holda $\eta_1 = \sqrt{\xi + 1}$ va $\eta_2 = \frac{1}{\alpha} \ln \xi$ t.m.larning zichlik funksiyalarini toping.

17. ξ t.m.ning zichlik funksiyasi $f_{\xi}(x) = \frac{1}{\pi} \frac{1}{1+x^2}$ bo'lsin. $\eta_1 = \arctg \xi$ va $\eta_2 = 1/\xi$ t.m.larning zichlik funksiyalarini toping.

18. $\xi \sim N(0, 1)$ bo'lsin, u holda $\eta_1 = \xi^3$ va $\eta_2 = |\xi|$ t.m.larning zichlik funksiyalarini toping.

$$19. \xi \text{ t.m.ning t.f. } F_{\xi}(x) = \begin{cases} 0, & x \leq -1, \\ a(x+1)^2, & -1 < x \leq 2, \\ 1, & x > 2 \end{cases} \text{ bo'lsin.}$$

$\eta_1 = \xi^2 + 1$ va $\eta_2 = 1/\xi$ t.m.larning zichlik funksiyalarini toping.

$$20. \xi \text{ t.m.ning zichlik funksiyasi } f(x) = \begin{cases} \frac{3}{2}x^2, & |x| \leq h, \\ 0, & |x| > h. \end{cases}$$

bo'lsin. $\eta_1 = \xi^2 + 1$ va $\eta_2 = 1/(1+\xi)$ t.m.larning zichlik funksiyalarini toping.

III. Ikki argumentning funksiyalari

4-misol. Agar ξ va η t.m.lar bog'liqsiz bo'lib, $\eta \sim N(0, 1)$, $\xi \sim N(0, 1)$ bo'lsa, $\zeta = \xi + \eta$ ning taqsimotini toping.

▷ (20) formulaga asosan:

$$\begin{aligned} f_{\zeta}(z) &= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-x)^2}{2}} dx = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-\frac{2x^2 - 2zx + z^2}{2}} dx = \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-\frac{2\left(x - \frac{z}{2}\right)^2 + \frac{z^2}{2}}}{2} dx = \frac{1}{2\pi} e^{-\frac{z^2}{4}} \int_{-\infty}^{+\infty} e^{-(x - \frac{z}{2})^2} d\left(x - \frac{z}{2}\right) = \\ &= \frac{1}{2\pi} e^{-\frac{z^2}{4}} \sqrt{\pi} = \frac{1}{\sqrt{2}\sqrt{2\pi}} e^{-\frac{z^2}{2(\sqrt{2})^2}}, \end{aligned}$$

ya'ni $f_{\xi+\eta}(z) = \frac{1}{\sqrt{2}\sqrt{2\pi}} e^{-\frac{z^2}{2(\sqrt{2})^2}}$. Demak, $\zeta \sim N(0, 2)$. ◁

Misolalar

1. ξ_1, \dots, ξ_n - t.m.lar bog'liqsiz va $\xi_i \sim R(1, b)$ bo'lsa, $\eta = \min\{\xi_1, \dots, \xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

2. $\xi \sim N(a_1; 1)$ $\eta \sim N(a_2; 1)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\zeta = \xi + \eta$ t.m.ning taqsimotini toping.

3. $\xi \sim E(1)$, $\eta \sim E(1/2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\zeta = \min\{\xi, 2\eta\}$ t.m.ning taqsimotini toping.

4. ξ_1, \dots, ξ_n - t.m.lar bog'liqsiz va $\xi_i \sim N(a_i, \sigma_i)$, $i = 1, \dots, n$ bo'lsa, $\eta = \sum_{i=1}^n c\xi_i$ t.m.ning taqsimotini toping.

5. $\xi \sim N(0; 1)$, $\eta \sim N(2; 4)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

6. $\xi \sim E(1)$, $\eta \sim E(1/2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta - 1$ t.m.ning taqsimotini toping.

7. ξ_1, ξ_2 - t.m.lar bog'liqsiz va $\xi_i \sim E(\lambda_i)$. $\eta = \max\{\xi_1, \xi_2\}$ t.m.ning zichlik va t.f.ni toping.

8. $\xi \sim R(0; 2)$, $\eta \sim R(0; 2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

9. $\xi \sim N(a_1; 1)$, $\eta \sim N(a_2; 4)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta - 5$ t.m.ning taqsimotini toping.

10. ξ_1, \dots, ξ_n - t.m.lar bog'liqsiz va $\xi_i \sim R(-1, 1)$ bo'lsa, $\eta = \min\{2\xi_1, \dots, 2\xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

11. $\xi \sim R[0; 1]$, $\eta \sim E(1)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

12. ξ_1, ξ_2, ξ_3 - t.m.lar bog'liqsiz va $\xi_i \sim R(0, 1)$ bo'lsa, $\eta = \xi_1 + \xi_2 + \xi_3$ t.m.ning taqsimot funksiyasini toping.

13. $\xi \sim \pi(\lambda_1)$, $\eta \sim \pi(\lambda_2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

14. $\xi \sim (\lambda, \alpha)$, $\eta \sim (\lambda, \alpha)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

15. ξ_1, ξ_2, ξ_3 t.m.lar bog'liqsiz va $\xi_i \sim E(\alpha)$ bo'lsa, $\eta = \xi_1 + \xi_2 + \xi_3$ t.m.ning zichlik funksiyasini toping.

16. ξ_1, \dots, ξ_n - t.m.lar bog'liqsiz va $\xi_i \sim R(a, 1)$ bo'lsa, $\eta = \max\{\xi_1, \dots, \xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

17. $\xi \sim Bi(n_1, p)$, $\eta \sim Bi(n_2, p)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi + \eta$ t.m.ning taqsimotini toping.

18. $\xi \sim N(a_1; 1)$, $\eta \sim N(a_2; 1)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \xi - \eta$ t.m.ning taqsimotini toping.

19. ξ_1, \dots, ξ_n - t.m.lar bog'liqsiz va $\xi_i \sim E(\lambda_i)$ bo'lsa, $\eta = \min\{\xi_1, \dots, \xi_n\}$ t.m.ning zichlik va taqsimot funksiyalarini toping.

20. $\xi \sim R(0; 1)$, $\eta \sim R(0; 2)$ va ξ, η t.m.lar bog'liqsiz bo'lsa, $\varsigma = \eta - \xi$ t.m.ning taqsimotini toping.

IV. Ikki o'lchovlik diskret tasodifiy miqdorlar

5-misol. (ξ, η) tasodifiy vektorning birgalikdagi taqsimoti:

$\eta \setminus \xi$	0	1	Marginal taqsimotlarni, birgalikdagi t.f.ni toping.
0	1/6	2/6	
1	2/6	1/6	

▷ Marginal taqsimotlarini topamiz: $P\{\xi = 0\} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$,
 $P\{\xi = 1\} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$; $P\{\eta = 0\} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$, $P\{\eta = 1\} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$.

Demak, ξ va η t.m.larning alohida taqsimot qonunlari quyidagi ko'rinishga ega bo'ladi: $\left\{ \begin{array}{l} \xi: 0, 1 \\ p: \frac{1}{2}, \frac{1}{2} \end{array} \right.$ va $\left\{ \begin{array}{l} \eta: 0, 1 \\ p: \frac{1}{2}, \frac{1}{2} \end{array} \right.$.

(ξ, η) ikki o'lchovlik t.m.ning $F(x, y)$ taqsimot funksiyasini $F(x, y) = \sum_{x_i < x} \sum_{y_j < y} p_{ij}$ formulaga ko'ra hisoblaymiz:

$F(x, y)$	$x \leq 0$	$0 < x \leq 1$	$x > 1$
$y \leq 0$	0	0	0
$0 < y \leq 1$	0	1/6	1/2(=1/6+2/6)
$y > 1$	0	1/2(=1/6+2/6)	1(=1/6+2/6+2/6+1/6)

◁

Misollar

 (ξ, η) tasodifiy vektorning birgalikdagi taqsimoti berilgan:1.

$\xi \setminus \eta$	1	2	3
0.1	0.12	0.08	0.40
0.2	0.16	0.10	0.14

2.

$\xi \setminus \eta$	-1	0	1
0	0.1	0.2	0.1
1	0.2	0.3	0.1

3.

$\xi \setminus \eta$	-1	0	1
1	2/9	1/9	0
2	1/9	0	1/9
3	2/9	1/9	1/9

4.

$\xi \setminus \eta$	-1	0	1	2
-1	0.07	0.04	0.1	0.12
0	0.08	0.1	0.07	0.08
1	0.09	0.13	0.1	0.02

5.

$\xi \backslash \eta$	1	2	3	4
1	0.07	0.04	0.11	0.11
2	0.08	0.11	0.06	0.08
3	0.09	0.13	0.1	0.02

6.

$\xi \backslash \eta$	0	1
0	1/8	0
1	1/4	1/8
2	1/8	3/8

7.

$\xi \backslash \eta$	0	1	2
-1	1/4	1/8	1/8
1	1/8	1/8	1/4

8.

$\xi \backslash \eta$	1	2	3
0	C	0.15	0.15
1	0.2	0.3	C

9.

$\xi \backslash \eta$	-1	0	1	2
-1	0.05	0.3	0.15	0.05
1	0.1	0.05	0.25	0.05

10.

$\xi \backslash \eta$	1	2	3
1	2/9	1/9	0
2	1/9	0	1/9
3	2/9	1/9	1/9

11.

$\xi \backslash \eta$	-1	1
0	1/8	0
1	1/4	1/8
2	1/8	3/8

12.

$\xi \backslash \eta$	2	4	6	8
-1	0.1	0.3	0.1	0.1
1	0.1	0	0.25	0.25

13.

$\xi \backslash \eta$	-1	0	1
0	0	0.1	0.4
1	0.2	0.2	0.1

14.

$\xi \backslash \eta$	-1	0	1
0	0.15	0.3	0.3
1	0.1	0.05	0.1

15.

$\xi \backslash \eta$	0	1	2	3
-1	0.05	0.12	0.08	0.04
1	0.09	0.3	0.11	0.21

16.

$\xi \backslash \eta$	4	5
3	0.17	0.1
10	0.13	0.3
12	0.25	0.05

17.

$\xi \backslash \eta$	2	10	12	20
0	0.05	0.11	0.08	0.05
1	0.1	0.3	0.11	0.20

18.

$\xi \backslash \eta$	10	20	30
-10	0.1	0.25	0.15
10	0.15	0.1	0.25

19.

$\xi \setminus \eta$	1	2	3	4
1	0.05	0.04	0.1	0.11
2	0.1	0.1	0.1	0.1
3	0.11	0.1	0.04	0.05

20.

$\xi \setminus \eta$	0	1	2
0	1/4	1/3	1/8
1	1/8	1/16	0
2	C	0	0

Marginal taqsimotlarni, birgalikdagi t.f. $F(x, y)$ ni, $\eta/\xi = x$; shartli taqsimotlarni toping va sonli xarakteristikalarini hisoblang.

V. Ikki o'lchovlik uzluksiz tasodifiy miqdorlar

6-misol.

(ξ, η) ikki o'lchovli t.m.ning birgalidagi zichlik funksiyasi berilgan

$$f(x, y) = \begin{cases} Ce^{-x-y}, & \text{agar } x \geq 0, y \geq 0, \\ 0, & \text{aks holda.} \end{cases}$$

Quyidagilarni toping: 1) C ; 2) $F(x, y)$; 3) F_ξ va F_η ; 4) f_ξ va f_η ; 5) ξ va η t.m.lar bog'liqsizmi?

▷ 1) $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) dx dy = 1$ tenglikdan

$$C \int_0^{+\infty} \int_0^{+\infty} e^{-x-y} dx dy = C \int_0^{+\infty} e^{-x} dx \int_0^{+\infty} e^{-y} dy = C = 1.$$

$$2) F(x, y) = \begin{cases} \int_0^x \int_0^y e^{-u-v} du dv = \int_0^x e^{-u} du \int_0^y e^{-v} dv = \\ = (1 - e^{-x})(1 - e^{-y}), & x \geq 0, y \geq 0, \\ 0, & \text{aks holda.} \end{cases}$$

Demak, $F(x, y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}), & x \geq 0, y \geq 0, \\ 0, & \text{aks holda.} \end{cases}$

3) $x \geq 0$ da:

$$F_\xi(x) = F(x, +\infty) = \int_0^x \left(\int_0^{+\infty} e^{-u} e^{-v} dv \right) du = \int_0^x e^{-u} du = 1 - e^{-x},$$

demak,

Aynan shunday,

$$F_{\eta}(x) = \begin{cases} 1 - e^{-y}, & y \geq 0, \\ 0, & y < 0. \end{cases}$$

$$4) f_{\xi}(x) = F'_{\xi}(x) = \begin{cases} (1 - e^{-x})'_x, & x \geq 0, \\ 0, & x < 0, \end{cases} = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

va shu kabi $f_{\eta}(y) = \begin{cases} e^{-y}, & y \geq 0, \\ 0, & y < 0. \end{cases}$

$$5) f(x, y) = \begin{cases} e^{-x-y}, & x \geq 0, y \geq 0, \\ 0, & \text{aks holda,} \end{cases} \quad f_{\xi}(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

$f_{\eta}(y) = \begin{cases} e^{-y}, & y \geq 0, \\ 0, & y < 0, \end{cases}$ $f(x, y) = f_{\xi}(x)f_{\eta}(y)$ tenglik o'rinli, demak, ξ va η t.m.lar bog'liqsiz. \triangleleft

Misollar

a) (ξ, η) tasodifiy vektorning birgalikdagi zichlik funksiyalari berilgan:

1. $f(x, y) = \begin{cases} C(x^2 + y^2), & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$
2. $f(x, y) = \begin{cases} C(x^3 + y^3), & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$
3. $f(x, y) = \begin{cases} C(x^4 + y^4), & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$
4. $f(x, y) = \begin{cases} C(x^2 + y), & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$
5. $f(x, y) = \begin{cases} C(2x^2 + y^2), & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$
6. $f(x, y) = \begin{cases} Cxy, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$
7. $f(x, y) = \begin{cases} Cx^2y^2, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$
8. $f(x, y) = \begin{cases} Cx^3y^3, & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{aks holda.} \end{cases}$

$$9. f(x, y) = \begin{cases} C(x+y)(1-x)(1-y), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

$$10. f(x, y) = \begin{cases} Cxy, 0 \leq x \leq 1, 0 \leq y \leq 1-x, \\ 0, \text{ aks holda.} \end{cases}$$

$$11. f(x, y) = \begin{cases} C(1+xy), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

$$12. f(x, y) = \begin{cases} C(1-xy), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

$$13. f(x, y) = \begin{cases} C(1+x^2y^2), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

$$14. f(x, y) = \begin{cases} C(1-x^2y^2), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

$$15. f(x, y) = \begin{cases} C(x-y)^2, 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

$$16. f(x, y) = \begin{cases} Ce^{-(x+2y)}, x \geq 0, y \geq 0 \\ 0, \text{ aks holda.} \end{cases}$$

$$17. f(x, y) = \begin{cases} Ce^{-2(x+y)}, x \geq 0, y \geq 0 \\ 0, \text{ aks holda.} \end{cases}$$

$$18. f(x, y) = \begin{cases} Ce^{-(2x+y)}, x \geq 0, y \geq 0 \\ 0, \text{ aks holda.} \end{cases}$$

$$19. f(x, y) = \begin{cases} Cxy(x+y), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

$$20. f(x, y) = \begin{cases} Cxy(x-2y), 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, \text{ aks holda.} \end{cases}$$

Quyidagilarni hisoblang: $C, f(x), f(y), F(x, y), M\xi, M\eta, D\xi, D\eta, \rho(\xi, \eta)$, va ξ, η t.m.larni bog'liqsizlikga tekshiring.

7-misol. (ξ, η) tasodifiy vektor D sohada tekis taqsimlangan bo'lsin: D: $x^2 + y^2 \leq 1, y \geq 0$ doiraning yarmi. Birgalikdagi zichlik

funksiyasi $f(x, y)$ ni tuzing va $\rho(\xi, \eta)$ ni hisoblang.

\triangleright (ξ, η) tasodifiy vektor D sohada tekis taqsimlangan bo'lsa, uning zichlik funksiyasi $f(x, y) = \begin{cases} \frac{1}{S(D)}, & (x, y) \in D, \\ 0, & (x, y) \notin D \end{cases}$ bo'ladi.

Bu yerda $S(D)$ - D sohaning yuzi. Demak, $S(D) = \frac{1}{2}\pi R^2$, $R = 1$ va $f(x, y) = \begin{cases} \frac{2}{\pi}, & (x, y) \in D, \\ 0, & (x, y) \notin D. \end{cases}$ Endi (ξ, η) t.m.ning momentlarini hisoblaymiz:

$$M\xi = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x f(x, y) dx dy = \int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} x \frac{2}{\pi} dy =$$

$$-\frac{1}{\pi} \int_0^1 \sqrt{1-x^2} d(1-x^2) = -\frac{2}{3} \frac{1}{\pi} (1-x)^{3/2} \Big|_0^1 = \frac{2}{3\pi}.$$

$$M\eta = \int_{-1}^1 \int_0^{\sqrt{1-x^2}} y \frac{2}{\pi} dx dy = \frac{1}{\pi} \int_{-1}^1 [y^2]_0^{\sqrt{1-x^2}} dx = \frac{1}{\pi} \int_{-1}^1 (1-x^2) dx =$$

$$= \frac{2}{\pi} \int_0^1 (1-x^2) dx = \frac{2}{\pi} \left[x - \frac{x^3}{3} \right]_0^1 = \frac{2}{\pi} \left(1 - \frac{1}{3} \right) = \frac{4}{3\pi}.$$

$$D\xi = M(\xi - M\xi)^2 = \int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x - M\xi)^2 \frac{2}{\pi} dx dy = \frac{2}{\pi} \int_{-1}^1 \int_0^{\sqrt{1-x^2}} x^2 dx dy =$$

$$= \frac{2}{\pi} \int_{-1}^1 x^2 \left(\int_0^{\sqrt{1-x^2}} dy \right) dx = \frac{2}{\pi} \int_{-1}^1 x^2 [y]_0^{\sqrt{1-x^2}} dx = \frac{2}{\pi} \int_{-1}^1 x^2 \sqrt{1-x^2} dx =$$

$$= \left[\begin{array}{l} x = \sin t \\ dx = \cos t dt \\ \pm 1 \rightarrow \pm \frac{\pi}{2} \end{array} \right] = \frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t \cos t \cos t dt = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 \sin^2 t \cos^2 t dt =$$

$$\begin{aligned}
&= \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 2t dt = \frac{1}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \cos 4t) dt = \frac{1}{4\pi} \left[t - \frac{1}{4} \sin 4t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \\
&= \frac{1}{4\pi} \left[\left(\frac{\pi}{2} - 0 \right) - \left(-\frac{\pi}{2} - 0 \right) \right] = \frac{1}{4\pi} = \frac{1}{4}.
\end{aligned}$$

$$\begin{aligned}
D\eta &= M(\eta - M\eta)^2 = M\eta^2 - (M\eta)^2 = \int_{-1}^1 \int_0^{\sqrt{1-x^2}} y^2 \frac{2}{\pi} dx dy - \left(\frac{4}{3\pi} \right)^2 = \\
&= \frac{2}{\pi} \int_{-1}^1 \frac{y^3}{3} \Big|_0^{\sqrt{1-x^2}} dx - \frac{4^2}{(3\pi)^2} \frac{2}{3\pi} \int_{-1}^1 (1-x^2) \sqrt{1-x^2} dx - \frac{16}{3^2\pi^2} = \\
&= \left[\begin{array}{l} x = \sin t \\ dx = \cos t dt \\ \pm 1 \rightarrow \pm \frac{\pi}{2} \end{array} \right] = \frac{2}{3\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \sin^2 t) \sqrt{1 - \sin^2 t} \cos t dt - \frac{16}{9\pi^2} = \\
&= \frac{2}{3\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t \cos t \cos t dt - \frac{16}{9\pi^2} = \frac{2}{3\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos^2 t)^2 dt - \frac{16}{9\pi^2} = \\
&= \frac{1}{6\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos 2t)^2 dt - \frac{16}{9\pi^2} = \frac{1}{6\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + 2\cos 2t + \cos^2 2t)^2 dt - \\
&- \frac{16}{9\pi^2} = \frac{1}{6\pi} t \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{6\pi} \sin 2t \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{6\pi} \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos 4t) dt - \frac{16}{9\pi^2} = \\
&= \frac{1}{6\pi} \pi + \frac{1}{12\pi} \pi + \frac{1}{12\pi} \frac{1}{4} \sin 4t \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{16}{9\pi^2} = \frac{1}{6} + \frac{1}{12} - \frac{16}{9\pi^2} = \\
&= \frac{1}{4} - \frac{16}{9\pi^2} = \frac{9\pi^2 - 64}{36\pi^2}.
\end{aligned}$$

$$\begin{aligned}
 M\xi\eta &= \int_{-1}^1 \int_0^{\sqrt{1-x^2}} xy \frac{2}{\pi} dx dy = \frac{2}{\pi} \int_{-1}^1 x \left(\int_0^{\sqrt{1-x^2}} y dy \right) dx = \\
 &= \frac{2}{\pi} \int_{-1}^1 x \left[\frac{y^2}{2} \right]_0^{\sqrt{1-x^2}} dx = \frac{1}{\pi} \int_{-1}^1 x [y^2]_0^{\sqrt{1-x^2}} dx = \\
 &= \frac{1}{\pi} \int_{-1}^1 x(1-x^2) dx = \frac{1}{2\pi} \int_{-1}^1 (1-x^2) dx^2 = \\
 &= -\frac{1}{2\pi} \int_{-1}^1 (1-x^2) d(1-x^2) = -\frac{1}{2\pi} \left[\frac{(1-x^2)^2}{2} \right]_{-1}^1 = \\
 &= -\frac{1}{2\pi} (0-0) = 0.
 \end{aligned}$$

$$cov(\xi; \eta) = 0. \quad \rho(\xi; \eta) = \frac{cov(\xi; \eta)}{\sqrt{D\xi}\sqrt{D\eta}} = \frac{0}{\sqrt{\frac{1}{4}}\sqrt{\frac{9\pi^2-64}{36\pi^2}}} = 0.$$

Misollar

- b) (ξ, η) tasodifiy vektor D sohada tekis taqsimlangan bo'lsin:
1. $D: x^2 + y^2 \leq 1, x \geq 0, y \geq 0$ -doiraning to'rtidan biri.
 2. $D: \frac{x^2}{9} + \frac{y^2}{4} \leq 1, x \geq 0, y \geq 0$ -ellipsning to'rtidan biri.
 3. $D: \frac{x^2}{4} + y^2 \leq 1, x \geq 0, y \geq 0$ -ellipsning to'rtidan biri.
 4. $D: \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1, x \geq 0, y \geq 0$ -ellipsning to'rtidan biri.
 5. $D: x^2 + (y-1)^2 \leq 1, y \geq 1$ - doiraning yarmi.
 6. $D: x^2 + y^2 \leq 1, x \geq 0$ -doiraning yarmi.
 7. $D: x^2 + y^2 \leq 1$ -doira.
 8. D : uchlari $O(0,0)$, $A(1,0)$, $B(0,1)$ nuqtalarda bo'lgan uchburchak.
 9. D : uchlari $A(1,0)$, $B(1,1)$, $C(0,1)$ nuqtalarda bo'lgan uchburchak.
 10. D : uchlari $A(-1,0)$, $O(0,0)$, $B(0,-1)$ nuqtalarda bo'lgan uchburchak.

11. D: uchlari A(1,1), B(-1,1), C(1,-1), D(-1,-1) nuqtalarda bo'lgan to'rtburchak.

12. D: uchlari O(0,0), A(1,0), B(1,1), C(0,1) nuqtalarda bo'lgan to'rtburchak.

13. D: uchlari O(0,0), A(0,1), B(-1,1), C(-1,0) nuqtalarda bo'lgan to'rtburchak.

14. D: uchlari O(0,0), A(1,1), B(0,1), C(1,0) nuqtalarda bo'lgan to'rtburchak.

15. D: $y = x^2, y = 0, x = 1$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

16. D: $y = x^2, y = 1, x = 0$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

17. D: $y = \sqrt{x}, y = 0, x = 1$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

18. D: $y = \sqrt{1-x}, y = 0, x = 0$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

19. D: $y = x^3, y = 0, x = 1$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

20. D: $y = x^3, y = 1, x = 0$ chiziqlar bilan chegaralangan egri chiziqli uchburchak.

Birgalikdagi zichlik funksiyasi $f(x, y)$ ni tuzing va $\rho(\xi, \eta)$ ni hisoblang.

VI. Katta sonlar qonuni

Agar $\forall \varepsilon > 0$ son uchun

$$\lim_{n \rightarrow \infty} P\{|X_n - A| < \varepsilon\} = 1$$

munosabat o'rinli bo'lsa, u holda $X_1, X_2, \dots, X_n, \dots$ t.m.lar o'zgarmas son A ga ehtimollik bo'yicha yaqinlashadi deyiladi. Ehtimollik bo'yicha yaqinlashish $X_n \xrightarrow{P} A$ orqali belgilanadi.

$X_1, X_2, \dots, X_n, \dots$ t.m.lar ketma-ketligi mos ravishda $MX_1, MX_2, \dots, MX_n, \dots$ matematik kutilmalarga ega bo'lib, $\forall \varepsilon > 0$ son uchun $n \rightarrow \infty$ da

$$\lim_{n \rightarrow \infty} P\left\{\left|\frac{1}{n} \sum_{i=1}^n X_i - \frac{1}{n} \sum_{i=1}^n MX_i\right| < \varepsilon\right\} = 1$$

munosabat bajarilsa, u holda $X_1, X_2, \dots, X_n, \dots$ t.m.lar ketma-ketligi katta sonlar qoniniga bo'ysunadi deyiladi.

Teorema(Chebishev). Agar bog'liqsiz $X_1, X_2, \dots, X_n, \dots$ t.m.lar ketma-ketligi uchun shunday $\exists C > 0$ bo'lib, $DX_i \leq C, i = 1, 2, \dots$ tengsizliklar o'rinli bo'lsa, u holda $\forall \varepsilon > 0$ son uchun

$$\lim_{n \rightarrow \infty} P \left\{ \left| \frac{1}{n} \sum_{i=1}^n X_i - \frac{1}{n} \sum_{i=1}^n MX_i \right| < \varepsilon \right\} = 1 \quad (21)$$

munosabat o'rinli bo'ladi.

Bu teoremani isbotlashda quyidagi Chebishev tengsizligidan foydalaniladi: $\forall \varepsilon > 0$ uchun quyidagi tengsizlik o'rinli:

$$P \{ |X - MX| \geq \varepsilon \} \leq \frac{DX}{\varepsilon^2}. \quad (22)$$

Agar $X_1, X_2, \dots, X_n, \dots$ bog'liqsiz va bir xil taqsimlangan t.m.lar, hamda $MX_i = a, DX_i = \sigma^2$ bo'lsa, u holda $\forall \varepsilon > 0$ son uchun quyidagi munosabat o'rinli

$$\lim_{n \rightarrow \infty} P \left\{ \left| \frac{1}{n} \sum_{i=1}^n X_i - a \right| < \varepsilon \right\} = 1. \quad (23)$$

Bernulli teoremasi KSQ ning sodda shakli hisoblanadi. U nisbiy chastotaning turg'unligini asoslaydi.

Teorema(Y. Bernulli). Agar A hodisaning bitta tajribada ro'y berishi ehtimolligi p bo'lib, n ta bog'liqsiz tajribada bu hodisa n_A marta ro'y bersa, u holda $\forall \varepsilon > 0$ son uchun

$$\lim_{n \rightarrow \infty} P \left\{ \left| \frac{n_A}{n} - p \right| < \varepsilon \right\} = 1 \quad (24)$$

munosabat o'rinlidir.

8-misol. $\{\xi_k\}$ - bog'liqsiz t.m.lar ketma-ketligi bo'lib, uning taqsimoti $P(\xi_k = a) = \frac{k}{2k+1}, P(\xi_k = -a) = \frac{k+1}{2k+1}$ bo'lsa, $\{\xi_k\}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

▷ KSQ o'rinli ekanligini Chebishev teoremasidan foydalanib tekshiramiz. Buning uchun DX_k ni hisoblaymiz:

$$MX_k = a \left(\frac{k}{2k+1} - \frac{k+1}{2k+1} \right) = \frac{-a}{2k+1};$$

$$M\xi_k^2 = a^2 \left(\frac{k}{2k+1} + \frac{k+1}{2k+1} \right) = a^2;$$

$$D\xi_k = M\xi_k^2 - (M\xi_k)^2 = a^2 - a^2 \frac{1}{(2k+1)^2} = \frac{a^2(4k^2+4k)}{(2k+1)^2} = \frac{4a^2k(k+1)}{(2k+1)^2}$$

$$D\xi_k = \frac{4a^2k^2(1+\frac{1}{k})}{k^2(2+\frac{1}{k})} \leq 2a^2, \quad k \rightarrow \infty.$$

Demak, $\{\xi_k\}$ t.m.lar ketma-ketligi uchun katta sonlar qonuni o'rinli. \triangleleft

Misollar

1. Agar $P(\xi_k = \pm k) = \frac{1}{2k^2}$, $P(\xi_k = 0) = 1 - \frac{1}{k^2}$ bo'lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQga bo'ysunadimi?

2. $\{\xi_k\}_{k=1}^{\infty}$ t.f. $F_{\xi_k}(x)$ bo'lgan bog'liqsiz t.m.lar ketma-ketligi bo'lsin: $F_{\xi_k}(x) = \begin{cases} 0, & x \leq 1; \\ 1 - x^{-k}, & x \geq 1. \end{cases}$ $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQga bo'ysunadimi?

3. ξ_1, ξ_2, \dots bir xil taqsimlangan bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = 0$, $D\xi_i = 1$ bo'lsin. $\eta_k = \xi_k + \xi_{k+1}$, $k = 1, 2, \dots$ t.m.lar ketma-ketligi uchun KSQ o'rinlimi?

4. $\{\xi_k\}$ bog'liqsiz t.m.lar va $P(\xi_k = \pm\sqrt{k}) = \frac{1}{2}$ bo'lsa, bu ketma-ketlik uchun KSQ o'rinlimi?

5. $\{\xi_k\}$ bog'liqsiz t.m.lar ketma-ketligi bo'lib, uning taqsimoti $P(\xi_k = a) = \frac{1}{k+1}$, $P(\xi_k = -a) = \frac{k}{k+1}$ bo'lsin. Chebishev tengsizligi yordamida $\{\xi_k\}$ bog'liqsiz t.m.lar ketma-ketligi KSQ ga bo'ysunishini ko'rsating.

6. $\{\xi_k\}$, $k = 1, 2, \dots$ bog'liqsiz t.m.lar ketma-ketligi va $\xi_k \sim R(0, 2n^\alpha)$ bo'lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

7. Agar $P(\xi_k = \pm\sqrt{k}) = \frac{1}{2k}$, $P(\xi_k = 0) = 1 - \frac{1}{k}$ bo'lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

8. Agar $P(\xi_k = \pm 2^k) = 2^{-(2k+1)}$, $P(\xi_k = 0) = 1 - 2^{-2k}$ bo'lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

9. ξ_1, ξ_2, \dots bir xil taqsimlangan bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = a, D\xi_i = 1$. α ning qanday qiymatlarida $\eta_k = \alpha(\xi_k + \xi_{k+1}), k = 1, 2, \dots$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

10. Agar $P(\xi_k = \pm 2^k) = \frac{1-2^{-k}}{2}, P(\xi_k = \pm 1) = 2^{-k-1}$ bo'lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

11. $\{\xi_k\}, k = 1, 2, \dots$ bog'liqsiz t.m.lar ketma-ketligi va $\xi_k \sim R(1, n^\alpha)$ bo'lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

12. Agar $P(\xi_k = \pm k) = 2^{-k}, P(\xi_k = 0) = 1 - 2^{-k+1}$ bo'lsa, u holda $\{\xi_k\}_{k=1}^{\infty}$ t.m.lar ketma-ketligi KSQ ga bo'ysunadimi?

13. ξ_1, ξ_2, \dots bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = 0, D\xi_i = Ck^\alpha, \alpha \geq 0, i = 1, 2, \dots$. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

14. $\{\xi_k\}, k = 1, 2, \dots$ - bog'liqsiz t.m.lar ketma-ketligi va $\xi_k \sim R(0, \frac{n^\alpha}{2})$ bo'lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

15. Agar $P(\xi_k = \pm 2^{\alpha k}) = 2^{-(2k+1)}, P(\xi_k = 0) = 1 - 2^{-2k}$ bo'lsa, α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

16. ξ_1, ξ_2, \dots bir xil taqsimlangan bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = 0, D\xi_i = 1$ bo'lsin. $\eta_k = \xi_k - \xi_{k+1}, k = 1, 2, \dots$ ketma-ketlik uchun KSQ o'rinli bo'ladimi?

17. $\{\xi_k\}_{k=1}^{\infty}$ - bog'liqsiz t.m.lar ketma-ketligi bo'lib, t.f. bo'lsin: $F_{\xi_k}(x) = \begin{cases} 0, & x \leq 1; \\ 1 - x^{-k}, & x \geq 1. \end{cases}$ $\{a\xi_k\}_{k=1}^{\infty}, a > 0$, ketma-ketlik uchun KSQ o'rinli bo'ladimi?

18. Agar $\{\xi_k\}$ ketma-ketlik taqsimoti $P(\xi_k = \alpha) = \frac{k}{2k+1}, P(\xi_k = -\alpha) = \frac{k+1}{2k+1}$ bo'lsa, α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

19. ξ_1, ξ_2, \dots bir xil taqsimlangan bog'liqsiz t.m.lar ketma-ketligi va $M\xi_i = a, D\xi_i = \sigma^2, \sigma < \infty$ bo'lsin. $\eta_k = 2\xi_k - \xi_{k+1}, k = 1, 2, \dots$ ketma-ketlik uchun KSQ o'rinlimi?

20. $\{\xi_k\}, k = 1, 2, \dots$ bog'liqsiz t.m.lar ketma-ketligi va $\xi_k \sim E(\alpha/2)$ bo'lsin. α ning qanday qiymatlarida $\{\xi_k\}$ ketma-ketlik uchun KSQ o'rinli bo'ladi?

2-§. STATISTIK TANLANMA. TANLANMA XARAKTERISTIKALARI

Statistik izlanish asosida kuzatilmalar, ya'ni statistik tajriba natijalari, sonli ma'lumotlar yotadi. Faraz qilaylik, biror tasodifiy tajribani kuzatish natijasida x_1, \dots, x_n sonlar olingan bo'lib, ularni bog'liq bo'lmagan va bir xil taqsimlangan X_1, \dots, X_n t.m.larning qiymatlari deb qaraylik. U holda $x^{(n)} = (x_1, \dots, x_n)$ vektor $X^{(n)} = (X_1, \dots, X_n)$ tasodifiy vektorning qiymati bo'ladi. Matematik statistikada X_1, \dots, X_n tasodifiy miqdorlar n hajmga ega bo'lgan *takroriy tanlanma* yoki shunchaki *tanlanma* deb ataladi. Bunda $X^{(n)}$ vektor tanlanma yoki kuzatilayotgan tasodifiy vektor deb ham ataladi. o'z navbatida esa X_i t.m.ni biror abstrakt ξ t.m.ning har bir tajribadagi amaliy qiymati deb ham qaraymiz. $X^{(n)}$ tanlanma taqsimot qonuni \mathcal{P} - taqsimotlar oilasiga tegishli bo'lgan bosh to'plamdan olingan deb hisoblanadi. \mathcal{P} oila parametrik (masalan, $N(\theta_1, \theta_2^2)$, $R(\theta_1, \theta_2), \dots$) yoki noparametrik tipda bo'lishi mumkin.

Statistika deb, tanlanmadan olingan ixtiyoriy o'lchovli funksiyaga aytiladi: $T(X^{(n)}) = T(X_1, \dots, X_n)$.

Quyidagi statistikalar, ya'ni tanlanma momentlari muhim ahamiyat kasb etadi:

$$\text{Tanlanma o'rta qiymati} - \bar{x} = \frac{1}{n} \sum_{i=1}^n X_i;$$

$$k\text{- tartibli boshlang'ich tanlanma momenti} - \bar{x}^k = \frac{1}{n} \sum_{i=1}^n X_i^k;$$

$$k\text{- tartibli markaziy tanlanma moment} - \mu_{kn} = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{x})^k;$$

$$\text{Tanlanma dispersiyasi} - S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{x})^2 \text{ (siljimagan tan}$$

$$\text{lanma dispersiyasi} - \overline{S^2} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{x})^2);$$

Bu statistikalaridan tashqari amaliyotda variatsion qator hadlaridan ham keng foydalaniladi: X_1, \dots, X_n tanlanmaning elementlarini kamaymaslik tartibida joylashtiramiz:

$$X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}, \quad (25)$$

hosil bo'lgan qator variatsion qator, elementlari esa tartiblangan statistikalar deyiladi.

$$X_{(1)} = \min\{X_1, \dots, X_n\},$$

$$X_{(n)} = \max\{X_1, \dots, X_n\}.$$

9-misol. X_1, \dots, X_n tanlanma $Bi(n; \theta)$ taqsimotdan olingan bo'lsa, $\bar{x} = \frac{1}{n} \sum X_i$ statistika uchun $M\bar{x}$, $D\bar{x}$ ni hisoblang.

▷ Har bir X_1, \dots, X_n ning zichlik funksiyasi $f(x; \theta) = C_n^x \theta^x (1 - \theta)^{n-x}$, $x = 0, 1, \dots, n$, $\theta \in (0, 1)$, hamda $MX_1 = n\theta$; $DX_1 = n\theta(1 - \theta)$ bo'ladi.

$$M\bar{x} = M\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{1}{n} \sum_{i=1}^n MX_i = \frac{1}{n} \sum_{i=1}^n n\theta = n\theta.$$

$$\begin{aligned} D\bar{x} &= D\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{1}{n^2} \sum_{i=1}^n DX_i = \\ &= \frac{1}{n^2} \sum_{i=1}^n n\theta(1 - \theta) = \frac{n\theta(1 - \theta)}{n} = \theta(1 - \theta) \end{aligned}$$

Demak, $M\bar{x} = MX_1 = n\theta$, $D\bar{x} = \frac{DX_1}{n} = \theta(1 - \theta)$ bo'lar ekan. ◁

Misollar

1. X_1, \dots, X_n tanlanma $R[\theta_1; \theta_2]$ taqsimotdan olingan bo'lsa, $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika uchun $M\bar{x}$, $D\bar{x}$ larni hisoblang.

2. X_1, \dots, X_n tanlanma $N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsa, $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika uchun $M\bar{x}$, $D\bar{x}$ larni hisoblang.

3. X_1, \dots, X_n tanlanma $R[0; \theta]$ taqsimotdan olingan bo'lsa, $X_{(1)} = \min\{X_1, \dots, X_n\}$ statistika uchun $MX_{(1)}$, $DX_{(1)}$ larni hisoblang.

4. X_1, \dots, X_n tanlanma $R[0; \theta]$ taqsimotdan olingan bo'lsa, $X_{(n)} = \max\{X_1, \dots, X_n\}$ statistika uchun $MX_{(n)}$, $DX_{(n)}$ larni hisoblang.

5. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x; \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta. \end{cases}$

$\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika uchun $M\bar{x}$, $D\bar{x}$ larni hisoblang.

6. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x; \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta. \end{cases}$ $X_{(1)} = \min\{X_1, \dots, X_n\}$ statistika uchun $MX_{(1)}$, $DX_{(1)}$ larni hisoblang.

7. X_1, \dots, X_n tanlanma $N(0, \theta^2)$ taqsimotdan olingan bo'lsa, $\overline{x^2} = \frac{1}{n} \sum_{i=1}^n X_i^2$ statistika uchun $M\overline{x^2}$, $D\overline{x^2}$ larni hisoblang.

8. X_1, \dots, X_n tanlanma $N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsa, $S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{x})^2$, $\overline{S^2} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{x})^2$ statistikalar uchun MS^2 , $M\overline{S^2}$ larni hisoblang.

9. X_1, \dots, X_n tanlanma t.f. $F(x)$ bo'lgan taqsimotdan olingan bo'lsa, $F_n(x) = \frac{1}{n} \sum_{i=1}^n I(X_i < x)$, $x \in R^1$ statistika uchun $MF_n(x)$, $DF_n(x)$ larni hisoblang.

10. X_1, \dots, X_n tanlanma $R[\theta_1; \theta_2]$ taqsimotdan olingan bo'lsa, $X_{(n)} = \max\{X_1, \dots, X_n\}$ statistika uchun $MX_{(n)}$, $DX_{(n)}$ larni hisoblang.

11. X_1, \dots, X_n tanlanma $R[\theta_1; \theta_2]$ taqsimotdan olingan bo'lsa, $X_{(1)} = \min\{X_1, \dots, X_n\}$ statistika uchun $MX_{(1)}$, $DX_{(1)}$ larni hisoblang.

12. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsa, $X_{(n)} = \max\{X_1, \dots, X_n\}$ statistika uchun $MX_{(n)}$, $DX_{(n)}$ larni hisoblang.

13. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsa, $X_{(1)} = \min\{X_1, \dots, X_n\}$ statistika uchun $MX_{(1)}$, $DX_{(1)}$ larni hisoblang.

14. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsa,

$\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika uchun $M\bar{x}$, $D\bar{x}$ larni hisoblang.

15. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsa, $S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{x})^2$ statistika uchun MS^2 , DS^2 larni hisoblang.

16. X_1, \dots, X_n tanlanma $Bi(n; \theta)$ taqsimotdan olingan bo'lsa, $T_n(x^{(n)}) = X_1(n - X_1)/(n(n - 1))$ statistika uchun $MT_n(x^{(n)})$, $DT_n(x^{(n)})$ larni hisoblang.

17. X_1, \dots, X_n tanlanma $\Gamma(1; \theta)$ taqsimotdan olingan bo'lsa, $T_n(x^{(n)}) = X_1 + X_n$ statistika uchun $MT_n(x^{(n)})$, $DT_n(x^{(n)})$ larni hisoblang.

18. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x; \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0; \theta], \theta > 0, \\ 0, & x \notin [0; \theta]. \end{cases}$
 $T_n(x^{(n)}) = \frac{3}{2}\bar{x}$ statistika uchun $MT_n(x^{(n)})$, $DT_n(x^{(n)})$ larni hisoblang.

19. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x; \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0; \theta], \theta > 0, \\ 0, & x \notin [0; \theta]. \end{cases}$
 $X_{(n)} = \max\{X_1, \dots, X_n\}$ statistika uchun $MX_{(n)}$, $DX_{(n)}$ larni hisoblang.

20. X_1, \dots, X_n tanlanma $\log N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsa, $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika uchun $M\bar{x}$, $D\bar{x}$ larni hisoblang.

3-§. EKSPONENSIAL MODEL

X_1, \dots, X_n bog'liqsiz t.m. lar ξ t.m. ning har bir tajribadagi amaliy qiymatlari deb qaraladi va demak, ular bog'liqsiz bir hil taqsimlangan t.m.lardir. ξ t.m. hosil qilgan ehtimollik fazosi $(\mathcal{X}, \mathcal{B}, P)$ bo'lsin, ya'ni \mathcal{X} - ξ t.m.ning qiymatlari to'plami, \mathcal{B} to'plam \mathcal{X} dan olingan to'plamlarning Borel σ -algebrasi va P esa $(\mathcal{X}, \mathcal{B})$ o'lchovli fazoda aniqlangan ehtimollik o'lchovidir. $X^{(n)}$ tanlanma hosil qilgan ehtimollik fazosi $(\mathcal{X}^{(n)}, \mathcal{B}^{(n)}, P^{(n)})$ tanlanma fazo (yoki statistik model) deb ataladi. Bu yerda $\mathcal{X}^{(n)} = \mathcal{X} \times \dots \times \mathcal{X}$, $\mathcal{B}^{(n)}$ - to'plam $\mathcal{X}^{(n)}$ dagi to'plamlarning σ -algebrasi, $P^{(n)}$ esa $X^{(n)}$

ning $B = B_1 \times \dots \times B_n$ da aniqlangan taqsimoti $B_i \in \mathcal{B}$. Qulaylik uchun $P^{(n)}$ ni P kabi yozib u \mathcal{P} oilaga tegishli deb ($\mathcal{P} = \{P\}$), statistik modelni $(\mathcal{X}^{(n)}, \mathcal{B}^{(n)}, \mathcal{P})$ kabi belgilaymiz.

$(\mathcal{X}^{(n)}, \mathcal{B}^{(n)}, \{P_\theta, \theta \in \Theta\})$ parametrik statistik model eksponensial statistik model deyiladi, agar P_θ ning $f(x; \theta)$ zichlik funksiyasi quyidagi ko'rinishda bo'lsa:

$$f(x; \theta) = h(x) \exp \left\{ \sum_{i=1}^n A_i(\theta) t_i(x) + B(\theta) \right\}.$$

Bu holda $X^{(n)}$ tanlanmaning zichlik funksiyasi $f_n(x^{(n)}, \theta) = \prod_{i=1}^n f(x_i, \theta)$ quyidagicha ko'rinishda bo'ladi:

$$f_n(x^{(n)}, \theta) = h_n(x^{(n)}) \exp \left\{ \sum_{i=1}^k A_i(\theta) T_i(x^{(n)}) + nB(\theta) \right\},$$

bu yerda $h_n(x^{(n)}) = \prod_{i=1}^n h(x_i)$; $T_i(x^{(n)}) = \sum_{j=1}^n t_i(x_j)$.

10-misol. $\Gamma(\theta_1, \theta_2)$ taqsimotlar oilasi eksponensial oilaga tegishlimi?

▷ $\Gamma(\theta_1, \theta_2)$ taqsimot zichlik funksiyasini yozib olamiz:

$$f(x, \theta) = \begin{cases} \frac{\theta_1^{-\theta_2}}{\Gamma(\theta_2)} x^{\theta_2-1} e^{-x/\theta_1}, & x \geq 0, \quad \theta_1, \theta_2 > 0, \\ 0, & x < 0. \end{cases}$$

Bu yerdan $f(x, \theta) = \exp \{x/\theta_1 + (\theta_2 - 1) \ln x + (-\theta_2 \ln \theta_1 - \ln \Gamma(\theta_2))\}$ - eksponensial oilaga tegishli ekanligini ko'rish mumkin: $h(x) \equiv 1$, $A_1(\theta) = 1/\theta_1$, $A_2(\theta) = \theta_2 - 1$, $t_1(x) = x$, $t_2(x) = \ln x$, $B(\theta) = -\theta_2 \ln \theta_1 - \ln \Gamma(\theta_2)$. ◁

Misollar

Quyidagi taqsimotlar oilasi eksponensial oilaga tegishlimi?

- | | |
|--------------------------------------|---|
| 1. $R[-\theta, \theta]$. | 2. $N(\theta_1, \theta_2^2)$. |
| 3. $W(\theta_1, \theta_2)$. | 4. $K(\theta, 1)$. |
| 5. $R[\theta_1, \theta_2]$. | 6. $P(\theta_1, \theta_2)$. |
| 7. $N(\theta, 2\theta)$. | 8. $f(x, \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \theta > 0, \\ 0, & x \notin [0, \theta]. \end{cases}$ |
| 9. $G(\frac{\theta}{2})$. | 10. $f(x; \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta \end{cases}$ |
| 11. $\overline{Bi}(r, \theta)$. | 12. $f(x, \theta) = \begin{cases} 3x^2\theta^{-3}e^{-(x/\theta)^3}, & x \geq 0, \theta > 0, \\ 0, & x < 0. \end{cases}$ |
| 13. $R[\theta - 1, \theta + 1]$. | 14. $f(x, \theta) = e^{-\frac{x-\theta}{2}}, x, \theta \in R$. |
| 15. $\log N(\theta_1, \theta_2^2)$. | 16. $F(x, \theta) = \begin{cases} 1 - e^{-\frac{x-\theta_1}{\theta_2}}, & x \geq \theta_1, \theta_2 > 0, \\ 0, & x < \theta_1. \end{cases}$ |
| 17. $Bi(n; \frac{\theta}{2})$. | 18. $f(x, \theta) = \begin{cases} \theta(\ln^{\theta-1}x)/x, & x \in [1, e], \theta > 0, \\ 0, & x \notin [1, e]. \end{cases}$ |
| 19. $E(2\theta)$. | 20. $f(x, \theta) = e^{-x+\theta}(1 + e^{-x+\theta})^{-2}, x, \theta \in R$ |

4-§. YETARLI STATISTIKALAR

$(\mathcal{X}^{(n)}, \mathcal{B}^{(n)}, \{P_\theta^{(n)}, \theta \in \Theta\})$ - parametrik statistik model va $\mathcal{X}^{(n)}$ dagi P_θ taqsimot yordamida tuzilgan $P_\theta\{X^{(n)} \in B/T(X^{(n)})\}$ shartli taqsimot bo'lsin, $B \in \mathcal{B}^{(n)}$.

$T(X^{(n)})$ statistika $\{P_\theta, \theta \in \Theta\}$ oila (yoki θ parametr) uchun yetarli statistika deyiladi, agar $P_\theta(X^{(n)} \in B/T)$ shartli taqsimotning θ ga bo'g'liq bo'lmagan varianti mavjud bo'lsa.

Yetarli statistikasining mavjudligini Neyman- Fisherning faktorlashtirish teoremasi yordamida osongina tekshirish mumkin. $\{P_\theta, \theta \in \Theta\} \ll \mu$ bo'lsin ($\mu^{(n)} = \mu \times \dots \times \mu$).

Teorema(Neyman - Fisher). T statistika θ parametr uchun yetarli statistika bo'lishi uchun $f_n(x^{(n)}; \theta)$ zichlik funksiasi $\mu^{(n)}$

o'lchovga nisbatan deyarli hamma yerda quyidagicha ifodalanishi zarur va yetarli:

$$f_n(x^{(n)}; \theta) = \Psi_n(T(x^{(n)}); \theta) h_n(x^{(n)}), \quad (26)$$

bu yerda Ψ_n -o'lchovli funksiya yetarli statistika va θ ga bog'liq, h_n esa θ ga bog'liq emas va $\psi_n, h_n \geq 0$.

11-misol. $\pi(\theta)$ taqsimot uchun etarli statistikani ko'rsating.

$$f(x, \theta) = \frac{e^{-\theta} \theta^x}{x!}, \quad x = 0, 1, \dots, \quad \theta \in (0, 1).$$

▷ Tanlanma zichlik funksiyasini topamiz:

$$f_n(x^{(n)}, \theta) = \prod_{i=1}^n f(x_i, \theta) = \frac{e^{-n\theta} \theta^{\sum x_i}}{\prod_{i=1}^n x_i!} = e^{-n\theta} \theta^{\sum x_i} \left(\prod_{i=1}^n x_i! \right)^{-1}$$

Neyman-Fisher faktorlashtirish kriteriysiga ko'ra:

$$f_n(x^{(n)}, \theta) = e^{-n\theta} \theta^{\sum x_i} \left(\prod_{i=1}^n x_i! \right)^{-1} = \Psi_\theta(T(x^{(n)}); \theta) h_n(x^{(n)});$$

$$\Psi_\theta(T(x^{(n)}); \theta) = e^{-n\theta} \theta^{\sum x_i}; \quad h_n(x^{(n)}) = \left(\prod_{i=1}^n x_i! \right)^{-1}$$

Demak, $\pi(\theta)$ taqsimot uchun $T(x^{(n)}) = \sum_{i=1}^n x_i$ - etarli statistika bo'lar ekan. ◁

Misollar

Quyidagi taqsimotlar uchun yetarli statistikalarini ko'rsating.

1. $R(\theta_1, \theta_2)$.
2. $N(\theta_1, \theta_2^2)$.
3. $P(\theta_1, \theta_2)$.
4. $\Gamma(\theta_1, \theta_2)$.
5. $W(\theta_1, \theta_2)$.
6. $Ge(\theta)$.
7. $Bi(n; \theta)$.
8. $\Gamma(1; 2\theta_2)$.
9. $\overline{Bi}(r, \theta)$.
10. $E(\theta)$.
11. $K(\theta)$.
12. $\log N(\theta_1, \theta_2^2)$.
13. $P(1; \theta_2)$.
14. $N(\theta, 2\theta)$.
15. $R[-\theta, \theta]$.

$$16. f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta. \end{cases}$$

$$17. F(x, \theta) = \begin{cases} 1 - e^{-\frac{x-\theta_1}{\theta_2}}, & x \geq \theta_1, \theta_2 > 0, \\ 0, & x < \theta_1. \end{cases}$$

$$18. f(x, \theta) = \begin{cases} \theta x^{\theta-1}, & x \in (0; 1), \theta > 0, \\ 0, & x \notin (0; 1). \end{cases}$$

$$19. f(x, \theta) = e^{-x+\theta} (1 + e^{-x+\theta})^2, \quad x, \theta \in \mathbb{R}.$$

$$20. f(x, \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \theta > 0, \\ 0, & x \notin [0, \theta]. \end{cases}$$

5-§. TO'LA STATISTIKA

$(\mathcal{Y}, \mathcal{U})$ o'lchovli fazoda aniqlangan $\{Q_\theta, \theta \in \Theta\}$ taqsimotlar oilasi to'la deb ataladi, agar ixtiyoriy \mathcal{U} o'lchovli $\varphi(y)$ funksiya uchun

$$\int \phi(y) Q_\theta(dy) = 0, \quad \theta \in \Theta \quad (27)$$

tenglikdan $\{Q_\theta\}$ ga nisbatan deyarli hamma yerda $\phi(y) = 0$ ekanligi kelib chiqsa.

$(\mathcal{X}^{(n)}, \mathcal{B}^{(n)}, \{P_\theta^{(n)}, \theta \in \Theta\})$ statistik modelda aniqlangan.

$T : (\mathcal{X}^{(n)}, \mathcal{B}^{(n)}) \rightarrow (\mathcal{Y}, \mathcal{U})$ statistika to'la deb ataladi, agar u yaratgan

$$Q_\theta(A) = P_\theta(x^{(n)} : T(x^{(n)}) \in A), \quad A \in \mathcal{U}$$

ehtimollar taqsimotlari oilasi to'la bo'lsa.

12-misol. $Bi(n; \theta)$ taqsimot uchun $\sum_{i=1}^n X_i$ statistika to'la statistika ekanini ko'rsating.

$\triangleright \theta \in \Theta = (0, 1)$ parametrlik binomial taqsimot uchun $T = \sum_{i=1}^n X_i$ statistikaning taqsimoti

$$Q_\theta(t) = P_\theta(T = t) = \begin{cases} C_n^t \theta^t (1 - \theta)^{n-t}, & t \in Y = \{0, 1, \dots, n\}, \\ 0, & \text{aks holda.} \end{cases}$$

bo'ladi. $\varphi = \varphi(t)$ haqiqiy funksiya uchun

$$\sum_{t=0}^n \varphi(t) Q_\theta(t) = (1 - \theta)^n \sum_{t=0}^n C_n^t \varphi(t) \left(\frac{\theta}{1 - \theta} \right)^t = 0, \quad \theta \in \Theta,$$

bo'lsin. Bu tenglamani chap tomonida darajasi n dan oshmagan $\frac{\theta}{1 - \theta}$ ning polinomi turibdi. Bu polinom ko'pi bilan n ta turli nuqtalarda no'lga teng bo'ladi. Ammo bu tenglik barcha $\theta \in \Theta$ lar uchun o'rinli bo'lmoqda. Bu esa, o'z navbatida barcha $t = 0, 1, \dots, n$ lar uchun $\varphi(t) = 0$ ekanini ko'rsatadi.

Demak, $\{Q_\theta(t), \theta \in \Theta\}$ oila va $T = \sum_{i=1}^n x_i$ statistika to'la ekan. \triangleleft

Misollar

1. $R[\theta; \theta + 1]$, $\theta \in R$ taqsimot uchun $(X_{(1)}, X_{(n)})$ yetarli statistika, ammo to'la statistika bo'lmasligini ko'rsating.

2. $R[\theta_1; \theta_2]$, $\theta_1 < \theta_2$ taqsimot uchun $(X_{(1)}, X_{(n)})$ vektor to'la statistika bo'lishini ko'rsating.

3. $E(\theta)$ taqsimot uchun $\sum_{i=1}^n X_i$ to'la statistika bo'lishini ko'rsating.

4. $N(0; \theta^2)$ taqsimot uchun $\overline{x^2} = \frac{1}{n} \sum_{i=1}^n X_i^2$ statistika to'la statistika bo'ladimi?

5. Zichlik funksiyasi $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimot uchun $X_{(1)}$ statistika to'la statistika bo'ladimi?

6. Zichlik funksiyasi $f(x; \theta) = \begin{cases} \theta^{-1} e^{-(x-\beta)/\theta}, & x \geq \beta, \\ 0, & x < \beta \end{cases}$ bo'lgan taqsimot uchun $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika to'la statistika bo'ladimi?

7. Zichlik funksiyasi $f(x; \theta) = \begin{cases} \alpha^{-1} e^{-(x-\theta)/\alpha}, & x \geq \theta, \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimot uchun $X_{(1)}$ statistika to'la statistika bo'ladimi?

8. $R[0; \theta]$ taqsimot uchun $X_{(n)}$ statistika to'la statistika bo'ladimi?

9. $R[\theta; 2\theta]$ taqsimot uchun $(X_{(1)}, X_{(n)})$ statistika to'la statistika bo'ladimi?

10. $E(\frac{1}{\theta})$ taqsimot uchun $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika to'la statistika bo'ladimi?

11. Zichlik funksiyasi $f(x, \theta) = \begin{cases} \theta x^{\theta-1}, & x \in (0; 1), \theta > 0, \\ 0, & x \notin (0; 1) \end{cases}$ bo'lgan taqsimot uchun $\overline{\ln x} = \frac{1}{n} \sum_{i=1}^n \ln X_i$ statistika to'la statistika bo'ladimi?

12. $\pi(\theta)$ taqsimot uchun $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika to'la statistika bo'ladimi?

13. $Ge(\theta)$ taqsimot uchun $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika to'la statistika bo'ladimi?

14. $W(\lambda; \theta)$ taqsimot uchun $\overline{x^\lambda} = \frac{1}{n} \sum_{i=1}^n X_i^\lambda$ statistika to'la statistika bo'ladimi?

15. Zichlik funksiyasi $f(x; \theta) = \begin{cases} \frac{2x}{\theta} e^{-x^2/\theta}, & x \geq 0, \theta > 0, \\ 0, & x < 0 \end{cases}$ bo'lgan taqsimot uchun $\overline{x^2} = \frac{1}{n} \sum_{i=1}^n X_i^2$ statistika to'la statistika bo'ladimi?

16. $N(\theta; \sigma^2)$ taqsimot uchun $\sum_{i=1}^n X_i$ statistika to'la statistika bo'ladimi?

17. Zichlik funksiyasi $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimot uchun $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika to'la statistika bo'ladimi?

18. $\log N(\theta; \sigma^2)$ taqsimot uchun $e^{\bar{x}}$ statistika to'la statistika bo'ladimi?

19. Zichlik funksiyasi $f(x, \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \theta > 0, \\ 0, & x \notin [0, \theta] \end{cases}$ bo'lgan taqsimot uchun $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika to'la statistika bo'ladimi?

20. $N(a; \theta^2)$ taqsimot uchun $S_n^2 = \frac{1}{n} \sum_{i=1}^n (X_i - a)^2$ statistika to'la statistika bo'ladimi?

6-§. BAHOLAR VA ULARNING KOSSALARI

Agar $T_n(X^{(n)}) = T(X_1, \dots, X_n) \in \Theta$ bo'lsa, u holda $T_n(X^{(n)})$ statistika noma'lum θ parametr uchun *baho* deb ataladi.

Agar statistik bahoning matematik kutilmasi noma'lum parametrga teng, ya'ni $MT_n(X^{(n)}) = MT(X_1, \dots, X_n) = \theta$ bo'lsa, statistik baho θ uchun *siljimagan baho* deyiladi.

Agar $T(X_1, \dots, X_n)$ statistika noma'lum parametr θ ga ehtimollik bo'yicha yaqinlashsa, ya'ni $\forall \varepsilon > 0$ son uchun:

$$\lim_{n \rightarrow \infty} \{P|T(X_1, \dots, X_n) - \theta| < \varepsilon\} = 1 \quad (28)$$

munosabat o'rinni bo'lsa, u holda $T(X_1, \dots, X_n)$ statistik baho *asosli baho* deyiladi.

$T_1(X_1, \dots, X_n)$ va $T_2(X_1, \dots, X_n)$ lar noma'lum θ parametr uchun baholar bo'lib,

$$M(T_1 - \theta)^2 < M(T_2 - \theta)^2 \quad (29)$$

munosabat bajarilsa, u holda $T_1(X_1, \dots, X_n)$ baho $T_2(X_1, \dots, X_n)$ bahoga nisbatan *effektiv* deyiladi. Agar $T_1(X_1, \dots, X_n)$ va

$T_2(X_1, \dots, X_n)$ lar noma'lum θ parametr uchun siljimagan baholar bo'lsa, u holda (29) tenglikni $DT_1 < DT_2$ ko'rinishda yozish mumkin.

13-misol. X_1, \dots, X_n tanlanma $R[0, \theta]$ taqsimotdan olingan bo'lsin, noma'lum parametr θ uchun $2\bar{x}$ bahoning siljimaganligi va asoslilikini ko'rsating.

▷ *Siljimaganlik:*

$$M(2\bar{x}) = 2M\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = 2 \frac{1}{n} \sum_{i=1}^n MX_i = \frac{2}{n} \sum_{i=1}^n \frac{\theta}{2} = \frac{2\theta}{n} n = \theta.$$

Asoslilik:

$$D(2\bar{x}) = 4D\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{4}{n^2} \sum DX_i = \frac{4}{n^2} \frac{\theta^2}{12} n = \frac{\theta^2}{3n} \xrightarrow{n \rightarrow \infty} 0,$$

Chebisev tengsizligiga ko'ra, $\forall \varepsilon > 0$ uchun:

$$P\{|2\bar{x} - \theta| < \varepsilon\} \geq 1 - \frac{D(2\bar{x})}{\varepsilon^2} = 1 - \frac{\theta^2}{3\varepsilon n} \xrightarrow{n \rightarrow \infty} 1.$$

◁

Misollar

1. X_1, \dots, X_n tanlanma $R[0; \theta]$ taqsimotdan olingan bo'lsin, noma'lum parametr θ uchun $\frac{n+1}{n} X_{(n)}$ bahoni siljimaganlik va asoslilikka tekshiring.

2. X_1, \dots, X_n tanlanma $N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsin, \bar{x} va S^2 larni mos ravishda θ_1 va θ_2^2 lar uchun siljimaganlik va asoslilikka tekshiring.

3. X_1, \dots, X_n tanlanma zichligi $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimotdan olingan bo'lsin, noma'lum θ parametr uchun quyidagi baholarni siljimaganlik va asoslilikka tekshiring: a) $X_{(1)}$; b) $\bar{x} - 1$.

4. X_1, \dots, X_n tanlanma $N(\theta, \sigma^2)$ bo'lgan taqsimotdan olingan bo'lsin (bu yerda σ -ma'lum), $T(x^{(n)}) = \bar{x}^2 - \frac{\sigma^2}{n}$ statistika $g(\theta) = \theta^2$ uchun siljimagan baho bo'lishini ko'rsating.

5. X_1, \dots, X_n tanlanma $R(\theta_1, \theta_2)$ taqsimotdan olingan bo'lsa, $T(X^{(n)}) = \frac{1}{2}(X_{(n)} + X_{(1)})$ statistika $g(\theta) = \frac{1}{2}(\theta_1 + \theta_2)$ uchun siljimagan baho bo'lishini ko'rsating.

6. X_1, \dots, X_n tanlanma $R(\theta_1, \theta_2)$ taqsimotdan olingan bo'lsa, $T(X^{(n)}) = \frac{n+1}{n-1}(X_{(n)} + X_{(1)})$ statistika $g(\theta) = \theta_2 - \theta_1$ uchun siljimagan baho bo'lishini ko'rsating.

7. X_1, \dots, X_n tanlanma $MX_1 = a$ ma'lum va MX_1^2 chekli bo'lgan taqsimotdan olingan bo'lsin. $T(X^{(n)}) = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{x})^2$ statistika noma'lum dispersiya uchun siljimagan va asosli baho bo'ladimi?

8. X_1, \dots, X_n tanlanma $MX_1 = a$ ma'lum va MX_1^2 chekli bo'lgan taqsimotdan olingan bo'lsin. $T(X^{(n)}) = \bar{x}^2 - a^2$ statistika noma'lum dispersiya uchun siljimagan va asosli baho bo'ladimi?

9. X_1, \dots, X_n tanlanma $MX_1 = a$ ma'lum va MX_1^2 chekli bo'lgan taqsimotdan olingan bo'lsin. $T(X^{(n)}) = \frac{1}{n} \sum_{i=1}^n (X_i - a)^2$ statistika noma'lum dispersiya uchun siljimagan va asosli baho bo'ladimi?

10. X_1, \dots, X_n tanlanma $P(\theta, \beta)$ taqsimotdan olingan bo'lsin (bu yerda β -ma'lum). Noma'lum parametr θ uchun $T(X^{(n)}) = X_{(1)}$ statistika siljimagan va asosli baho bo'ladimi?

11. X_1, \dots, X_n tanlanma zichlik funksiyasi $f(x; \theta) = e^{-x+\theta} (1 + e^{-x+\theta})^{-2}$, $-\infty < x < +\infty$, $\theta \in (-\infty, +\infty)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T(X^{(n)}) = \bar{x}$ statistika siljimagan baho bo'lishini ko'rsating.

12. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T(X^{(n)}) = \frac{1}{\bar{x}}$ statistika siljimagan va asosli baho bo'ladimi?

13. X_1, \dots, X_n tanlanma $E(\frac{1}{\sqrt{\theta}})$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T(X^{(n)}) = (\bar{x})^2$ statistika siljimagan va asosli baho bo'ladimi?

14. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T(X^{(n)}) = \frac{k}{\sqrt{k!} \bar{x}^k}$, $k = 1, 2, \dots$ statistika siljimagan va asosli baho bo'ladimi?

15. X_1, \dots, X_n tanlanma $Bi(1; \sqrt{\theta})$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $T(X^{(n)}) = (\bar{x})^2$ statistika siljimagan va

asosli baho bo'ladimi?

16. X_1, \dots, X_n tanlanma $P(\alpha; \theta)$ taqsimotdan olingan bo'lsin (bu yerda α -ma'lum). Noma'lum parametr θ uchun $T(X^{(n)}) = \frac{1}{\ln x - \ln x_{(1)}}$ statistika siljimagan va asosli baho bo'ladimi?

17. X_1, \dots, X_n tanlanma $W(\alpha; \theta)$ taqsimotdan olingan bo'lsin (bu yerda α -ma'lum). Noma'lum parametr θ uchun $T(X^{(n)}) = 1/(\bar{x}^\alpha)$ statistika siljimagan va asosli baho bo'ladimi?

18. X_1, \dots, X_n tanlanma $Bi(n; \theta)$ taqsimotdan olingan bo'lsin. $T(X^{(n)}) = \frac{n\bar{x} + \alpha}{n + \beta}$, $\alpha \geq 0$; $\beta \geq 0$ baholar oilasini ko'raylik. $MT(X^{(n)})$ va $\sqrt{DT(X^{(n)})}$ larni hisoblang.

19. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x; \theta) = \begin{cases} \alpha^{-1} e^{-(x-\theta)/\alpha}, & x \geq \theta, \\ 0, & x < \theta \end{cases}$ (bu yerda α -ma'lum). Noma'lum parametr θ uchun $T(X^{(n)}) = X_{(1)}$ statistika siljimagan va asosli baho bo'ladimi?

20. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin: $f(x; \theta) = \begin{cases} \theta^{-1} e^{-(x-\beta)/\theta}, & x \geq \beta, \\ 0, & x < \beta \end{cases}$ (bu yerda β -ma'lum). Noma'lum parametr θ uchun $T(X^{(n)}) = \bar{x} + \beta$ statistika siljimagan va asosli baho bo'ladimi?

7-§. RAO-KRAMER TENGSIZLIGI. EFFEKTIV BAHOLASH

$(\mathcal{X}^{(n)}, \mathcal{B}^{(n)}, \{P_\theta^{(n)}, \theta \in \Theta\})$, $\Theta \subseteq R$ - parametrik statistik modolni qaraylik. Har bir X_i kuzatilmaning $f(x, \theta)$ umumlashgan zichlik funksiyasi uchun Rao-Kramer *regulyarlik shartlarini* kiritamiz:

(I) $N_f = \{x : f(x, \theta) > 0\}$ - to'plam θ ga bog'liq emas;

(II) $\Theta = R$ yoki Θ - to'plam R dagi biror interval;

(III) $\frac{\partial}{\partial \theta} f(x, \theta)$ - xususiy hosila mavjud va $\{P_\theta, \theta \in \Theta\}$ ga nisbatan deyarli hamma yerda $\forall \theta \in \Theta$ uchun chekli;

(IV) $\forall \theta \in \Theta$ va $i = 1, 2$ uchun $\int \left| \frac{\partial^i}{\partial \theta^i} f(x, \theta) \right| \mu(dx) < \infty$;

(V) $\forall \theta \in \Theta$: $0 < I(\theta) = M_\theta \left[\frac{\partial}{\partial \theta} \ln f(\xi, \theta) \right]^2 < \infty$.

Biz $x^{(n)}$ tanlanmaning $f_n(X^{(n)}, \theta) = \prod_{i=1}^n f(x_i, \theta)$ - zichlik

funksiyasini qarayotganimizda (I) - (V) shartlarni f o'rnida f_n ni ishlatamiz va integrallar $\mathcal{X}^{(n)}$ to'plam bo'yicha tushuniladi. $I(\theta)$ funksiya ξ tasodifiy miqdordagi θ parametr haqidagi Fisher informat-siyasi deyiladi. $X^{(n)}$ tanlanmaga mos Fisher informat-siyasini $I_n(\theta)$ orqali belgilaymiz.

Ushbu

$$l_n(x^{(n)}, \theta) = \frac{\partial}{\partial \theta} \ln f_n(x^{(n)}, \theta) = \sum_{i=1}^n l(x_i, \theta),$$

$$l(x_i, \theta) = \frac{\partial}{\partial \theta} \ln f(x_i, \theta), \quad i = 1, \dots, n,$$

- funksiyalar *informantlar* deb ataladi.

$\{f_n(x^{(n)}, \theta), \theta \in \Theta\}$ uchun (I) - (V) shartlar bajarilsin. U holda

$$I_n(\theta) = nI(\theta), \quad \theta \in \Theta. \quad (30)$$

Teorema (Rao-Kramer). $\{f_n(x^{(n)}, \theta), \theta \in \Theta\}$ oila uchun (I) - (V) shartlar bajarilsin va differensiallanuvchi $g(\theta)$ funksiyaga siljimagan $\hat{g}_n(X^{(n)})$ baho uchun barcha $\forall \theta \in \Theta$ larda

(VI) $\int |\hat{g}_n(x^{(n)}) \frac{\partial}{\partial \theta} f_n(x^{(n)}, \theta)| \mu^{(n)}(dx^{(n)}) < \infty$ va $D_\theta \hat{g}_n < \infty$ bo'lsin. U holda $\forall \theta \in \Theta$ uchun

$$D_\theta \hat{g}_n(X^{(n)}) \geq \frac{[g'(\theta)]^2}{nI(\theta)}. \quad (31)$$

(31) da tenglik bajarilishi uchun $f_n(x^{(n)}; \theta)$ - zichlik funksiyasi quyidagi maxsus eksponensial oilaga tegishli bo'lishi zarur va yetar-lidir:

$$f_n(x^{(n)}; \theta) = \exp \left\{ \hat{g}_n(x^{(n)}) \Psi_1(\theta) + \Psi_2(\theta) + k_n(x^{(n)}) \right\}, \quad (32)$$

bu yerda $\Psi_1(\theta) \neq 0$. Agar (31) da tenglik o'rinli bo'lsa, \hat{g}_n baho $g(\theta)$ uchun Rao-Kramer ma'nosida effektiv (yoki shunchaki effektiv) baho deb ataladi.

14-misol. $X_1, \dots, X_n \sim N(\theta, \sigma^2)$ bo'lsin. $\hat{\theta} = \bar{x}$ effektiv baho bo'ladimi?

$\triangleright f(x, \theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\theta)^2}{2\sigma^2}\right\}$ funksiya uchun regulyarlik shartlari o'rinli. Fisher informatsiyasini hisoblaymiz:

$$\ln f(X_1; \theta) = -\ln(2\pi\sigma^2)^{1/2} - \frac{(X_1 - \theta)^2}{2\sigma^2},$$

$$\frac{\partial \ln f(X_1, \theta)}{\partial \theta} = \frac{X_1 - \theta}{\sigma^2},$$

$$\begin{aligned} I(\theta) &= M_{\theta} \left(\frac{\partial \ln f(X_1, \theta)}{\partial \theta} \right)^2 = M_{\theta} \left(\frac{X_1 - \theta}{\sigma^2} \right)^2 = \frac{1}{\sigma^4} M_{\theta} (X_1 - \theta)^2 = \\ &= \frac{1}{\sigma^4} M_{\theta} (X_1 - M X_1)^2 = \frac{1}{\sigma^4} D_{\theta} X_1 = \frac{1}{\sigma^2}. \end{aligned}$$

Demak, $I(\theta) = \frac{1}{\sigma^2}$. Endi \bar{x} ning dispersiyasini hisoblaymiz:

$$D_{\theta} \bar{x} = D_{\theta} \left(\frac{1}{n} \sum_{i=1}^n X_i \right) = \frac{1}{n^2} \sum_{i=1}^n D_{\theta} X_i = \frac{1}{n^2} n \sigma^2 = \frac{\sigma^2}{n}.$$

Demak, $D_{\theta} \bar{x} = \frac{1}{nI(\theta)}$, ya'ni \bar{x} baho θ uchun effektiv ekan. Bu yerda $f_n(x^{(n)}; \theta)$ uchun (32) formula o'rinlidir:

$$f_n(x^{(n)}; \theta) = \exp \left\{ \bar{x} \frac{n\theta}{\sigma^2} + \left(-\theta^2 \frac{n}{2\sigma^2} \right) + \left(-\frac{\bar{x}^2 n}{2\sigma^2} - \frac{n}{2} \ln 2\pi - n \ln \sigma \right) \right\}.$$

◁

Misollar

Quyidagi modellarda keltirilgan baholarni effektivlikka tekshiring.

Model

1. $N(\theta; \sigma^2)$

2. $N(a; \theta^2)$

3. $\pi(\theta)$

4. $Bi(n; \theta)$

5. $\pi(\theta)$

6. $E(1/\theta)$

7. $\pi(\theta)$

8. $Ge(\theta)$

9. $\Gamma(\theta, \lambda)$

10. $f(x, \theta) = e^{-x+\theta} (1 + e^{-x+\theta})^{-2}; x, \theta \in R$

11. $f(x; \theta) = \begin{cases} \frac{2x}{\theta} e^{-x^2/\theta}, & x \geq 0, \theta > 0, \\ 0, & x < 0 \end{cases}$

12. $\overline{Bi}(r, \theta)$

13. $\Gamma(\theta, \lambda)$

14. $E(\theta)$

15. $Bi(n; \theta)$

16. $N(a; \theta^2)$

17. $f(x, \theta) = \begin{cases} \theta (\ln^{\theta-1} x)/x, & x \in [1, e], \theta > 0, \\ 0, & x \notin [1, e] \end{cases}$

18. $E(\theta)$

19. $Ge(\theta)$

20. $N(\theta; \sigma^2)$

Baholar

$\hat{\theta} = X_1$

$\hat{\theta}^2 = S^2$

$\hat{\theta} = \frac{1}{2}(X_1 + X_2)$

$\hat{\theta} = \bar{x}/n$

$\hat{\theta} = \bar{x}$

$\hat{\theta} = \bar{x}$

$\hat{\theta} = X_1$

$\hat{\theta} = 1/(1 + \bar{x})$

$\hat{\theta} = \bar{x}/\lambda$

$\hat{\theta} = \bar{x}$

$\hat{\theta} = \bar{x}^2$

$\hat{\theta} = \bar{x}/(r + \bar{x})$

$\hat{\theta} = \frac{1}{\bar{x}}(1 - 1/n)$

$\hat{\theta} = \frac{1}{\bar{x}}$

$\hat{\theta} = X_1$

$\hat{\theta} = \bar{x}^2 - a^2$

$\hat{\theta} = \bar{x}$

$\hat{\theta} = \frac{n-1}{n\bar{x}}$

$\hat{\theta} = \frac{1}{\bar{x}}$

$\hat{\theta}^2 = (\bar{x})^2$

8-§. MOMENTLAR USULI

X t.f. $F(x, \theta)$ noma'lum parametr $\theta = (\theta_1, \dots, \theta_r)$ ga bog'liq bo'lgan t.m. bo'lsin. Faraz qilaylik, X t.m.ning birinchi r ta $\alpha_k = MX^k$, $k = 1, \dots, r$ momentlari mavjud bo'lsin. Tabiiyki, ular noma'lum θ parametrning $\alpha_k = \alpha_k(\theta)$ funksiyalari bo'ladi. $A_{nk} = \sum_{i=1}^n X_i^k$, $k = 1, \dots, r$ tanlanma momentlarini KSQ ga asoslangan holda mos ravishda α_k , $k = 1, \dots, r$ larga tenglashtirib, yetarlicha katta n larda r ta tenglamalar sistemasini tuzib olamiz:

$$\begin{cases} \alpha_1(\theta) = A_{n1}, \\ \alpha_2(\theta) = A_{n2}, \\ \vdots \\ \alpha_r(\theta) = A_{nr}. \end{cases} \quad (33)$$

Mana shu tenglamalar sistemasini $\theta_1, \dots, \theta_r$ larga nisbatan yechib, $\tilde{\theta}_k = \tilde{\theta}_k(X_1, \dots, X_n)$, $k = 1, \dots, r$ yechimlarga ega bo'lamiz. Shunday topilgan $\tilde{\theta}_k$, $k = 1, \dots, r$ statistikalar momentlar usuli bilan noma'lum θ_k , $k = 1, \dots, r$ parametrlar uchun tuzilgan statistik baholar deb ataladi.

15-misol. $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta \end{cases}$ taqsimot noma'lum parametrini momentlar usulida baholang.

▷ Buning uchun matematik kutilmani hisoblaymiz:

$$\begin{aligned} M_{\theta} X &= \int_{\theta}^{+\infty} x e^{\theta-x} dx = \left| \begin{array}{l} x = u \\ e^{\theta-x} dx = d\nu \quad \nu = -e^{\theta-x} \end{array} \right| = -x e^{\theta-x} \Big|_{\theta}^{+\infty} + \\ &+ \int_{\theta}^{+\infty} e^{\theta-x} dx = \theta - e^{\theta-x} \Big|_{\theta}^{+\infty} = \theta + 1. \end{aligned}$$

Nazariy momentni mos tanlanma momentga tenglashtirib MUBni topamiz: $\theta = \bar{x} - 1 <$

Misollar

Quyidagi taqsimotlar noma'lum parametrlarini momentlar usulida bahosini tuzing.

1. $\Gamma(\theta_1, \theta_2)$.

2. $\overline{Bi}(r; \theta)$.

3. $N(\theta_1, \theta_2^2)$.

4. $R[\theta_1, \theta_2]$.

5. $P(\theta_1, \theta_2)$.

6. $\Gamma(2\theta, 1)$.

7.
$$F(x, \theta) = \begin{cases} 1 - e^{-\frac{x-\theta_1}{\theta_2}}, & x \geq \theta_1, \theta_2 > 0, \\ 0, & x < \theta_1. \end{cases}$$

8. $P(2\theta, 1)$.

9.
$$f(x, \theta) = \begin{cases} \theta x^{\theta-1}, & x \in [0, 1], \\ 0, & x \notin [0, 1]. \end{cases}$$

10. $W(\theta_1, \theta_2)$.

11. $f(x, \theta) = e^{-\frac{x-\theta}{2}}, x > \theta, \theta > 0$.

12. $Ge(\theta)$.

13. $f(x, \theta) = e^{-x+\theta}(1 + e^{-x+\theta})^{-2}, x, \theta \in R$.

14. $R[-2\theta, 2\theta]$.

15.
$$f(x, \theta) = \begin{cases} \theta (\ln^{\theta-1} x) / x, & x \in [1, e], \theta > 0, \\ 0, & x \notin [1, e]. \end{cases}$$

16. $N(\theta, 2\theta)$.

17.
$$f(x, \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \theta > 0, \\ 0, & x \notin [0, \theta]. \end{cases}$$

18. $\log N(\theta_1, \theta_2^2)$.

19.
$$f(x, \theta) = \begin{cases} 3x^2\theta^{-3}e^{-(x/\theta)^3}, & x \geq 0, \theta > 0, \\ 0, & x < 0. \end{cases}$$

20. $E\left(\frac{2}{\theta}\right)$.

9-§. HAQIQATGA MAKSIMAL O'XSHASHLIK USULI

Kuzatilmalari X_1, \dots, X_n lardan va umumlashgan zichlik funksiyasi $f(x, \theta)$ bo'lgan X t.m.ni olaylik:

$$f(x, \theta) = \begin{cases} P_{\theta}\{X = x\} & \text{agar } X \text{ diskret t.m. bo'lsa,} \\ \frac{\partial F(x, \theta)}{\partial x}, & \text{agar } X \text{ uzluksiz t.m. bo'lsa.} \end{cases}$$

Quyidagi funksiyaga

$$L(x_1, \dots, x_n, \theta) = f_n(x^{(n)}, \theta) = f(x_1, \theta) \cdot \dots \cdot f(x_n, \theta)$$

haqiqatga maksimal o'xshashlik funksiyasi deyiladi. Faraz qilaylik, $L(x_1, \dots, x_n, \theta)$ funksiya $\theta \in \Theta$ yopiq sohada biror θ^* nuqtada eng katta qiymatga erishsin:

$$L(x_1, \dots, x_n, \theta^*) = \max_{\theta \in \Theta} L(x_1, \dots, x_n, \theta).$$

Haqiqatga maksimal o'xshashlik funksiyasi eng katta qiymatga erishadigan θ^* qiymat noma'lum θ parametr uchun haqiqatga maksimal o'xshashlik usuli bilan tuzilgan statistik baho deb ataladi. Ularni ma'lum shartlarda quyidagi tenglamalar sistemasidan ham topish mumkin:

$$\left. \frac{\partial L(x_1, \dots, x_n, \theta)}{\partial \theta_k} \right|_{\theta=\theta^*} = 0, \quad k = 1, \dots, r. \quad (34)$$

(34) sistema haqiqatga maksimal o'xshashlik tenglamalari sistemasini deyiladi. Ko'p hollarda (34) tenglamalar sistemasini o'rniga quyidagi unga ekvivalent tenglamalar sistemasini yechish qulay bo'ladi:

$$\left. \frac{\partial \ln L(x_1, \dots, x_n, \theta)}{\partial \theta_k} \right|_{\theta=\theta^*} = 0, \quad k = 1, \dots, r. \quad (35)$$

16-misol. $R[-\theta, \theta]$ taqsimot noma'lum parametrini haqiqatga maksimal o'xshashlik usuli bilan baholang.

▷ $R[-\theta, \theta]$ taqsimot zichlik funksiyasini yozib olamiz:

$$f(x, \theta) = \begin{cases} \frac{1}{2\theta}, & x \in [-\theta, \theta] \\ 0, & x \notin [-\theta, \theta] \end{cases} = \frac{1}{2\theta} I(-\theta \leq x \leq \theta).$$

Haqiqatga o'xshashlik funksiyasini tuzamiz:

$$\begin{aligned} f_n(x^{(n)}, \theta) &= \frac{1}{(2\theta)^n} \prod_{i=1}^n I(-\theta \leq X_i \leq \theta) = \\ &= \frac{1}{(2\theta)^n} I(-\theta \leq X_{(1)} \leq X_{(n)} \leq \theta). \end{aligned}$$

Demak, $\begin{cases} \hat{\theta} = -X_{(1)} - \text{quyi chegara uchun} \\ \hat{\theta} = X_{(n)} - \text{yuqori chegara uchun} \end{cases}$ – HMO'UB bo'ladi, ya'ni yagona θ uchun ikkita baho $-X_{(1)}$ va $X_{(n)}$ lar HMO'UB bo'lar ekan. \triangleleft

17-misol. $\pi(\theta)$ taqsimot noma'lum parametrini haqiqatga maksimal o'xshashlik usuli bilan baholang.

\triangleright Puasson taqsimoti $p_k = P(\xi = k) = \frac{e^{-\theta} \theta^k}{k!}$, $k = 0, 1, \dots; \theta > 0$ dan haqiqatga o'xshashlik funksiyasini tuzamiz:

$$f_n(x^{(n)}, \theta) = \frac{e^{-\theta n} \theta^{\sum_{i=1}^n x_i}}{\prod_{i=1}^n x_i!} = \frac{e^{-\theta n} \theta^{n\bar{x}}}{\prod_{i=1}^n x_i!}.$$

Bu funksiya θ bo'yicha uzluksiz differensiallanuvchi ekanligidan, θ bo'yicha ekstremumini topish mumkin. Buning uchun bu funksiyaning logarifmidan foydalanamiz:

$$\ln f_n(x^{(n)}, \theta) = \ln \left(\frac{e^{-\theta n} \theta^{\sum_{i=1}^n x_i}}{\prod_{i=1}^n x_i!} \right) = n\bar{x} \ln \theta - \ln \prod_{i=1}^n x_i! - \theta n.$$

U holda $\frac{\partial \ln f_n(x^{(n)}, \theta)}{\partial \theta} = \frac{n\bar{x}}{\theta} - n = 0$ tenglamadan θ uchun \bar{x} HMO'UB bo'lar ekan. Demak, $\hat{\theta} = \bar{x}$. \triangleleft

Misollar

Quyidagi taqsimotlar noma'lum parametrlarini haqiqatga maksimal o'xshashlik usuli bilan baholang.

1. $\Gamma(\theta_1, \theta_2)$.
2. $\overline{Bi}(r, \theta)$.
3. $N(\theta_1, \theta_2^2)$.
4. $f(x, \theta) = \frac{\theta}{2} e^{-\theta x}$, $\theta > 0$.
5. $E(2\theta)$.
6. $R(-\theta, \theta)$.
7. $N(\theta, 2\theta)$.
8. $R[\theta - 1, \theta + 1]$.
9. $R[\theta_1, \theta_2]$.
10. $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta. \end{cases}$
11. $P(\theta_1, \theta_2)$.
12. $F(x, \theta) = \begin{cases} 1 - e^{-\frac{x-\theta_1}{\theta_2}}, & x \geq \theta_1, \theta_1, \theta_2 > 0, \\ 0, & x < \theta_1. \end{cases}$
13. $\log N(\theta_1, \theta_2^2)$.
14. $f(x, \theta) = \begin{cases} 3x^2\theta^{-3}e^{-(x/\theta)^3}, & x \geq 0, \theta > 0, \\ 0, & x < 0. \end{cases}$
15. $W(2, \theta)$.
16. $f(x, \theta) = \begin{cases} \theta(\ln^{\theta-1}x)/x, & x \in [1, e], \theta > 0, \\ 0, & x \notin [1, e]. \end{cases}$
17. $G(\frac{\theta}{2})$.
18. $f(x, \theta) = e^{-\frac{(x-\theta)}{2}}$, $x, \theta \in R$.
19. $Bi(n; \frac{\theta}{2})$.
20. $f(x, \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \theta > 0, \\ 0, & x \notin [0, \theta]. \end{cases}$

10-§. BAYES BAHOLASH USULI

Bayescha baholashning mohiyati shundan iboratki, unda noma'lum parametr θ tasodifiy miqdor deb qaraladi. θ t.m. ning zichlik funksiyasi $q(\theta)$ *aprior zichlik* deb ataladi. $f_n(x^{(n)}, \theta)$ tanlanmaning zichlik funksiyasi bo'lsin.

$$f(\theta; x_1, \dots, x_n) = f_n(x^{(n)}, \theta) q(\theta)$$

funksiya $R^{(n)} \times \Theta$ dagi biror taqsimotning zichlik funksiyasi (yoki $x^{(n)}$ va θ ning) birgalikdagi zichlik funksiyasi bo'ladi. Noma'lum parametr θ ning x_1, \dots, x_n tanlanma bo'yicha *Bayes bahosi*

$$\theta_n^* = M(\theta/x_1, \dots, x_n) = \int_{\Theta} \theta q(\theta/x_1, \dots, x_n) \lambda(d\theta)$$

ifoda orqali topiladi. Bu yerda $q(\theta/x_1, \dots, x_n)$ noma'lum parametr θ ning *aposterior zichligi* bo'lib, u quyidagicha hisoblanadi

$$q(\theta/x_1, \dots, x_n) = \frac{f_n(x^{(n)}, \theta) q(\theta)}{\int_{\Theta} f_n(x^{(n)}, s) q(s) \lambda(ds)}$$

18-misol. $X_1, \dots, X_n \sim N(\theta; 1)$ va θ parametr taqsimoti $N(0; \sigma^2)$ (σ^2 - ma'lum) bo'lsin. Noma'lum parametr θ ning Bayes bahosini tuzing.

▷ Noma'lum parametr taqsimoti $N(0; \sigma^2)$ bo'lganligi uchun uning zichlik funksiyasi $q(\theta) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\theta^2/2\sigma^2}$, $\theta \in R$, $\sigma > 0$ bo'ladi. $x^{(n)}$ tanlanmaning zichlik funksiyasi esa

$$f_n(x^{(n)}, \theta) = \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2} \sum_{i=1}^n (x_i - \theta)^2}$$

$q(\theta/x_1, \dots, x_n)$ aposterior zichlik $q(\theta) f_n(x^{(n)}, \theta)$ ga proporsional:

$$\exp \left\{ -\frac{\theta^2}{2\sigma^2} - \frac{1}{2} \sum_{i=1}^n (x_i - \theta)^2 \right\} = \exp \left\{ -\frac{\theta^2 (1/\sigma^2 + n)}{2} + \bar{x}n\theta - \frac{n\bar{x}^2}{2} \right\}$$

Har ikkala tomon darajalarini tenglashtirsak,

$$\frac{\theta^2}{2} \left(\frac{1}{\sigma^2} + n \right) + \bar{x}n\theta = -\frac{1}{2} \left(\frac{1}{\sigma^2} + n \right) \left(\theta - \frac{\bar{x}n}{1/\sigma^2 + n} \right)^2 + \frac{(\bar{x}n)}{2(1/\sigma^2 + n)}$$

hosil bo'ladi. Bu tenglikdan $q(\theta/x_1, \dots, x_n)$ aposterior zichlik $N\left(\frac{\bar{x}n\sigma^2}{1+n\sigma^2}; \frac{\sigma^2}{1+n\sigma^2}\right)$ taqsimotga ega ekanligi kelib chiqadi. Demak, θ uchun Bayes baho

$$\theta_n^* = \int_{\Theta} \theta q(\theta/x_1, \dots, x_n) d\theta = \frac{\bar{x}n\sigma^2}{1+n\sigma^2}.$$

◁

Misollar

1. $X_1, \dots, X_n \sim N(\theta; 1)$ va θ parametr $N(a; \sigma^2)$ (a, σ^2 – ma'lum) bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

2. $X_1, \dots, X_n \sim Bi(1; \theta)$ va θ parametr $R[0, 1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

3. $X_1, \dots, X_n \sim N(\theta; 1)$ va θ parametr $Bi(1; 1/2)$ bo'lsin. Noma'lum parametr θ ning bayes bahosini toping.

4. $X_1, \dots, X_n \sim \pi(\theta)$ va θ parametr $Bi(1; 1/2)$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

5. X_1, \dots, X_n tanlanma zichlik funksiyasi $f(x; \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimotdan olingan va θ parametr taqsimoti $R[0, 1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

6. $X_1, \dots, X_n \sim R[0, \theta]$ va θ parametr $R[0, 1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

7. $X_1, \dots, X_n \sim R[0, \theta]$ va θ parametr zichlik funksiyasi $q(\theta) = 1/\theta^2$, $\theta \geq 1$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

8. $X_1, \dots, X_n \sim R[0, \theta]$ va θ parametr taqsimoti: $\begin{cases} \theta : & 1 & 2 \\ P_{\theta} : & \frac{1}{2} & \frac{1}{2} \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

9. $X_1, \dots, X_n \sim E(\theta)$ va θ parametr $E(1)$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

10. $X_1, \dots, X_n \sim E(\theta)$ va θ parametr $R[0, 1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

11. X_1, \dots, X_n tanlanma zichlik funksiyasi $f(x; \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta] \\ 0, & x \notin [0, \theta] \end{cases}$ bo'lgan taqsimotdan olingan va θ parametr taqsimoti $P(\beta, 1)$ (β – ma'lum) bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

12. X_1, \dots, X_n tanlanma zichlik funksiyasi $f(x; \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \\ 0, & x \notin [0, \theta] \end{cases}$ va θ parametr zichlik funksiyasi $q(\theta) = 3\theta^2$, $\theta \in [0, 1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

13. X_1, \dots, X_n tanlanma zichlik funksiyasi $f(x; \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \\ 0, & x \notin [0, \theta] \end{cases}$ va θ parametr zichlik funksiyasi $R[0, 1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

14. $X_1, \dots, X_n \sim Bi(1; \theta)$ va θ parametr taqsimoti $\begin{cases} \theta: 1/2 & 1/3 \\ P_\theta: 1/2 & 1/2 \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

15. $X_1, \dots, X_n \sim Bi(1; \theta)$ va θ parametr zichlik funksiyasi $q(\theta) = \lambda\theta^{\lambda-1}$, $\theta \in [0, 1]$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

16. $X_1, \dots, X_n \sim \pi(\theta)$ va θ parametr taqsimoti $\begin{cases} \theta: 1 & 2 \\ P_\theta: 1/3 & 2/3 \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

17. $X_1, \dots, X_n \sim G(\theta)$ va θ parametr taqsimoti $\begin{cases} \theta: \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\ P_\theta: \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

18. $X_1, \dots, X_n \sim G(\theta)$ va θ parametr taqsimoti $\begin{cases} \theta: 1 & 2 \\ P_\theta: \frac{1}{3} & \frac{2}{3} \end{cases}$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

19. $X_1, \dots, X_n \sim \pi(\theta)$ va θ parametr $Bi(1; \frac{1}{3})$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

20. $X_1, \dots, X_n \sim G(\theta)$ va θ parametr $Bi(1; \frac{1}{2})$ bo'lsin. Noma'lum parametr θ ning Bayes bahosini toping.

11-§. BAHOLARNING ASIMPTOTIK NORMALLIGI

Agar $\sqrt{n}(\theta_n - \theta) \Rightarrow N(0; D(\theta_n))$ (yoki $\frac{\sqrt{n}(\theta_n - \theta)}{D(\theta_n)} \Rightarrow N(0; 1)$) shart bajarilsa, θ_n baho noma'lum parametr θ uchun *asimptotik normal* baho deyiladi.

Agar θ_n noma'lum parametr θ uchun a.n. baho bo'lsa, u holda θ_n asosli baho bo'ladi.

19-misol. X_1, \dots, X_n tanlanma $R[0; \theta]$ taqsimotdan olingan bo'lsin. $2\bar{x}$ statistika noma'lum θ parametr uchun asimptotik normal baho bo'lishini ko'rsating. $\triangleright MX_1 = \theta/2$ va $DX_1 = \theta^2/12$ ekanligini e'tiborga olib, quyidagini yoza olamiz:

$$\begin{aligned} \sqrt{n}(\theta_n - \theta) &= \sqrt{n}(2\bar{x} - \theta) = \sqrt{n} \left(2 \frac{\sum_{i=1}^n X_i}{n} - \theta \right) = \\ &= \frac{\sum_{i=1}^n 2X_i - n\theta}{\sqrt{n}} = \frac{\sum_{i=1}^n 2X_i - n \cdot M \cdot (2X_1)}{\sqrt{n}}. \end{aligned}$$

Markaziy limit teoreмага asosan:

$$\frac{\sum_{i=1}^n 2X_i - nM(2X_1)}{\sqrt{n}} \Rightarrow N(0; D(2X_1)) = N(0; 4DX_1) = N(0; \theta^2/3),$$

ya'ni $2\bar{x}$ asimptotik normal baho bo'lar ekan. \triangleleft

Misollar

1. X_1, \dots, X_n tanlanma chekli dispersiyaga ega taqsimotdan olingan bo'lsin. $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ statistika noma'lum $\theta = MX_1$ parametr uchun asimptotik normal baho bo'lishini ko'rsating.

2. X_1, \dots, X_n tanlanma $N(0, \theta^2)$ taqsimotdan olingan bo'lsin. $\bar{x}^2 = \frac{1}{n} \sum_{i=1}^n X_i^2$ statistika noma'lum θ^2 parametr uchun asimptotik normal baho bo'lishini ko'rsating.

3. Agar MX_1^4 chekli bo'lsa, $S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{x})^2$ statistika noma'lum $\theta = DX_1$ uchun asimptotik normal baho bo'lishini ko'rsating.

4. X_1, \dots, X_n tanlanma $R[0, \theta]$ taqsimotdan olingan bo'lsin. $\sqrt[k]{(k+1)x^k}$, $k \geq 1$ statistika noma'lum parametr θ uchun asimptotik normal baho bo'lishini ko'rsating.

5. X_1, \dots, X_n tanlanma $R\left[\frac{\theta}{2}, \theta\right]$ taqsimotdan olingan bo'lsin. $\ln(4\bar{x}/3)$ statistika noma'lum parametrlar $\ln \theta$ uchun asimptotik normal baho bo'lishini ko'rsating.

6. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. $\sqrt[k]{k!/x^k}$ statistika noma'lum parametrlar $\ln \theta$ uchun asimptotik normal baho bo'lishini ko'rsating.

7. X_1, \dots, X_n tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. $\ln \bar{x}$ statistika noma'lum parametrlar $\ln \theta$ uchun asimptotik normal baho bo'lishini ko'rsating.

8. X_1, \dots, X_n tanlanma zichligi $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametrlar θ uchun $\bar{x} - 1$ statistika asimptotik normal baho bo'ladimi?

9. X_1, \dots, X_n tanlanma zichligi $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametrlar θ uchun $X_{(1)}$ statistika asimptotik normal baho bo'ladimi?

10. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin:

$$f(x, \theta) = \begin{cases} \theta^{-1} e^{-(x-b)/\theta}, & x \geq b, b \in R, \theta > 0 \\ 0, & x < b. \end{cases}$$

Noma'lum parametrlar θ uchun \bar{x} statistika asimptotik normal baho bo'ladimi?

11. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin:

$$f(x, \theta) = \begin{cases} a^{-1} e^{-(x-\theta)/a}, & x \geq \theta, \theta \in R, a > 0 \\ 0, & x < \theta. \end{cases}$$

Noma'lum parametrlar θ uchun $\bar{x} - S$ statistika asimptotik normal baho bo'ladimi?

12. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin:

$$f(x, \theta) = \begin{cases} \theta^{-1} e^{-(x-b)/\theta}, & x \geq b, b \in R, \theta > 0 \\ 0, & x < b. \end{cases}$$

Noma'lum parametrlar θ uchun $\bar{X} - X_{(1)}$ statistika asimptotik normal baho bo'ladimi?

13. X_1, \dots, X_n tanlanma zichlik funksiyasi quyidagicha bo'lgan taqsimotdan olingan bo'lsin:

$$f(x, \theta) = \begin{cases} a^{-1} e^{-(x-\theta)/a}, & x \geq \theta, \theta \in R, a > 0 \\ 0, & x < \theta. \end{cases}$$

Noma'lum parametr θ uchun $X_{(1)}$ statistika asimptotik normal baho bo'ladimi?

14. X_1, \dots, X_n tanlanma $P(\theta, \beta)$ (β - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $\frac{1}{\ln x - \ln X_{(1)}}$ statistika asimptotik normal baho bo'ladimi?

15. X_1, \dots, X_n tanlanma $P(\alpha, \theta)$ (α - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $X_{(1)}$ statistika asimptotik normal baho bo'ladimi?

16. X_1, \dots, X_n tanlanma $W(\alpha, \theta)$ (α - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $1/\sqrt{x^\alpha}$ statistika asimptotik normal baho bo'ladimi?

17. X_1, \dots, X_n tanlanma $R[0, \theta]$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $\frac{n+1}{n} X_{(n)}$ statistika asimptotik normal baho bo'ladimi?

18. X_1, \dots, X_n tanlanma $\pi(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun \bar{x} statistika asimptotik normal baho bo'ladimi?

19. X_1, \dots, X_n tanlanma $N(a, \theta^2)$ (a - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ^2 uchun $\frac{1}{n} \sum_{i=1}^n (X_i - a)^2$ statistika asimptotik normal baho bo'ladimi?

20. X_1, \dots, X_n tanlanma $\Gamma(\theta, 1)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $(\bar{x})^{-1}$ statistika asimptotik normal baho bo'ladimi?

21. X_1, \dots, X_n tanlanma $\Gamma(\theta, 1)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun $\left(\frac{1}{2n} \sum_{i=1}^n X_i^2\right)^{-1/2}$ statistika asimptotik normal baho bo'ladimi?

12-§. BAHOLARNI SOLISHTIRISH

20-misol. $F(x, \theta) = \begin{cases} 1 - e^{\theta-x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$, taqsimot uchun $T_1(X^{(n)}) = \bar{x} - 1$ va $T_2(X^{(n)}) = X_{(1)}$ baholarni o'rtacha kvadratik ma'noda effektivlikka tekshiring.

▷ Buning uchun avval quyidagi momentlarni hisoblaymiz:

$$MX_1 = \int_{\theta}^{+\infty} x e^{\theta-x} dx = \left| \begin{array}{l} x = u \quad dx = du \\ e^{\theta-x} dx = dv \quad v = -e^{\theta-x} \end{array} \right| =$$

$$-x e^{\theta-x} \Big|_{\theta}^{+\infty} + \int_{\theta}^{+\infty} e^{\theta-x} dx = \theta - e^{\theta-x} \Big|_{\theta}^{+\infty} = \theta + 1;$$

$$MX_1^2 = \int_{\theta}^{+\infty} x^2 e^{\theta-x} dx = \left| \begin{array}{l} x^2 = u \quad 2x dx = du \\ e^{\theta-x} dx = dv \quad v = -e^{\theta-x} \end{array} \right| =$$

$$-x^2 e^{\theta-x} \Big|_{\theta}^{+\infty} + 2 \int_{\theta}^{+\infty} x e^{\theta-x} dx = \theta^2 + 2(\theta + 1);$$

$MT_1(X^{(n)}) = M(\bar{x} - 1) = MX_1 - 1 = \theta$, ya'ni $T_1(X^{(n)})$ siljimgan baho bo'lganligi uchun $M(T_1(X^{(n)}) - \theta)^2 = DT_1(X^{(n)})$ bo'ladi. Demak, $DT_1(X^{(n)}) = D(\bar{x} - 1) = D\bar{x} = \frac{1}{n} DX_1 = \frac{1}{n} (\theta^2 + 2(\theta + 1) - (\theta + 1)^2) = \frac{1}{n}$.

Endi $T_2(X^{(n)}) = X_{(1)}$ bahoning momentlarini hisoblaymiz:

$$MX_{(1)} = \int_{\theta}^{+\infty} x n e^{(\theta-x)(n-1)} e^{\theta-x} dx = n \int_{\theta}^{+\infty} x e^{n(\theta-x)} dx =$$

$$= \left| \begin{array}{l} x = u \quad dx = du \\ e^{n(\theta-x)} dx = dv \quad v = -\frac{e^{n(\theta-x)}}{n} \end{array} \right| =$$

$$\begin{aligned}
 &= n \left[-x \frac{e^{n(\theta-x)}}{n} \Big|_{\theta}^{+\infty} + \frac{1}{n} \int_{\theta}^{+\infty} e^{n(\theta-x)} dx \right] = \\
 &= \theta + \int_{\theta}^{+\infty} e^{n(\theta-x)} dx = \theta - \frac{e^{n(\theta-x)}}{n} \Big|_{\theta}^{+\infty} = \theta + \frac{1}{n};
 \end{aligned}$$

$$\begin{aligned}
 MX_{(1)}^2 &= \int_{\theta}^{+\infty} x^2 n e^{n(\theta-x)} dx = \left| \begin{array}{ll} x^2 = u & 2x dx = du \\ e^{n(\theta-x)} dx = dv & v = -\frac{e^{n(\theta-x)}}{n} \end{array} \right| = \\
 &= n \left[-x^2 \frac{e^{n(\theta-x)}}{n} \Big|_{\theta}^{+\infty} + \frac{2}{n} \int_{\theta}^{+\infty} x e^{n(\theta-x)} dx \right] = \theta^2 + \frac{2}{n} \left(\theta + \frac{1}{n} \right).
 \end{aligned}$$

$$M(X_{(1)} - \theta)^2 = MX_{(1)}^2 - 2\theta MX_{(1)} + \theta^2 = \theta^2 + \frac{2\theta}{n} + \frac{2}{n^2} - 2\theta^2 - \frac{2\theta}{n} + \theta^2 = \frac{2}{n^2};$$

Demak, $n \geq 3$ uchun $\frac{1}{n} > \frac{2}{n^2}$ bo'lgani uchun $X_{(1)}$ baho $\bar{x} - 1$ bahoga nisbatan effektiv baho ekan. \triangleleft

Misollar

Quyidagi taqsimotlar uchun keltirilgan baholarni o'rtacha kvadratik ma'noda effektivlikka tekshiring:

1. $N(a, \theta^2)$: $S^2 = \frac{1}{n-1} \sum (X_i - \bar{x})^2$;
 $S_2^2 = \frac{1}{n} \sum (X_i - a)^2$;
2. $R[0; \theta]$: $T_1(X^{(n)}) = 2\bar{x}$;
 $T_2(X^{(n)}) = \frac{n+1}{n} X_{(n)}$;
3. $R[0, \theta]$: $T_1(X^{(n)}) = X_{(n)}$;
 $T_2(X^{(n)}) = X_{(1)} + X_{(n)}$;
4. $R[\theta, \theta + 1]$: $T_1(X^{(n)}) = \bar{x} - \frac{1}{2}$;
 $T_2(X^{(n)}) = X_{(n)} - 1$;

5. $R[\theta, \theta + 1]$: $T_1(X^{(n)}) = \bar{x} - \frac{1}{2}$;
 $T_2(X^{(n)}) = X_{(1)}$;
6. $f(x, \theta) = \begin{cases} \frac{2x}{\theta^2}, & x \in [0, \theta], \theta > 0, \\ 0, & x \notin [0, \theta]; \end{cases}$ $T_1(X^{(n)}) = \frac{3}{2}\bar{x}$;
 $T_2(X^{(n)}) = X_{(n)}$;
7. $F(x, \theta) = \begin{cases} 1 - e^{\theta-x}, & x \geq \theta, \\ 0, & x < \theta; \end{cases}$ $T_1(X^{(n)}) = \bar{x} - 1$;
 $T_2(X^{(n)}) = X_{(1)} - \frac{1}{n}$
8. $f(x, \theta) = \begin{cases} \theta^{-1}, e^{-\frac{(x-b)}{\theta}}, & x \geq b, \theta > 0. \\ 0, & x < b; \end{cases}$ $T_1(X^{(n)}) = S$;
 $T_2(X^{(n)}) = \bar{x} - X_{(1)}$,
9. $N(a; \theta^2)$: $T_1(X^{(n)}) = \bar{x}^2 - a^2$;
 $T_2(X^{(n)}) = \frac{1}{n-1} \sum_{i=1}^n (X_i - a)^2$;
10. $P(\theta; 1)$: $T_1(X^{(4)}) = X_{(1)}$;
 $T_2(X^{(4)}) = \bar{x}$.

13-§. TANLANMA XARAKTERISTIKALARI TAQSIMOTI

21-misol. $X_1, \dots, X_n \sim R[0; 1]$ va $R = X_{(n)} - X_{(1)}$ tanlanma qulochi bo'lsin. $2n(1 - R)$ ning limit taqsimotini toping.

▷ $X_{(1)}$ va $X_{(n)}$ larning birgalikdagi zichlik funksiyasi ([9]):

$$f(x; y) = \begin{cases} n(n-1)(y-x)^{n-2}, & 0 < x < y < 1, \\ 0, & \text{aks holda.} \end{cases}$$

U holda R va $X_{(n)}$ larning birgalikdagi zichlik funksiyasi

$$g(x) = \begin{cases} n(n-1)x^{n-2}, & 0 < x < 1, \\ 0, & \text{aks holda,} \end{cases}$$

bo'ladi. Bu zichlik yordamida R ning zichlik funksiyasini hisoblaymiz: agar $0 < x < 1$ bo'lsa,

$$\varphi_R(x) = \int g(x) dt = \int_0^1 n(n-1)x^{n-2} dt = n(n-1)x^{n-2}(1-x),$$

aks holda $\varphi_R(x) = 0$. R ning zichlik funksiyasi yordamida $2n(1 - R)$ t.m.ning taqsimotini topamiz:

$$\begin{aligned} H_{2n(1-R)}(x) &= P(2n(1 - R) < x) = P\left(R > 1 - \frac{x}{2n}\right) = \\ &= 1 - P\left(R \leq 1 - \frac{x}{2n}\right) = 1 - G_R\left(1 - \frac{x}{2n}\right). \end{aligned}$$

$$\begin{aligned} h_n(x) &= (H_{2n(1-R)}(x))' = g_R\left(1 - \frac{x}{2n}\right) \frac{1}{2n} = \\ &= n(n-1)\left(1 - \frac{x}{2n}\right)^{n-2} \left(1 - \left(1 - \frac{x}{2n}\right)\right) \frac{1}{2n} = \frac{n-1}{4n} x \left(1 - \frac{x}{2n}\right)^{n-2}, \\ 0 < x < 2n. \quad &2n(1 - R) \text{ ning zichlik funksiyasi} \end{aligned}$$

$$h_n(x) = \begin{cases} \frac{n-1}{4n} x \left(1 - \frac{x}{2n}\right)^{n-2}, & 0 < x < 2n, \\ 0, & \text{aks holda.} \end{cases}$$

va $\lim_{n \rightarrow \infty} \left(1 - \frac{x}{2n}\right)^{n-2} = e^{-x/2}$ ekanligini hisobga olsak,

$$\lim_{n \rightarrow \infty} h_n(x) = 4^{-1} x e^{x/2} I(0 < x < \infty)$$

ni hosil qilamiz.

Demak, $2n(1 - R)$ ning limit taqsimoti

$$f(x) = \begin{cases} 4^{-1} x e^{x/2}, & 0 < x < \infty, \\ 0, & \text{aks holda,} \end{cases}$$

ya'ni χ_4^2 taqsimot ekan (ozodlik darajasi 4 bo'lgan xi-kvadrat taqsimot). \triangleleft

Misollar

1. Agar $X_1, \dots, X_n \sim N(a, \sigma^2)$ bo'lsa, $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ ning taqsimotini toping.
2. Agar $X_1, \dots, X_2 \sim N(0, 1)$ bo'lsa, $Z = X_1^2 + X_2^2$ ning taqsimotini toping.
3. $N(a; \theta_2)$ (a - ma'lum), modelda $\sqrt{n} \frac{\bar{x}-a}{S}$ statistikaning taqsimotini toping.
4. $N(a; \sigma^2)$ modelda $n \frac{(\bar{x}-a)^2}{\sigma^2}$ statistikaning taqsimotini toping.
5. $N(a, \sigma^2)$ modelda $S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{x})^2$ statistikaning taqsimotini toping.
6. $R[a, b]$ modelda $X_{(1)}$ va $X_{(n)}$ statistikalarining taqsimotlarini toping.
7. $E(\alpha)$ modelda $X_{(1)}$ va $X_{(n)}$ statistikalarining taqsimotlarini toping.
8. $N(a, \sigma^2)$ modelda $\frac{nS^2}{\sigma^2}$ statistikaning taqsimotini toping.
9. Agar $X_1, \dots, X_n \sim \pi(\lambda)$, $Y_1, \dots, Y_m \sim \pi(\lambda)$ va $X_i \perp Y_i$ bo'lsa, $\bar{x} + \bar{y}$ ning taqsimotini toping.
10. Agar $X_1, \dots, X_n \sim N(a_x; \sigma)$, $Y_1, \dots, Y_m \sim N(a_y; \sigma)$ va $X_i \perp Y_i$ bo'lsa, $\bar{x} - \bar{y} - (a_x - a_y)$ ning taqsimotini toping.
11. Agar $X_1, \dots, X_n \sim \Gamma(\alpha; \lambda)$ bo'lsa, $\sum_{i=1}^n X_i$ ning taqsimotini toping.
12. Agar $X_1, \dots, X_n \sim N(a; \sigma^2)$ bo'lsa, $\sqrt{n} \frac{\bar{x}-a}{\sigma}$ ning taqsimotini toping.
13. Agar $X_1, \dots, X_n \sim N(a; \sigma^2)$ bo'lsa, $\sum_{i=1}^n \left(\frac{X_i - a}{\sigma} \right)^2$ ning taqsimotini toping.
14. Agar $X_1, \dots, X_n \sim N(a; \sigma^2)$ bo'lsa, $\sum_{i=1}^n \left(\frac{X_i - \bar{x}}{\sigma} \right)^2$ ning taqsimotini toping.
15. Agar $X_1, \dots, X_n \sim \pi(\lambda)$ bo'lsa, $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ ning taqsimotini toping.

16. Agar $X_1, \dots, X_n \sim B_i(1; p)$ bo'lsa, $\sum_{i=1}^n X_i$ ning taqsimotini toping.

17. Agar $X_1, \dots, X_n \sim E(\alpha)$ bo'lsa, $\bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$ ning taqsimotini toping.

18. Agar $X_1, \dots, X_n \sim R[0; b]$ bo'lsa, $X_{(2)}$ ning taqsimotini toping.

19. Agar $X_1, \dots, X_n \sim R[0; \theta]$ bo'lsa, $n \rightarrow \infty$ da $nX_{(1)}/\theta$ ning limit taqsimotini toping.

20. Agar $X_1, \dots, X_n \sim R[0; \theta]$ bo'lsa, $n \rightarrow \infty$ da $n(\theta - X_{(n)})/\theta$ ning limit taqsimotini toping.

14-§. PARAMETRLARNI INTERVAL BAHOLASH

ξ t.m.ning t.f. $F(x; \theta)$, $\theta \in \Theta$ va $X^{(n)} = (X_1, \dots, X_n)$ tanlanma shu taqsimotdan olingan bo'lsin. $T_1(x^{(n)})$ va $T_2(x^{(n)})$, $T_1 < T_2$ statistikalar uchun berilgan $\gamma \in (0; 1)$ da barcha $\theta \in \Theta$ uchun

$$P(T_1 \leq \theta \leq T_2) \geq \gamma$$

tengsizlik o'rinli bo'lsa, $[T_1(x^{(n)}), T_2(x^{(n)})]$ oraliq γ *ishonchlilik oraliq'i (intervali)* deyiladi. γ esa ishonchlilik darajasi deyiladi. Odatda γ ni 1 ga yaqin qilib tanlanadi ($\gamma = 0, 95; 0, 99; \dots$).

$T_1(x^{(n)})$ va $T_2(x^{(n)})$, $T_1 < T_2$ statistikalar uchun berilgan $\gamma \in (0; 1)$ da barcha $\theta \in \Theta$ uchun

$$\liminf_{n \rightarrow \infty} P(T_1 \leq \theta \leq T_2) \geq \gamma$$

tengsizlik o'rinli bo'lsa, $[T_1(x^{(n)}), T_2(x^{(n)})]$ oraliq *asimptotik ishonchlilik oraliq'i* deyiladi.

Umumiy holda ishonchlilik intervalini tuzishni keltiramiz:

$X^{(n)} = (X_1, \dots, X_n)$ tanlanma va noma'lum parametr θ ga bog'liq, hamda quyidagi shartlarni qanoatlantiruvchi $G(x^{(n)}; \theta)$ funksiya berilgan bo'lsin:

- 1) $G(x^{(n)}; \theta)$ ning taqsimoti noma'lum parametr θ ga bog'liq emas;
- 2) $G(x^{(n)}; \theta)$ uzluksiz va θ bo'yicha monoton.

Bunday $G(x^{(n)}; \theta)$ t.m. *markaziy statistika* deyiladi. $f_G(t)$ funksiya

$G(x^{(n)}; \theta)$ ning zichlik funksiyasi bo'lsin, u holda ixtiyoriy $\gamma \in (0; 1)$ uchun $g_1 < g_2$ sonlarni shunday tanlash mumkinki,

$$P\left(g_1 \leq G(x^{(n)}; \theta) \leq g_2\right) = \int_{g_1}^{g_2} f_G(t) dt = \gamma$$

o'rinli bo'lsin. g_1, g_2 lar $G(x^{(n)}; \theta)$ ning taqsimoti kvantillaridir. Odatda g_1, g_2 lar

$$P\left(G(x^{(n)}; \theta) \leq g_1\right) = P\left(G(x^{(n)}; \theta) \geq g_2\right) = \gamma$$

shartlardan tanlanadi. $g_1 \leq G(x^{(n)}; \theta) \leq g_2$ tengsizlikni θ ga nisbatan yechib, $T_1 \leq \theta \leq T_2$ ishonchlik oralig'ini hosil qilamiz.

22-misol. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $R[\theta, 2\theta]$, $\theta > 0$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlik intervalini tuzing.

▷ Agar $X_i \sim R[\theta, 2\theta]$ bo'lsa, u holda $Y_i = \frac{X_i}{\theta} - 1 \sim [0, 1]$ bo'ladi. Bu taqsimot noma'lum parametrining HMO'UB $X_{(n)}$ bo'lgani uchun, markaziy statistikani quyidagi ko'rinishda olamiz:

$$Y_{(n)} = \max\{Y_1, \dots, Y_n\} = \frac{\max\{X_1, \dots, X_n\}}{\theta} - 1 = \frac{X_{(n)}}{\theta} - 1 = G(x^{(n)}; \theta).$$

$$\text{Uning taqsimoti } F_{Y_{(n)}}(y) = P_{\theta}(\eta < y) = \begin{cases} 0, & y < 0, \\ y^n, & y \in [0, 1], \\ 1, & y > 0. \end{cases}$$

Ixtiyoriy musbat g_1 va g_2 lar uchun:

$$\begin{aligned} P_{\theta}(g_1 \leq G(X_i; \theta) \leq g_2) &= P_{\theta}\left(g_1 \leq \frac{X_{(n)}}{\theta} - 1 \leq g_2\right) = \\ &= P_{\theta}\left(\frac{X_{(n)}}{g_2 + 1} \leq \theta \leq \frac{X_{(n)}}{g_1 + 1}\right). \end{aligned}$$

Intervalning uzunligi $\frac{X_{(n)}(g_1 - g_2)}{(g_1 + 1)(g_2 + 1)}$ miqdor g_1, g_2 lar kattalashsa, hamda bir-biriga yaqinlashsa kichrayadi. $Y_{(n)}$ t.m.ning zichlik funksiyasi $[0, 1]$

oraliqda ny^{n-1} ga teng va u monoton o'suvchi. Shuning uchun bir-biriga yaqin g_1 va g_2 kvantillar eng katta qiymatlariga $g_2 = 1$ va g_1 :

$$\gamma = P_\theta (g_1 \leq Y_{(n)} \leq 1) = F_{Y_{(n)}}(1) - F_{Y_{(n)}}(g_1) = 1 - g_1^n = \gamma,$$

bo'lsa erishadi. Bu yerdan $g_1 = \sqrt[n]{1-\gamma}$ bo'ladi. Demak,

$$\gamma = P_\theta \left(\sqrt[n]{1-\gamma} \leq Y_{(n)} \leq 1 \right) = P_\theta \left(\frac{X_{(n)}}{2} \leq \theta \leq \frac{X_{(n)}}{1 + \sqrt[n]{1-\gamma}} \right).$$

◁

23-misol. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $\pi(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlik intervalini tuzing.

▷ MLT ga ko'ra:

$$\frac{\sum_{i=1}^n X_i - nMX_1}{\sqrt{nDX_1}} = \sqrt{n} \frac{\bar{x} - \theta}{\sqrt{\theta}} \Rightarrow \eta \sim N(0; 1).$$

$n \rightarrow \infty$ da $P\left(-c < \sqrt{n} \frac{\bar{x} - \theta}{\sqrt{\theta}} < c\right) \rightarrow P(-c < \eta < c) = \gamma$, $c = t_{\frac{1+\gamma}{2}}$ -normal taqsimot kvantili. Sust yaqinlashish xossasiga ko'ra, agar $\xi_n \xrightarrow{p} 1$ va $\eta_n \Rightarrow \eta$ bo'lsa, u holda $\xi_n \eta_n \Rightarrow \eta$. $\theta_n = \bar{x}$ asosli baho ekanligidan $\frac{\theta}{\bar{x}} \xrightarrow{p} 1$. Bundan

$$\sqrt{\frac{\theta}{\bar{x}}} \sqrt{n} \frac{\bar{x} - \theta}{\sqrt{\theta}} = \sqrt{n} \frac{\bar{x} - \theta}{\sqrt{\bar{x}}} \Rightarrow \eta.$$

Shuning uchun,

$$P\left(-t_{\frac{1+\gamma}{2}} < \sqrt{n} \frac{\bar{x} - \theta}{\sqrt{\bar{x}}} < t_{\frac{1+\gamma}{2}}\right) \rightarrow P\left(-t_{\frac{1+\gamma}{2}} < \eta < t_{\frac{1+\gamma}{2}}\right) = \gamma.$$

Tengsizlikni noma'lum parametr θ ga nisbatan yechsak, $P\left(\bar{x} - \frac{t_{\frac{1+\gamma}{2}} \sqrt{\bar{x}}}{\sqrt{n}} < \theta < \bar{x} + \frac{t_{\frac{1+\gamma}{2}} \sqrt{\bar{x}}}{\sqrt{n}}\right) \rightarrow \gamma$, $n \rightarrow \infty$, asimptotik ishonchlik intervalini hosil qilamiz.

Demak, noma'lum parametr θ uchun asimptotik ishonchlilik intervali $\left(\bar{x} - \frac{t_{1+\frac{\gamma}{2}} \sqrt{\bar{x}}}{\sqrt{n}}; \bar{x} + \frac{t_{1+\frac{\gamma}{2}} \sqrt{\bar{x}}}{\sqrt{n}} \right)$ bo'lar ekan. \triangleleft

Misollar

1. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $N(\theta, \sigma^2)$ (σ^2 - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

2. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $N(a, \theta^2)$ (a - ma'lum) taqsimotdan olingan bo'lsin. Noma'lum parametr θ^2 uchun ishonchlilik intervalini tuzing.

3. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ_1 uchun ishonchlilik intervalini tuzing.

4. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $N(\theta_1, \theta_2^2)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ_2^2 uchun ishonchlilik intervalini tuzing.

5. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $R[0, \theta]$, $\theta > 0$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

6. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $\Gamma(\theta; \lambda)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

7. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $E(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

8. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $Bi(n; \theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

9. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $f(x, \theta) = \begin{cases} e^{\theta-x}, & x \geq \theta \\ 0, & x < \theta \end{cases}$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

10. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $Ge(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

11. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $R[0; \theta]$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun asimptotik ishonchlilik intervalini tuzing.

12. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $R[\theta, \theta + 1]$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

13. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $\Gamma(\theta; 1)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ uchun ishonchlilik intervalini tuzing.

15-§. STATISTIK GIPOTEZALARNI TEKSHIRISH

Kuzatilayotgan t.m. haqida aytilgan ixtiyoriy fikrga *statistik gipoteza* deyiladi. Tekshirilishi kerak bo'lgan gipoteza *asosiy gipoteza* deyiladi va uni H_0 bilan belgilaymiz. Asosiy gipotezadan mantiqan qarama-qarshi bo'lgan ixtiyoriy gipotezaga *raqobatlashuvchi* yoki *alternativ gipoteza* deb ataladi. Statistik gipoteza sodda deyiladi, agar u taqsimotni bir qiymatli aniqlasa: $H_0 : F = F_0$, aks holda u murakkab deyiladi: $H_0 : F \in \mathcal{F}$. Matematik statistikada statistik gipotezalarni tekshirishda ikki xil xatolikka yo'l qo'yilishi mumkin. Statistik yechim asosida asosiy faraz u to'g'ri bo'lgan holda ham rad etilishi mumkin. Bunday xatolik *I tur xatolik* deyiladi. I tur xatolikning ehtimolligini α bilan belgilaymiz. U qiymatdorlik darajasi ham deyiladi. Statistik yechim asosida alternativ gipoteza to'g'ri bo'lsa ham rad etilishi mumkin. Bunday xatolik *II tur xatolik* deyiladi. II tur xatolikning ehtimolligini β bilan belgilaymiz.

Statistik gipotezalarni tekshirish statistik ma'lumotlarga asoslanadi. Faraz qilaylik, X_1, X_2, \dots, X_n lar X t.m.ning n ta bog'liqsiz tajribalardagi kuzatilmalari va X t.m.ning biron - bir xarakteristikasi haqidagi asosiy H_0 gipoteza ko'rilayotgan bo'lsin. Statistik ma'lumotlar asosida asosiy gipoteza H_0 ni qabul qilish yoki rad etish qoidasini tuzish kerak. Asosiy gipoteza H_0 ni qabul qilish yoki rad etish qoidasi - H_0 gipotezani tekshirishning statistik kriteriyasi deyiladi. Statistik gipotezalarni tekshirish - statistik ma'lumotlar asosida asosiy gipotezani tasdiqlash yoki uni rad etishdan iborat bo'ladi.

Endi statistik alomatlarini tuzish qoidalari bilan tanishamiz. Odatda statistik kriteriyani qurish empirik ma'lumotlarni asosiy H_0

gipoteza bo'yicha tavsiflovchi statistika $T = T(X_1, \dots, X_n)$ ni tanlashdan boshlanadi. Bunday tanlashda ikki xossa bajarilishi talab etiladi: a) statistika manfiy qiymatlar qabul qilmaydi; b) asosiy gipoteza to'g'ri bo'lganda statistikaning aniq, H_0 - gipotezaviy taqsimoti ma'lum bo'lishi kerak.

Faraz qilaylik, bunday statistika topilgan bo'lib, $S = \{t : t = T(X_1, \dots, X_n), (X_1, \dots, X_n) - \text{tanlanma fazosiga tegishli}\}$ - statistikaning qiymatlar to'plami bo'lsin. Amaliyotda har ikki xatoliklarni bir vaqtda minimallashtirish murakkabdir. Shu boisdan, dastlab I tur xatolik α ni fiksirlab, β ni minimallashtiriladi (yoki kriteriy quvvati $1-\beta$ maksimallashtiriladi). Oldindan $0 < \alpha < 1$ - sonini tayinlaylik. Endi S sohani shunday kesishmaydigan S_α va $S \setminus S_\alpha$ sohalariga ajratamizki, bunda asosiy gipoteza H_0 to'g'ri bo'lganida $T(X_1, \dots, X_n) \in S_\alpha$ tasodifiy hodisaning ro'y berish ehtimoli α dan oshmasin:

$$P \{T(X_1, \dots, X_n) \in S_\alpha / H_0\} \leq \alpha.$$

Asosiy gipoteza H_0 ni tekshirish qoidasi quyidagicha bo'ladi: $x = (x_1, \dots, x_n)$ t.m. X ning biror tanlanmasi qiymati bo'lsin. Agar $t = T(x)$ miqdor S_α sohaga tegishli bo'lsa: $T(x) \in S_\alpha$, u holda asosiy gipoteza H_0 to'g'ri bo'lganida rad etiladi. Aks holda, ya'ni $T(x) \notin S_\alpha$ bo'lsa, asosiy gipoteza H_0 qabul qilinadi, chunki bu holda statistik ma'lumotlar asosida qilingan xulosalar asosiy gipotezani rad etmaydi. Shuni ta'kidlash lozimki, $t \in S \setminus S_\alpha$ bo'lishi asosiy gipoteza H_0 ni albatta to'g'ri bo'lishini tasdiqlamaydi, balki bu holat statistik ma'lumotlar va nazariy gipotezaning yetarli darajada muvofiqligini ko'rsatadi xalos. Yuqorida keltirilgan qoidada $T = T(X_1, \dots, X_n)$ statistikasi kriteriy statistikasi, S_α - soha esa kritik soha deyiladi. Odatda α ning qiymatlari uchun 0.1; 0.05; 0.01 sonlari qabul qilinadi. Amaliyotda kritik sohalar $\{t \geq t_\alpha\}$ yoki $\{|t| \geq t_\alpha\}$ ko'rinishida bo'ladi.

Faraz qilaylik, S_α kritik soha bo'lsin. U holda H gipoteza to'g'ri bo'lganida statistikaning qiymati kritik sohaga tegishli bo'lishi ehtimolligi

$$W(H) = P \{T(X_1, \dots, X_n) \in S_\alpha / H\},$$

alamatning quvvat funksiyasi deyiladi. Alomat quvvati $H =$

H_1 bo'lganida, ya'ni $W(H_1)$ ehtimollik asosiy gipoteza noto'g'ri bo'lganida to'g'ri yechimni qabul qilishi ehtimolligini anglatadi. Alomatning siljimagalik xossasi muhim o'rin tutadi va bu xossa

$$P\{T(X_1, \dots, X_n) \in S_\alpha/H_1\} \leq \alpha$$

tengsizlik bilan aniqlanadi.

Mavjud statistik gipotezalarni o'rganilayotgan taqsimotlar oilasiga qarab ikki guruhga ajratish mumkin: parametrik va noparametrik gipotezalar. T.m.larning taqsimot funksiyasi parametrlari taqsimotlar oilasiga tegishli bo'lsin. Ammo, taqsimotning parametrlari $\theta = (\theta_1, \dots, \theta_n)$ noma'lumdur. Masalan, t.m. normal qonunlar oilasiga tegishli bo'lsa, uning taqsimot funksiyasi ikkita: o'rta qiymat va dispersiya orqali to'liq aniqlanadi va H_0 gipoteza bu holda matematik kutilma hamda dispersiya qiymatlari haqida bo'ladi. Demak, asosiy H_0 gipoteza noma'lum parametr qiymatlari haqida bo'lar ekan. Bunday statistik gipotezaga *parametrik gipoteza* deb ataladi.

Agarda t.m.ning taqsimot funksiyasi umuman noma'lum bo'lsa, noparametrik gipoteza qaraladi. Noparametrik gipoteza taqsimot funksiyasining ma'lum xossalarga ega ekanligi haqida bo'lishi mumkin.

Endi parametrik statistik alomatlarini qaraylik. X t.m.ning asl taqsimot funksiyasi quyidagi taqsimotlar oilasiga tegishli bo'lsin:

$$\mathcal{F} = \{F(\cdot, \theta), \theta \in \Theta\}.$$

Bu yerda $\theta = (\theta_1, \dots, \theta_r)$ - r - o'lchovli vektor, $\Theta \subseteq R^r$ parametrlar qiymati to'plami bo'lsin. U holda asosiy gipoteza H_0 ga asosan $\theta \in \Theta_0$, alternativ gipotezaga asosan esa $\theta \in \Theta_1 = \Theta \setminus \Theta_0$. Asosiy gipoteza H_0 ni tekshirish uchun S_α va S_α^* ikkita kritik to'plamlar bo'lib, ular har birining qiymatdorlik darajasi α bo'lsin. Faraz qilaylik,

$$W(S_\alpha^*, \theta) \leq W(S_\alpha, \theta), \quad \theta \in \Theta_0, \quad (36)$$

va

$$W(S_\alpha^*, \theta) \geq W(S_\alpha, \theta), \quad \theta \in \Theta_1, \quad (37)$$

bo'lsin. Aytaylik, (37) tengsizlikda hech bo'lmaganda θ ning bitta qiymati uchun qat'iy tengsizlik o'rinni bo'lsin. U holda S_α^* ga asoslangan statistik alomat S_α nikiga nisbatan tekis quvvatliroq deyiladi. Tabiiyki, bu holda S_α^* ga asoslangan statistik alomatni S_α nikiga nisbatan afzal ko'rmoq maqsadga muvofiq bo'ladi.

Agarda (36) va (37) munosabatlar ixtiyoriy S_α uchun o'rinni bo'lsa, S_α^* ga mos alomat *tekis eng quvvatli kriteriy* deyiladi.

Asosiy va alternativ gipotezalar sodda bo'lgan holda t.e.q. kriteriy qurishni ko'ramiz. ξ t.m. va shu t.m. yordamida $X^{(n)} = (X_1, \dots, X_n)$ tanlanma olingan bo'lsin. Faraz qilaylik, ξ t.m. uzluksiz va H_0 gipotezaga asosan $f_0(x)$, H_1 gipotezaga asosan esa $f_1(x)$ zichlik funksiyalarga ega bo'lsin.

$$\text{Demak, } P_{H_0}(x^{(n)}) = \prod_{i=1}^n f_0(X_i) \text{ va } P_{H_1}(x^{(n)}) = \prod_{i=1}^n f_1(X_i).$$

Quyidagi nisbatni ko'raylik

$$l(x^{(n)}) = \frac{P_{H_1}(x^{(n)})}{P_{H_0}(x^{(n)})} = \frac{\prod_{i=1}^n f_1(x_i)}{\prod_{i=1}^n f_0(x_i)}. \quad (38)$$

(38) tenglikka *haqiqatga o'xshashlik nisbati statistikasi* deyiladi.

Teorema(Neyman-Pirson). Sodda gipoteza H_0 ni unga alternativ bo'lgan sodda gipoteza H_1 bo'lgan holda tekshirish uchun t.e.q. kriteriy mavjud va uning kritik sohasi quyidagicha aniqlanadi:

$$S_\alpha = \left\{ x^{(n)} : l(x^{(n)}) = \frac{\prod_{i=1}^n f_1(x_i)}{\prod_{i=1}^n f_0(x_i)} \geq c \right\}, \quad (39)$$

bu yerda c kritik nuqta $P_{H_0}(l(x^{(n)}) \geq c) = \alpha$ shartdan aniqlanadi.

24-misol. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $N(\theta; \sigma^2)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz:

$$H_0 : \theta = \theta_0 \text{ va } H_1 : \theta = \theta_1, (\theta_1 > \theta_0).$$

I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

▷ (39) kriteriyning kritik sohasini quyidagidan aniqlaymiz:

$$l(x^{(n)}) = \frac{\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \exp\left\{-\sum_{j=1}^n \frac{(X_j - \theta_1)^2}{2\sigma^2}\right\}}{\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \exp\left\{-\sum_{j=1}^n \frac{(X_j - \theta_0)^2}{2\sigma^2}\right\}} \geq C$$

yoki

$$l(x^{(n)}) = \exp\left\{\frac{n\bar{x}(\theta_1 - \theta_0)}{\sigma^2} - \frac{n}{2\sigma^2}(\theta_1^2 - \theta_0^2)\right\} \geq C.$$

Tengsizlikni ikki tomonini logarifmlab, $\theta_1 > \theta_0$ ekanligini hisobga olsak, quyidagi ekvivalent tengsizlik hosil bo'ladi:

$$\bar{x} \geq C_1,$$

bu yerda $C_1 = \frac{1}{2}(\theta_0 + \theta_1) + \frac{\sigma^2 \ln C}{(\theta_1 - \theta_0)n}$.

Bu yerda C_1 - kritik nuqta va u I-tur xatolik α orqali aniqlanadi. Endi C_1 va β ni berilgan α orqali topamiz. Ma'lumki, $\bar{x} \sim N\left(\theta_0, \frac{\sigma^2}{n}\right)$.

Demak,

$$\alpha = P_{H_0}(\bar{x} > C_1) = P_{H_0}\left(\frac{\bar{x} - \theta_0}{\sigma/\sqrt{n}} \geq \frac{C_1 - \theta_0}{\sigma/\sqrt{n}}\right) = 1 - \Phi\left(\frac{(C_1 - \theta_0)\sqrt{n}}{\sigma}\right).$$

Agar U_α orqali α - kvantilni belgilasak, $\Phi(U_\alpha) = \alpha$, u holda

$$U_{1-\alpha} = \frac{C_1 - \theta_0}{\sigma} \sqrt{n}.$$

Bulardan foydalanib, C_1 - kritik chegarani topamiz:

$$C_1 = \theta_0 + \frac{U_{1-\alpha}\sigma}{\sqrt{n}}.$$

Demak, C_1 soni θ_0 ga bog'liq, lekin θ_1 ga bog'liq emas. H_1 gipoteza o'rinligida $\bar{x} \sim N\left(\theta_1, \frac{\sigma^2}{n}\right)$ ekanligidan

$$\beta = P_{H_1}(\bar{x} < C_1) = P_{H_1}\left(\frac{\bar{x} - \theta_1}{\sigma/\sqrt{n}} < \frac{C_1 - \theta_1}{\sigma/\sqrt{n}}\right) =$$

$$= \Phi \left(\frac{C_1 - \theta_1}{\sigma} \sqrt{n} \right) = \Phi \left(\sqrt{n} \frac{\theta_0 - \theta_1}{\sigma} + U_{1-\alpha} \right)$$

bo'ladi. \triangleleft

Misollar

1. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $N(a; \theta^2)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($\theta_1 > \theta_0$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

2. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $Bi(n; \theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($\theta_1 > \theta_0$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

3. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma $\pi(\theta)$ taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($\theta_1 > \theta_0$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

4. ξ t.m.ning taqsimoti $F(x)$ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : F(x) = R[-a; a]$ va $H_1 : F(x) = N(0; \sigma^2)$. I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

5. ξ t.m.ning taqsimoti $F(x)$ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : f(x) = \frac{1}{2}, |x| \leq 1$ va $H_1 : F(x) = N(0; 1)$. I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

6. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma zichlik funksiyasi $f(x; \theta) = e^{-(x-\theta)}$, $x > \theta$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($\theta_1 > \theta_0$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

7. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma zichlik funksiyasi $f(x; \theta) = \theta x^{-2}$, $x > \theta$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum

parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($\theta_1 \neq \theta_0$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

8. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma zichlik funksiyasi $f(x; \theta) = 2\theta^{-2}(\theta - x)$, $x \in (0; \theta)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($\theta_1 > \theta_0$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

9. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma zichlik funksiyasi $f(x; \theta) = 2(\theta x + (1 - \theta)(1 - x))$, $x \in (0; 1)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($0 \leq \theta_1 < \theta_0 \leq 1$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

10. $X^{(n)} = (X_1, \dots, X_n)$ tanlanma zichlik funksiyasi $f(x; \theta) = 2x/\theta^2$, $x \in (0; \theta)$ bo'lgan taqsimotdan olingan bo'lsin. Noma'lum parametr θ to'g'risida quyidagi ikki sodda gipotezani ko'ramiz: $H_0 : \theta = \theta_0$ va $H_1 : \theta = \theta_1$ ($\theta_1 > \theta_0$). I tur xatoligi α bo'lgan t.e.q. kriteriy quring va II tur xatolik β ni hisoblang.

16-§. MUVOFIQLIK KRITERIYLARI

I. Kolmogorov kriteriyasi

$X^{(n)} = (X_1, \dots, X_n)$ tanlanma F taqsimotdan olingan bo'lsin. $H_0 : F = F_0$ va $H_1 : F \neq F_0$ gipotezalarni ko'raylik. Agar F_0 uzluksiz bo'lsa, Kolmogorov kriteriyasidan foydalanish mumkin. Quyidagi Kolmogorov statistikasini ko'ramiz:

$$D_n = \sqrt{n} \sup_x \left| \widehat{F}_n(x) - F_0(x) \right|. \quad (40)$$

Agar H_0 gipoteza to'g'ri bo'lsa, u holda X_i larning taqsimoti F_0 bo'ladi. Kolmogorov teoremasiga ko'ra $D_n \Rightarrow \eta$, bu yerda η t.m.ning taqsimoti Kolmogorov taqsimotidir;

η t.m. taqsimot funksiyasi $K(x) = \sum_{j=-\infty}^{\infty} (-1)^j e^{-2j^2 x^2}$, $x > 0$ bo'lsa, u Kolmogorov taqsimotiga ega deyiladi. Bu taqsimot jadval-

lashtirilgan, ya'ni berilgan α da $K(x_\alpha) = 1 - \alpha$ dan x_α ning qiymatini topish mumkin. Quyidagi jadvalda Kolmogorov taqsimotining x_α kritik qiymatlari jadvali keltirilgan:

α	0,4	0,3	0,2	0,1	0,05	0,025	0,01	0,005	0,001
x_α	0,89	0,97	1,07	1,22	1,36	1,48	1,63	1,73	1,95

Kolmogorov kriteriyasi bo'yicha gipotezani tekshirish quyidagicha amalga oshiriladi:

a) $\hat{F}_n(x)$ empirik va faraz qilinayotgan $F_0(x)$ nazariy taqsimot funksiyalar quriladi;

b) D_n statistikaning qiymati hisoblanadi;

c) Agar $D_n \leq x_\alpha$ bo'lsa, u holda H_0 gipoteza tajriba natijalariga zid emas, ya'ni H_0 gipotezani qabul qilish mumkin. Aks holda, ya'ni $D_n > x_\alpha$ bo'lsa, H_0 gipoteza rad etiladi.

25-misol. Quyida berilgan interval variatsion qator uchun $\alpha = 0.05$ deb hisoblab, Kolmogorov kriteriyasi yordamida $H_0: X \sim N(119, 2; 87, 48)$ gipotezani tekshiring.

$a_{i-1} - a_i$	94-100	100-106	106-112	112-118	118-124	124-130
n_i	3	7	11	20	28	19

$a_{i-1} - a_i$	130-136	136-142
n_i	10	2

▷ Berilgan tanlanma bo'yicha empirik va nazariy taqsimotlarni hisoblaymiz: Nazariy taqsimotni hisoblashda $F(x) = \frac{1}{2} + \Phi\left(\frac{x-a}{\sigma}\right) = \frac{1}{2} + \Phi\left(\frac{x-119,2}{9,35}\right)$ formuladan foydalanamiz. Bu yerda $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$ - Laplas funksiyasi. Bu funksiyaning qiymatlari jadvali ilovada keltirilgan (1-jadval). Masalan,

$$F(100) = \frac{1}{2} + \Phi\left(\frac{100 - 119,2}{9,35}\right) = \frac{1}{2} + \Phi(-2,05) = 0,5 - 0,47982 = 0,02018 \approx 0,02.$$

Huddi shunday qolgan qiymatlarini ham hisoblaymiz va quyidagi jadvalni hosil qilamiz:

x	100	106	112	118	124	130	136	142
$\hat{F}_n(x)$	0,03	0,1	0,21	0,41	0,69	0,88	0,98	1
$F(x)$	0,02	0,08	0,221	0,449	0,625	0,878	0,964	0,99

Ushbu jadvaldan ko'rinadiki,

$$D_{100} = \sqrt{100} \sup \left| \hat{F}_n(118) - F(118) \right| = 10 \cdot 0,039 = 0,39.$$

$\alpha = 0,05$ dagi Kolmogorov kriteriysining $x_{0,05}$ kritik qiymati 1,36 ga teng. Demak, $D_{100} = 0,39 < 1,36 = x_{0,05}$ va H_0 gipoteza tajriba natijalariga zid emas. \triangleleft

II. Pirsonning xi-kvadrat kriteriysi

χ^2 - kriteriy guruhlangan ma'lumotlarga asoslanadi. ξ t.m.ning taqsimoti F bo'lsin. $H_0 : F = F_0$ va $H_1 : F \neq F_1$ gipotezalarni ko'raylik. ξ ning qiymatlari to'plami X ni l ta kesishmaydigan $\Delta_1, \Delta_2, \dots, \Delta_l$ intervallarga ajratamiz. Δ_i intervalga to'g'ri kelgan x_i lar soni (chastotani) n_i deb, F_0 taqsimot bo'yicha $np_i = n \cdot P(X_j \in \Delta_i)$ - nazariy chastotani hisoblaymiz va quyidagi statistikani ko'ramiz:

$$\chi_n^2 = \sum_{k=1}^l \frac{(n_k - np_k)^2}{np_k}. \quad (41)$$

(41) statistika Pirsonning xi-kvadrat statistikasi deyiladi. $n \rightarrow \infty$ da (41) statistika ozodlik darajasi $\nu = l - 1$ bo'lgan xi-kvadrat taqsimotga egadir:

$$\chi_n^2 = \sum_{k=1}^l \frac{(n_k - np_k)^2}{np_k} \Rightarrow \chi_{l-1}^2.$$

Xi-kvadrat kriteriysi bo'yicha gipotezani tekshirish quyidagicha amalga oshiriladi:

a) Berilgan tanlanma yordamida interval variatsion qator tuzib olinadi;

b) χ_n^2 statistikaning qiymati hisoblanadi;

c) Berilgan α ga asosan xi-kvadrat taqsimot jadvalidan $\chi_{\nu, \alpha}^2$ kvan-til topiladi. Agar $\chi_n^2 < \chi_{\nu, \alpha}^2$ bo'lsa, u holda H_0 gipoteza tajriba

natijalariga zid emas, ya'ni H_0 gipotezani qabul qilish mumkin. Aks holda, ya'ni $\chi_n^2 > \chi_{\nu, \alpha}^2$ bo'lsa, H_0 gipoteza rad etiladi.

26-misol. Yuqorida ko'rilgan misolda $H_0 : X \sim N(119,2; 87,48)$ gipotezani xi-kvadrat kriteriysi yordamida tekshiring.

▷ Buning uchun avval p_i : ξ ni $[a_{i-1}, a_i]$ ga tushish ehtimolliklarini hisoblab olamiz. Normal taqsimot xossasiga ko'ra:

$$p_i = P(a_{i-1} \leq \xi \leq a_i) = \Phi\left(\frac{a_i - a}{\sigma}\right) - \Phi\left(\frac{a_{i-1} - a}{\sigma}\right) =$$

$$= \Phi\left(\frac{a_i - 119,2}{9,35}\right) - \Phi\left(\frac{a_{i-1} - 119,2}{9,35}\right), \quad i = 1, 2, \dots, 8.$$

Masalan,

$$p_1 = P(94 \leq \xi \leq 100) = \Phi\left(\frac{100 - 119,2}{9,35}\right) - \Phi\left(\frac{94 - 119,2}{9,35}\right) =$$

$$= \Phi(-2,05) - \Phi(-2,69) = 0,0166.$$

Qolgan p_i , $i = \overline{2,8}$ ehtimolliklar ham xuddi shunday hisoblanadi. χ_n^2 statistikani hisoblash uchun quyidagi jadvalni tuzib olamiz:

$[x_i, x_{i+1}]$	n_i	p_i	np_i	$(np_i - n)^2$	$\frac{(np_i - n)^2}{np_i}$
94 - 100	3	0.017	1,7	5.76	0.758
100 - 106	7	0.059	5,9		
106 - 112	11	0.141	14.1	9.61	0.682
112 - 118	20	0.228	22.8	7.84	0.344
118 - 124	28	0.247	24.7	10.89	0.441
124 - 130	19	0.182	18.2	0.64	0.035
130 - 136	10	0.087	8,7	0.16	0.014
136 - 142	2	0.029	2,9		
Σ	100	0,99	99		$\chi_n^2 = 2,27$

n_1 va n_8 chastotalar 5 dan kichik bo'lganligi sababli qo'shni interval-lar bilan birlashtirildi.

Demak, $\chi_n^2 = 2,27$. Birlashtirish hisobiga intervallar soni $l = 6$ ekanligini inobatga olib, $\chi_{5;0,05}^2 = 11,07$ ni aniqlaymiz.

$\chi_n^2 = 2,27 < 11,07 = \chi_{5;0,05}^2$ ekanligidan $H_0 : X \sim N(119, 2; 87, 48)$ gipotezani qabul qilamiz. \triangleleft

Misollar

Quyidagi misollarga berilgan tanlanma bo'yicha muvofiqlik kriteriyalari yordamida H_0 gipotezani tekshiring.

1. $n = 100$ ta detal uzunligini o'lchatib, quyidagi jadval tuzildi (mm da):

Uzunligi	98	98,5	99	99,5	100	100,5	101	101,5	102	102,5
Chastotasi	2	5	9	16	18	20	14	10	4	2

$H_0 : X \sim N(100, 25; 1)$.

2. Tasodifiy sonlar jadvalidan $n = 150$ ta son tanlanadi. $[10i, 10i + 9]$ ($i = 0, 1, \dots, 9$) oraliqqa tushgan sonlar chastotalari n_i : 16, 15, 19, 13, 14, 19, 14, 11, 13, 16 lardan iborat.

$H_0 : X \sim R[0, 100]$.

3. Telefon stansiyasida har minutda noto'g'ri ulanishlar soni ξ ustida kuzatishlar olib borilib 1 soat davomida quyidagi ma'lumotlar olindi: 3; 1; 3; 1; 4; 2; 2; 4; 0; 3; 0; 2; 2; 0; 2; 1; 4; 3; 3; 1; 4; 2; 2; 1; 1; 2; 1; 0; 3; 4; 1; 3; 2; 7; 2; 0; 0; 1; 3; 3; 1; 2; 4; 2; 0; 2; 3; 1; 2; 5; 1; 1; 0; 1; 1; 2; 2; 1; 1; 5. Muhimlik me'yorini $\alpha = 0,05$ deb tanlab, bu tanlanmaning nazariy taqsimoti Puasson taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

4. $n = 200$ laboratoriyadagi jonivorlarning og'irligi o'lchanib quyidagi jadval tuzildi (og'irlik grammida):

Intervallar	[30;35]	[35;40]	[40;45]	[45;50]	[50;55]	[55;60]
Chastotalar	6	12	15	22	47	42

Intervallar	[60;65]	[65;70]	[75;80]
Chastotalar	28	17	11

$H_0 : X \sim N(55, 95)$.

5. Quyidagi jadvalda $[15^{00}; 15^{20}]$ vaqt oralig'ida 100 kun davomida telefon chaqiriqlari soni keltirilgan.

Chaqiriqlar soni	0	1	2	3	4	5	6	7	8	9	10
Kunlar soni	2	14	23	24	18	9	6	2	1	0	1

$H_0 : X \sim \pi(32)$.

6. Ma'lum mahsulotning narxi ξ ning dispersiyasi $\sigma^2 = 2,25$ ga teng va quyidagi statistik tanlanmaga ega:

Narxlar intervali	$[3,0;3,6]$	$[3,6;4,2]$	$[4,2;4,8]$	$[4,8;5,4]$	$[5,4;6,0]$
Chastotasi	2		35	43	22

Narxlar intervali	$[6,0;6,6]$	$[6,6;7,2]$
Chastotasi	15	5

$H_0 : X \sim N(5; 2,25)$.

7. 10 soat davomida avtomashinalarning AYOQT da benzin kolonkasi oldiga kelishi kuzatiladi:

Vaqt oralig'i (soatda):	8-9	9-10	10-11	11-12	12-13	13-14
Avtomashinalar soni:	12	40	22	16	28	6

Vaqt oralig'i (soatda):	14-15	15-16	16-17	17-18
Avtomashinalar soni:	11	33	18	14

$H_0 : X \sim R[8, 18]$.

8. Quyida berilgan interval variatsion qator uchun $\alpha = 0.05$ deb hisoblab, muvofiqlik kriteriyalari yordamida tanlanmaning nazariy taqsimoti Puasson taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

$a_{i-1} - a_i :$	2,3-2,5	2,5-2,7	2,7-2,9	2,9-3,1	3,1-3,3	3,3-3,5
$n_i :$	3	6	9	8	5	2

9. Quyida talabalarning imtixon topshirganliklari to'g'risidagi ma'lumot berilgan:

Topshirilgan imtixonlar soni:	0	1	2	3	4	
Talabalar soni:		1	1	1	3	35

$\alpha = 0.05$ deb hisoblab, muvofiqlik kriteriyalari yordamida tan-

lanmaning nazariy taqsimoti Binomial taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

10. Quyida berilgan guruhlangan variatsion qator uchun $\alpha = 0.05$ deb hisoblab, muvofiqlik kriteriyalari yordamida tanlanmaning nazariy taqsimoti Puasson taqsimotidan iboratligi haqidagi H_0 gipotezani tekshiring.

x_i	0	1	2	3	4	5	6
n_i	405	366	175	40	8	4	2

TESTLAR

(New York Universiteti 2-kurs bakalavrlariga 1990 yilda berilgan test savollari)

1. R, S va T bog'liqsiz hodisalar va ular bir xil $\frac{1}{3}$ ehtimollikka ega. $P(R \cup S \cup T)$ ni toping?

A. $\frac{1}{27}$; B. $\frac{2}{3}$; C. $\frac{19}{27}$; D. $\frac{26}{27}$; E. 1.

2. X va Y t.m. larning birgalikdagi zichlik funksiyasi berilgan bo'lsin: $f(x, y) = \begin{cases} 25, & 0 \leq x \leq 2, x - 2 \leq y \leq x \\ 0, & \text{aks holda.} \end{cases}$ MX^3Y ni toping.

A. $\frac{6}{5}$; B. $\frac{4}{3}$; C. 2; D. 4; E. $\frac{24}{5}$.

3. X_1, X_2, \dots, X_{36} va Y_1, Y_2, \dots, Y_{49} bog'liqsiz t.m. lar mos matematik kutilmasi $MX = 30.4$ va $MY = 32.1$, o'rtacha kvadratik tarqoqligi $\sigma_X = 12$ va $\sigma_Y = 14$ bo'lgan taqsimotga ega bo'lsin. $P(\bar{x} > \bar{y})$ ni hisoblang.

A. 27; B. 34; C. 50; D. 66; E. 73.

4. X va Y diskret t.m. larning birgalikdagi taqsimot qonuni

berilgan:	$Y \setminus X$	-1	0	1
	0	1	1	2
	1	1	3	2

$Cov(X, Y)$ ni hisoblang. A. -02; B. 0; C. 02; D. 10; E. 12.

5. Nomerlangan kub toki 2 raqami tushmagunga qadar tashlanadi. X t.m. tashlashlar soni bo'lsa, $P(X \leq x) \geq \frac{1}{2}$ ni qanoatlantiruvchi eng kichik x ning qiymatini toping.

A. 2; B. 3; C. 4; D. 5; E. 6.

6. X va Y t.m. lar matematik kutilmasi μ dispersiyasi $\sigma^2 > 0$ bo'lgan normal taqsimotga ega va ρ bu t.m. lar korrelyatsiya koeffitsienti. Quyidagi mulohazalarning qaysilari to'g'ri?

I. agar $\rho = 0$ bo'lsa, uholda X va Y t.m. lar bog'liqsiz bo'ladi;

II. $Y - X$ normal taqsimlangan bo'ladi, faqat va faqat $\rho > 0$ bo'lsa;

III. $D(X + Y) < 2\sigma^2$ faqat va faqat $\rho < 0$ bo'lsa;

A. faqat I va II; B. faqat I va III; C. faqat II va III; D. I va III; E. to'g'ri javob yo'q.

7. Zichlik funksiyasi $f(x) = \begin{cases} 0, & \text{agar } x \leq 0 \\ 2\lambda e^{-2\lambda x}, & \text{agar } x > 0 \end{cases}$ bo'lgan ξ tasodifiy miqdor uchun $M\xi$ ni toping.

A. 1; B. $\frac{1}{\lambda}$; C. $\frac{1}{2}$; D. λ ; E. $\frac{1}{2\lambda}$.

8. $X_1, \dots, X_4 \sim N(3; \sigma^2)$, σ^2 - noma'lum bo'lsin. Agar tanlanmaning tajribadagi qiymatlari 4,8,5 va 3 bo'lsa, σ^2 uchun HMO'UB ni qiymatini toping.

A. $\frac{7}{2}$; B. $\frac{9}{2}$; C. $\frac{14}{3}$; D. 5; E. $\frac{15}{2}$.

9. X va Y t.m. larning birgalikdagi zichlik funksiyasi berilgan:

$f(x, y) = \begin{cases} \frac{1}{6}, & x = 1, 2, 3, y = 2, 3, \\ 0, & \text{aks holda.} \end{cases}$ $U = X + Y$ bo'lsin, u holda U

t.m. ning zichlik funksiyasini toping.

A. $g(u) = \begin{cases} \frac{1}{4}, & u = 3, 4, 5, 6, \\ 0, & \text{aks holda;} \end{cases}$ B. $g(u) = \begin{cases} \frac{2}{3}, & u = 3, \\ \frac{1}{5}, & u = 4, 5, 6, \\ 0, & \text{aksholda;} \end{cases}$

C. $g(u) = \begin{cases} \frac{1}{6}, & u = 3, 6, \\ \frac{1}{3}, & u = 4, 5; \\ 0, & \text{aksholda;} \end{cases}$ D. $g(u) = \begin{cases} \frac{1}{5}, & u = 2, 3, 4, 5, 6, \\ 0, & \text{aks holda;} \end{cases}$

E. $g(u) = \begin{cases} \frac{1}{4}, & u = 2, 3, 4, 5, 6, \\ 0, & \text{aks holda.} \end{cases}$

10. $X_1, X_2 \sim R[0, 1]$ va $Y = \min(X_1, X_2)$ bo'lsa Y t.m. ning zichlik funksiyasini toping.

A. 1, agar $0 \leq y \leq 1$; B. $2y(1-y)$, agar $0 \leq y \leq 1$;

C. $2y$, agar $0 \leq y \leq 1$; D. $(1-y)$, agar $0 \leq y \leq 1$;

E. $2(1-y)$, agar $0 \leq y \leq 1$

11. X_1, \dots, X_n matematik kutilmasi MX va dispersiyasi $DX > 0$ va Y_1, \dots, Y_n esa matematik kutilma MY va dispersiya $DY > 0$ taqsimotlardan olingan bo'lsin. Asosiy $H_0 : MX = MY$ gipotezani va alternativ $H_1 : MX \neq MY$ gipotezani ko'raylik. $Z_i = X_i - Y_i$ ga asoslangan St'yudent kriteriysi bo'yicha H_0 gipotezani tekshirishda quyida keltirilgan qaysi mulohazadan foydalaniladi?

I. Z_i normal taqsimotga ega;

II. $\sigma_X^2 = \sigma_Y^2$;

III. $Cov(X_i, Y_i) = 0$, $i = 1, \dots, n$

A. I,II va III; B. I; C. II; D. III; E. to'g'ri javob yo'q.

12. X_1, X_2 va X_3 diskret t.m. lar bo'lib, quyidagi zichlik funksiyaga ega bo'lsin: $g(x) = \begin{cases} \frac{x}{10}, & \text{agar } x = 1, 2, 3, \\ 0, & \text{aks holda.} \end{cases}$

$P(X_1 < X_2 < X_3)$ ni hisoblang.

A. 0.030; B. 0.050; C. 0.167; D. 0.250; E. 0.350.

13. X uzluksiz t.m. zichlik funksiyasi berilgan:

$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{agar } x > 0, \\ 0, & \text{aks holda.} \end{cases}$ Agar bu taqsimotning medianasi $\frac{1}{3}$ bo'lsa, λ ni toping.

A. $\frac{1}{3} \ln \frac{1}{2}$; B. $\frac{1}{3} \ln 2$; C. $2 \ln \frac{3}{2}$; D. $3 \ln 2$; E. 3.

14. $M_X(t)$ funksiya X t.m.ning hosil qiluvchi funksiyasi bo'lsin. Quyida keltirilgan mulohazalardan qaysilari to'g'ri?

I. $M_X(0) = 1$; II. $\frac{d^2 M_X(t)}{dt^2} |_{t=0} = \text{Var}[X]$; III. $M_X(t)$ X t.m.ning taqsimotini o'zaro bir qiymatli aniqlaydi.

A. I va II ; B. I va III; C. II va III; D. I,II va III; E. to'g'ri javob yo'q.

15. X va Y uzluksiz t.m. larning birgalikdagi zichlik funksiyasi berilgan: $f(x, y) = \begin{cases} \frac{12}{25}(x + y^2), & \text{agar } 1 < x < y < 2 \\ 0, & \text{aks holda.} \end{cases}$ Y t.m. ning

$f(y)$ marginal zichlik funksiyasini toping.

A. $\frac{6}{25}(2y^3 - y^2 - 1)$, agar $1 < y < 2$;

B. $\frac{6}{25}(3 + 2y^2)$, agar $1 < y < 2$;

C. $\frac{6}{25}[y^2(1 + 2y)]$, agar $1 < y < 2$;

D. $\frac{3(x+y^2)}{8+6x-3x^2-x^3}$, agar $1 < x < y < 2$;

E. $\frac{3(x+y^2)}{7+3x}$, agar $1 < x, y < 2$.

16. Noma'lum θ parametr uchun uchta bog'liqsiz ishonchlik intervallari tuzilgan. Agar har bir interval ishonchligi 0.98 bo'lsa, bu intervallardan birortasi ham θ ni o'z ichiga olmasligi ehtimolligini toping.

A. 0.0192; B. 0.0297; C. 0.0588; D. 0.9412; E. 0.9703.

17. $X_1, \dots, X_8 \sim N(MX, 9)$ va $Y_1, \dots, Y_8 \sim N(MY, 9)$ bo'lsin. Asosiy gipoteza $H_0 : MX = MY$ ni $H_1 : MX > MY$ alternativ gipotezaga qarshi tekshirishda $W = \bar{x} - \bar{y}$ statistikadan foyalaniladi. Agar $W > 3$ da H_0 rad etilsa, $MX - MY = 4.5$ bo'lganda kriteriy

quvvatini toping.

A. 05; B. 07; C. 16; D. 84; E. 93.

18. $X_1, X_2 \sim \pi(\theta)$ bo'lsin. Asosiy gipoteza $H_0 : \theta = 5$ ni $H_1 : \theta \neq 5$ alternativ gipotezaga qarshi tekshirishda $\bar{x} = \frac{X_1 + X_2}{2}$ statistikadan foyalaniladi. Agar $|\bar{x} - 5| \geq 4$ kritik soha bo'lsa, 1-tur xatolik ehtimolligini toping.

A. $1 - \sum_{y=2}^8 \frac{e^{-5.5y}}{y!}$; B. $1 - \sum_{y=1}^9 \frac{e^{-5.5y}}{y!}$; C. $1 - \sum_{y=2}^8 \frac{e^{-10.10y}}{y!}$; D. $1 - \sum_{y=0}^{17} \frac{e^{-10.10y}}{y!}$;

E. $1 - \sum_{y=3}^{17} \frac{e^{-10.10y}}{y!}$.

19. X uzluksiz t.m. ning birgalikdagi zichlik funksiyasi berilgan:

$f(x) = \begin{cases} \frac{1}{9}x(4-x) & \text{agar } 0 < x < 3, \\ 0, & \text{aks holda.} \end{cases}$ X ning modasini hisoblang:

A. $\frac{4}{9}$; B. 1; C. $\frac{3}{2}$; D. $\frac{7}{4}$; E. 2.

20. X va Y t.m. larning birgalikdagi taqsimot qonuni berilgan:

$Y \setminus X$	1	5
2	$\theta_1 + \theta_2$	$\theta_1 + 2\theta_2$
4	$\theta_1 + 2\theta_2$	$\theta_1 + \theta_2$

θ_1 va θ_2 parametrlar birgalikdagi taqsimot xossalari va $-0.25 \leq \theta_1 \leq 0.25$, $0 \leq \theta_2 \leq 0.35$ shartlarni qanoatlantiradi. (θ_1, θ_2) ning qanday qiymatida X va Y t.m.lar bog'liqsiz bo'ladi?

A. $(0, \frac{1}{6})$; B. $(\frac{1}{4}, 0)$; C. $(-\frac{1}{4}, \frac{1}{3})$; D. $(-\frac{1}{8}, \frac{1}{4})$; E. $(\frac{1}{16}, \frac{1}{8})$.

21. X, Y va Z bog'liqsiz normal taqsimlangan tasodifiy miqdorlar va $MX = 2$, $MY = 1$, $MZ = 2$ va $DX > 0$ bo'lsin. Agar $W = c \left[\frac{4(X-2)^2}{(Y-1)^2 + (Z-2)^2} \right]$ bo'lsa, c ning qanday qiymatida W t.m. $F_{1,2}$ Fisher taqsimotiga ega bo'ladi.

A. 0.25; B. 0.50; C. 1; D. 2; E. 4.

22. $X_1, \dots, X_{15} \sim N(\mu, \sigma^2)$ va $\bar{x} = \sum_{i=1}^{15} \frac{X_i}{15}$ va $T = \sum_{i=1}^{15} (X_i - \bar{x})^2$

bo'lsin. Asosiy gipoteza $H_0 : \sigma^2 \leq 10$ ga qarshi alternativ $H_1 : \sigma^2 > 10$ gipotezani ko'raylik. $1 - \alpha = 0.05$ bo'lganda kritik sohani toping.

A. H_0 rad etiladi, agar $T \geq 23.69$; B. H_0 rad etiladi, agar $T \geq 25.00$; C. H_0 rad etiladi, agar $T \geq 236.90$; D. H_0 rad etiladi, agar $T \geq 250.00$;

E. H_0 rad etiladi, agar $T \geq 261.20$.

23. Quyida keltirilgan hodisalarning qaysilari

$(B \cap C) \cup (A' \cap B \cap C')$ hodisaga teng kuchli:

I. $B \cap (A' \cup C)$; II. $(A' \cap B) \cup (B \cap C)$; III. $(A' \cap C') \cup (B \cap C)$.

A. I va II; B. I va III; C. II va III; D. I, II va III; E. to'g'ri javob yo'q;

24. Yashikda 100 shardan r tasi qizil, qolganlari esa oq rangda. Tavakkaliga 20 ta shar olinadi. Agar olingan sharlardan hech bo'lmasa 10 tasi qizil bo'lsa, asosiy gipoteza $H_0 : r = 30$ rad etilib, $H_1 : r > 30$ alternativ gipoteza qabul qilinadi. $r = 40$ bo'lsa, 2-tur xatolik ehtimolligini hisoblang.

$$A. \frac{\binom{40}{10}}{\binom{100}{20}}; \quad B. \sum_{k=10}^{20} \frac{\binom{30}{k} \binom{70}{20-k}}{\binom{100}{20}};$$

$$C. \sum_{k=0}^9 \frac{\binom{30}{k} \binom{70}{20-k}}{\binom{100}{20}}; \quad D. \sum_{k=10}^{20} \frac{\binom{40}{k} \binom{60}{20-k}}{\binom{100}{20}};$$

$$E. \sum_{k=0}^9 \frac{\binom{40}{k} \binom{60}{20-k}}{\binom{100}{20}}.$$

25. X va Y uzluksiz t.m. lar birgalikdagi zichlik funksiyasi berilgan:

$$f(x, y) = \begin{cases} 2(x+y) & \text{agar } 0 < x < y < 1 \\ 0, & \text{aks holda.} \end{cases} \quad MY \text{ ni hisoblang.}$$

A. $\frac{5}{12}$; B. $\frac{1}{2}$; C. $\frac{3}{4}$; D. 1; E. $\frac{2}{6}$.

26. X_1, X_2, \dots, X_n tanlanma zichlik funksiyasi

$$f(x) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \text{agar } \theta_1 \leq x \leq \theta_2 \\ 0, & \text{aks holda} \end{cases}, \quad \theta_2 > 0, \theta_1 < 0, \text{ bo'lgan taqsi-}$$

motdan olingan. Asosiy $H_0 : \theta_1 = -\theta_2$ gipoteza va unga alternativ $H_1 : \theta_1 \neq -\theta_2$ gipotezalarni ko'raylik. H_0 gipotezani tekshirish uchun qurilgan haqiqatga o'xshashlik nisbati statistikasining kritik sohasini toping.

A. $\frac{\max(X_i) - \min(X_i)}{\max(X_i)} \leq k$; B. $\frac{\max|X_i| - \min|X_i|}{\max(X_i)} \leq k$; C. $\frac{\max(X_i) - \min(X_i)}{\max|X_i|} \leq k$;

D. $\frac{\max(X_i)}{\max(X_i) - \min(X_i)} \leq k$; E. $\frac{\max|X_i|}{\max(X_i)} \leq k$.

27. X_1, X_2, X_3 lar bog'liqsiz va mos ravishda $\theta, 2\theta, 3\theta$ parametrlri Puasson taqsimoti bo'yicha taqsimlangan bo'lsin. Noma'lum parametr θ uchun HMO'UB toping.

A. $\frac{1}{2}\bar{x}$; B. \bar{x} ; C. $\frac{X_1 + 2X_2 + 3X_3}{6}$; D. $\frac{3X_1 + 2X_2 + X_3}{6}$; E. $\frac{6X_1 + 3X_2 + 2X_3}{11}$.

28. A va B hodisalarning ehtimolliklari mos ravishda 0.3 va 0.1 ga teng. o'tkazilgan bog'liqsiz 5 ta tajribada A 3 marta, 10 ta tajribada B hodisa 6 marta ro'y berishi ehtimoligini toping.

A. $\binom{5}{3} (0.3)^3 (0.7)^2 + \binom{10}{6} (0.1)^6 (0.9)^4$;

B. $\binom{5}{3} (0.3)^3 (0.7)^2 + \binom{10}{6} (0.1)^6 (0.9)^4$;

C. $\binom{15}{9} (0.4)^9 (0.6)^6$; D. $\frac{\binom{5}{3} \binom{10}{6}}{\binom{15}{9}}$; E. $(0.6)(0.9)$.

29. X va Y t.m.lar markazi $(0,0)$ nuqtada radiusi 1 bo'lgan doirada tekis taqsimlangan bo'lsin. Y t.m.ning $X = x$ dagi shartli zichlik funksiyasini toping.

A. $\frac{1}{2\pi\sqrt{1-x^2}}$, agar $-1 < y < 1$;

B. $\frac{1}{2\sqrt{1-x^2}}$, agar $-1 < y < 1$;

C. $\frac{1}{\pi}$, agar $-\sqrt{1-x^2} < y < \sqrt{1-x^2}$;

D. $\frac{1}{2\pi\sqrt{1-x^2}}$, agar $-\sqrt{1-x^2} < y < \sqrt{1-x^2}$;

E. $\frac{1}{2\sqrt{1-x^2}}$, agar $-\sqrt{1-x^2} < y < \sqrt{1-x^2}$.

30. Agar $M\xi = a$, $D\xi = \sigma^2$ bo'lib, $\eta = \frac{\xi - a}{\sigma}$ bo'lsa, $M\xi$ va $D\eta$ ni toping.

A. $a; 1$; B. $0; 1$; C. $0; \sigma^2$; D. $a; \frac{1}{\sigma}$; E. $0; 2$.

31. $X_1, \dots, X_n \sim N(\mu, 50)$ bo'lsin. $\bar{x} \geq 13.75$ kritik soha yordamida asosiy gipoteza $H_0: \mu = 10$ alternativ gipoteza $H_1: \mu = 15$ qarshi tekshiriladi. 2 - tur xatolik 0.31 dan kichik yoki teng bo'lishi uchun eng kichik tanlanma hajmini toping.

A. 2; B. 4; C. 5; D. 8; E. 20.

32. X_1, \dots, X_n tanlanma zichlik funksiyasi

$$f(x) = \begin{cases} \theta(a-x)^{\theta-1} & \text{agar } a-1 \leq x \leq a \\ 0, & \text{aks holda.} \end{cases} \quad a > 0, \theta > 0,$$

bo'lgan taqsimotdan olingan bo'lsin. Asosiy gipoteza $H_0 : \theta = \theta_0$ va unga alternativ gipoteza $H_1 : \theta > \theta_0$ bo'lsin. Uning uchun eng quvvatli kritik sohani toping.

A. $\sum_{i=1}^n \ln(a - X_i) \geq k$; B. $\sum_{i=1}^n \ln(a - X_i) \leq k$; C. $\sum_{i=1}^n X_i \geq k$;

D. $\sum_{i=1}^n X_i \leq k$; E. $\prod_{i=1}^n \ln(X_i - a) \leq k$.

33. X t.m. zichlik funksiyasi:

$$f(x; \theta) = \begin{cases} \left(\frac{1}{\theta}\right) x^{(1-\theta)/\theta} & \text{agar } 0 < x < 1 \\ 0, & \text{aks holda.} \end{cases} \quad \theta > 0 \text{ bo'lsin. Noma'lum}$$

parametr θ uchun MUB ni toping.

A. $\frac{1-\bar{x}}{\bar{x}}$; B. $\frac{\bar{x}-1}{\bar{x}}$; C. $\frac{\bar{x}}{1-\bar{x}}$; D. $\frac{\bar{x}}{\bar{x}-1}$; E. $\frac{1}{1+\bar{x}}$

34. X va Y diskret t.m. lar birgalikdagi zichlik funksiyasi berilgan: $f(x, y) = \frac{x^2+y^2}{56}$ agar $x = 1, 2, 3$, va $y = 1, \dots, x$. $P(Y = 3/Y \geq 2)$ ni hisoblang.

A. $\frac{9}{28}$; B. $\frac{1}{3}$; C. $\frac{6}{13}$; D. $\frac{41}{54}$; E. $\frac{6}{7}$.

35. Quyida keltirilgan funksiyalarning qaysilari taqsimot funksiya bo'ladi:

$$\text{I. } F(x) = \begin{cases} 0 & \text{agar } x \leq 1 \\ x^2 - 2x + 1 & \text{agar } 1 < x \leq 2 \\ 1 & \text{agar } x > 2 \end{cases};$$

$$\text{II. } F(x) = \begin{cases} 0 & \text{agar } x \leq 0 \\ x^2 - 2x + 1 & \text{agar } 0 < x \leq 1 + \sqrt{2} \\ 1 & \text{agar } x > 1 + \sqrt{2} \end{cases};$$

$$\text{III. } F(x) = \begin{cases} 0 & \text{agar } x < 2 \\ \frac{1}{2} & \text{agar } x = 2 \\ x^2 - 2x + \frac{1}{2} & \text{agar } 2 < x \leq 1 + \frac{\sqrt{6}}{2} \\ 1 & \text{agar } x > 1 + \frac{\sqrt{6}}{2} \end{cases}$$

A. I va II; B. I va III; C. II va III; D. I, II va III; E to'g'ri javob yo'q.

36. $X_1, X_2 \sim N(0, 1)$ va $M(c|X_1 - X_2|) = 1$ bo'lsa, c ni toping.

A. $\sqrt{\pi}$; B. $\frac{1}{\sqrt{\pi}}$; C. $\frac{\sqrt{2\pi}}{4}$; D. $\frac{2}{\sqrt{\pi}}$; E. $\frac{\sqrt{\pi}}{2}$.

37. $X_1, \dots, X_9 \sim N(0, 4)$ va $Y_1, \dots, Y_8 \sim N(0, 9)$ bo'lsa, quyida

keltirilganlarning qaysilari $P \left[\frac{8.2 \sum_{i=1}^9 X_i^2}{\sum_{j=1}^8 Y_j^2} < 1 \right]$ ehtimollikka yaqin?

A. 0.010; B. 0.025; C. 0.050; D. 0.950; E. 0.975.

38. X va Y uzluksiz t.m. lar birgalikdagi zichlik funksiyasi $f(x, y)$, marginal zichlik funksiyalari $f_X(x)$, $f_Y(y)$ lar $(0,1)$ da 0 dan farqli bo'lsin. Quyida keltirilgan tengliklarning qaysi biri to'g'ri?

A. $M(X^2Y^3) = \left(\int_0^1 x^2 dx \right) \left(\int_0^1 y^3 dy \right)$; B. $M(X^2) = \int_0^1 x^2 f(x, y) dx$;

C. $M(X^2Y^3) = \left(\int_0^1 x^2 f(x, y) dx \right) \left(\int_0^1 y^3 f(x, y) dy \right)$;

D. $M(X^2) = \int_0^1 x^2 f_X(x) dx$; E. $M(Y^3) = \int_0^1 y^3 f_X(x) dx$.

39. $Y \sim R(0, 1)$ va $Z = -a \ln(1 - Y)$, $a > 0$ bo'lsa, Z ning taqsimoti?

A. Koshi; B. Lognormal; C. Normal; D. Ko'rsatkichli; E. Tekis.

40. Agar ξ t.m. t.f. $F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-cx}, & x \geq 0 \end{cases}$ bo'lib, $F\left(\frac{1}{3}\right) =$

$\frac{1}{2}$ bo'lsa, c ni toping.

A. $3 \ln 2$; B. $2 \ln \frac{3}{2}$; C. $\frac{1}{3} \ln 2$; D. 3; E. 1.

41. A, B va C hodisalar uchun $P(A/C) = 0.05$, $P(B/C) = 0.05$ o'rinli bo'lsa, quyida keltirilganlarning qaysi biri to'g'ri?

A. $P[A \cap B \setminus C] = (0.05)^2$; B. $P[A' \cap B' \setminus C] \geq 0.90$;

C. $P[A \cup B \setminus C] \leq 0.05$; D. $P[A \cup B \setminus C'] \geq 1 - (0.05)^2$;

E. $P[A \cup B \setminus C'] \geq 0.10$.

42. X, Y va Z lar bog'liqsiz va Puasson taqsimotiga ega bo'lsin.

$MX = 3, MY = 1$ va $MZ = 4$ bo'lsa, $P(X + Y + Z \leq 1)$?

A. $13e^{-12}$; B. $9e^{-8}$; C. $\frac{13}{12}e^{-1/12}$; D. $9e^{-1/8}$; E. $\frac{9}{8}e^{-1/8}$.

43. Agar ξ t.m. $N(2; 1)$ normal qonun bo'yicha taqsimlangan bo'lsa, $P(\xi < M\xi)$ ni toping. A. 1/2; B. 1/3; C. 1/4; D. 1/5; E. 1/6.

44. Agar X_1, \dots, X_n t.m. lar birgalikdagi zichlik funksiyasi

$$f(x) = \begin{cases} \theta(\theta + 1)x^{\theta-1}(1-x) & \text{agar } 0 < x < 1 \\ 0, & \text{aks holda,} \end{cases}$$

$\theta > 0$ bo'lsa, θ uchun yetarli statistikani toping.

A. $\sum_{i=1}^n X_i$; B. $\sum_{i=1}^n X_i^2$; C. $\prod_{i=1}^n X_i$; D. $\frac{\prod_{i=1}^n X_i}{\prod_{i=1}^n (1-X_i)}$; E. $\left(\prod_{i=1}^n X_i\right) \prod_{i=1}^n (1-X_i)$.

45. Agar X_1, \dots, X_{10} t.m.lar dispersiyasi $\sigma^2 > 0$ bo'lgan normal taqsimotga ega bo'lsa, 95% aniqlikda σ^2 ishonchlik intervalini tuzing.

A. (10.162, ∞); B. (8.589, ∞); C. (2.00, ∞); D. (1.848, ∞);
E. (1.720, ∞).

46. Idishda 4 ta qizil va 6 ta oq shar bor. Tavakkaliga 3 ta shar olinadi. Tanlangan sharlardan hech bo'lmasa 2 tasi oq rangda bo'lsa, 1 ta qizil va 2 ta oq shar olinishi ehtimolini toping.

A. $\frac{1}{2}$; B. $\frac{2}{3}$; C. $\frac{3}{4}$; D. $\frac{9}{11}$; E. $\frac{54}{55}$.

47. Anketa tarqatilganda 50% aholi tezda javob qaytaradi, 40% aholi esa 2-marta yuborilganda javob qaytaradi. Agar anketa 4 ta kishiga yuborilgan bo'lsa va 1 ta anketa takroran javob bermagan ixtiyoriy 4 kishidan 1 tasiga yuborilsa, kamida 3 ta kishi umuman javob bermasligi ehtimolligini toping.

A. $(0.3^4) + 4(0.3^3)(0.7)$; B. $4(0.3^3)(0.7)$; C. $0.1^4 + 4(0.1^3)(0.9)$;
D. $0.4(0.3)(0.7^3) + 0.7^4$; E. $0.9^4 + 4(0.9^3)(0.1)$

48. Mijozlar tasodifiy va bog'liqsiz ravishda xizmat ko'rsatish oynasiga borib murojaat qilishadi. Murojaat qilish orasidagi vaqt o'rta qiymati 12 minut bo'lgan ko'rsatkichli taqsimotga ega. X t.m. 1 soat ichida murojaat qilganlar soni bo'lsa, $P(X = 10)$ ni hisoblang.

A. $\frac{10e^{-12}}{10!}$; B. $\frac{10^{12}e^{-10}}{10!}$; C. $\frac{12^{10}e^{-10}}{10!}$; D. $\frac{12^{10}e^{-12}}{10!}$; E. $\frac{5^{10}e^{-5}}{10!}$.

49. X va Y uzluksiz t.m. lar birgalikdagi zichlik funksiyasi berilgan:

$$f(x, y) = \begin{cases} xy & \text{agar } 0 \leq x \leq 2 \text{ va } 0 \leq y \leq 1 \\ 0, & \text{aks holda.} \end{cases}$$

$P\left[\frac{X}{2} \leq Y \leq X\right]$ ni toping. A. $\frac{3}{32}$; B. $\frac{1}{8}$; C. $\frac{1}{4}$; D. $\frac{3}{8}$; E. $\frac{3}{4}$.

50. Agar ξ va η bog'liqsiz t.m.lar bo'lib, $M\xi = 1$, $M\eta = 2$, $D\xi = 1$, $D\eta = 4$ bo'lsa, $M(\xi + \eta + 1)^2$ ni toping.

A. 21; B. 10; C. 6; D. 11; E. 5.

ILOVA

1. Statistikadagi muhim taqsimotlar

1. χ^2 taqsimot

ξ_i t.m.lar bog'liqsiz va $\xi_i \sim N(0, 1)$, $i = 1, \dots, k$ bo'lsa, $\eta = \xi_1^2 + \dots + \xi_k^2$ t.m. ozodlik darajasi k bo'lgan xi-kvadrat taqsimlangan deyiladi va bu taqsimotni $\chi^2(k)$ kabi belgilanadi. χ^2 - taqsimot zichlik funksiyasi:

$$f_{\chi^2}(x) = \begin{cases} \frac{1}{2^{k/2} \Gamma(k/2)} \cdot x^{\frac{k-2}{2}} \cdot e^{-\frac{x}{2}}, & x > 0, k > 0, \\ 0, & x \leq 0, \end{cases}$$

bu yerda $\Gamma(k)$ gamma funksiya. η t.m. momentlari $M\eta = k$, $D\eta = 2k$.

Xi - kvadrat taqsimotining boshqa taqsimotlar bilan bog'liqligi

o $k = 2$ da xi-kvadrat taqsimot parametri $1/2$ bo'lgan eksponensial qonun bilan ustma-ust tushadi: $\chi^2(2) = E(1/2)$.

o Xi-kvadrat taqsimot parametrlari $(k/2; 1/2)$ bo'lgan gamma taqsimotning xususiy holdir: $\chi^2(k) = \Gamma(k/2; 1/2)$.

o Agar $\eta_1 \sim \chi^2(k_1)$, $\eta_2 \sim \chi^2(k_2)$ va $\eta_1 \perp \eta_2$ bo'lsa, $F = \frac{\eta_1/k_1}{\eta_2/k_2}$ bu t.m. ozodlik darajasi (k_1, k_2) bo'lgan Fisher taqsimoti bo'yicha taqsimlangan bo'ladi (Fisher taqsimoti quyida keltiriladi).

2. St'yudent taqsimoti

Bog'liqsiz va $\xi_0, \xi_1, \dots, \xi_n \sim N(0, 1)$ bo'lsa, $t = \frac{\xi_0}{\sqrt{\sum_{i=1}^n \xi_i^2}}$ t.m.

St'yudent taqsimotiga ega deyiladi va $t(n)$ kabi belgilanadi. St'yudent taqsimoti zichlik funksiyasi:

$$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right) \sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}, \quad x \in (-\infty, +\infty), \quad n > 0$$

bu yerda $\Gamma(n)$ gamma funksiya. Bu t.m. momentlari $Mt = 0$, agar $n > 2$ bo'lsa, $Dt = \frac{n}{n-2}$.

St'yudent taqsimotining boshqa taqsimotlar bilan bog'liqligi:

o Koshi taqsimoti St'yudent taqsimotining xususiy holdir: $t(1) = K(0, 1)$.

o Agar $\xi \sim t(n)$ bo'lsa, u holda $\xi^2 \sim F(1, n)$ Fisher taqsimoti bo'yicha taqsimlangan bo'ladi.

3. Fisher taqsimoti

Agar $\eta_1 \sim \chi^2(k_1)$, $\eta_2 \sim \chi^2(k_2)$ va $\eta_1 \perp \eta_2$ bo'lsa, $F = \frac{\eta_1/k_1}{\eta_2/k_2}$ bu t.m. ozodlik darajasi (k_1, k_2) bo'lgan Fisher taqsimoti bo'yicha taqsimlangan deyiladi va $F(k_1, k_2)$ kabi belgilanadi. Fisher taqsimoti zichlik funksiyasi:

$$f(x) = \begin{cases} 0, & x \leq 0 \\ \frac{\Gamma\left(\frac{k_1+k_2}{2}\right)}{\Gamma\left(\frac{k_1}{2}\right) \cdot \Gamma\left(\frac{k_2}{2}\right)} \left(\frac{k_1}{k_2}\right)^{\frac{k_1}{2}} \frac{x^{\frac{k_1}{2}-1}}{\left(1 + \frac{k_1}{k_2}\right)^{\frac{k_1+k_2}{2}}}, & x > 0 \end{cases}$$

Bu t.m. momentlari: agar $k_2 > 2$ bo'lsa, $MF = \frac{k_2}{k_2-2}$, va $k_2 > 4$ bo'lsa, $DF = \frac{2k_2^2(k_1+k_2-2)}{k_1(k_2-2)^2(k_2-4)}$.

II. Statistik jadvallar

1-jadval

$$\text{Normal taqsimot qiymatlari } \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$$

x	0	1	2	3	4	5	6	7	8	9
0.0	0.000	0.039	0.079	0.119	0.159	0.199	0.239	0.279	0.319	0.359
0.1	03983	04380	04776	05172	05567	05962	06356	06749	07142	07535
0.2	07926	08317	08706	09095	09483	09871	10257	10642	11026	11409
0.3	11791	12172	12552	12930	13307	13683	14058	14431	14803	15173
0.4	15542	15910	16276	16640	17003	17364	17724	18082	18439	18793
0.5	19146	19497	19847	20194	20540	20884	21226	21666	21904	22240
0.6	22575	22907	23237	23565	23891	24215	24537	24857	25175	25490
0.7	25804	26115	26424	26730	27035	27337	27637	27935	28230	28524
0.8	28814	29103	29389	29673	29955	30234	30511	30785	31057	31327
0.9	31594	31859	32121	32381	32639	32894	33147	33398	33646	33891
1.0	34134	34375	34614	34849	35083	35314	35543	35769	35993	36214
1.1	36433	36650	36864	37076	37286	37493	37698	37900	38100	38298
1.2	38493	38686	38877	39065	39251	39435	39617	39796	39973	40147
1.3	40320	40490	40658	40824	40988	41149	41308	41466	41621	41774
1.4	41924	42073	42220	42364	42507	42647	42785	42922	43056	43189
1.5	43319	43448	43574	43699	43822	43943	44062	44179	44293	44408
1.6	44520	44630	44738	44845	44950	45053	45154	45254	45352	45449
1.7	45543	45637	45728	45818	45907	45994	46080	46164	46246	46327
1.8	46407	46485	46562	46638	46712	46784	46856	46926	46995	47062
1.9	47128	47193	47257	47320	47381	47441	47500	47558	47615	47670
2.0	47725	47778	47831	47882	47932	47982	48030	48077	48124	48169
2.1	48214	48257	48300	48341	48382	48422	48461	48500	48537	48574
2.2	48610	48645	48679	48713	48745	48778	48809	48840	48870	48899
2.3	48928	48956	48983	49010	49036	49061	49086	49111	49134	49158
2.4	49180	49202	49224	49245	49266	49286	49305	49324	49343	49361

2.5	49379	49396	49413	49430	49446	49461	49477	49492	49506	49520
2.6	49534	49547	49560	49573	49585	49598	49609	49621	49632	49643
2.7	49653	49664	49674	49683	49693	49702	49711	49720	49728	49736
2.8	49744	49752	49760	49767	49774	49781	49788	49795	49801	49807
2.9	49813	49819	49825	49831	49836	49841	49846	49851	49856	49861

x	3.0	3.5	4.0	5.0
$\Phi_0(x)$	0.49865	0.49977	0.499968	0.49999997

2-jadval

$N(0, 1)$ - normal taqsimot kvantillari

p	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
U_p	1.282	1.645	1.960	2.326	2.576	3.090	3.291

3-jadval

$t_p(k)$ - St'yudent taqsimoti kvantili
 k - ozodlik darajasi; p - kvantil tartibi

$$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}, \quad x \in (-\infty, +\infty), \quad n > 0$$

$k \setminus p$	0.750	0.900	0.950	0.975	0.990	0.995	0.999
1	1.000	3.078	6.314	12.706	31.821	63.657	318
2	0.816	1.886	2.920	4.303	6.965	9.925	22.3
3	0.765	1.638	2.353	3.182	4.541	5.841	10.2
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785

8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

4-jadval

$\chi^2(k)$ - taqsimoti kvantili
k - ozodlik darajasi; *p* - kvantil tartibi

<i>k</i> \ <i>p</i>	0.90	0.95	0.99	19	27.20	30.14	36.19
1	2.71	3.84	6.63	20	28.41	31.14	37.57
2	4.61	5.99	9.21	21	29.62	32.67	38.93
3	6.25	7.81	11.34	22	30.81	33.92	40.29
4	7.78	9.49	13.28	23	32.01	35.17	41.64
5	9.24	11.07	15.09	24	33.20	36.42	42.98
6	10.64	12.59	16.81	25	34.38	37.65	44.31
7	12.02	14.07	18.48	26	35.56	38.89	45.64
8	13.36	15.51	20.09	27	36.74	40.11	46.96
9	14.68	16.92	21.67	28	37.92	41.34	48.28
10	15.99	18.31	23.21	29	39.09	42.56	49.59
11	17.28	19.68	24.72	30	40.26	43.77	50.89
12	18.55	21.03	26.22	40	51.80	55.76	63.69
13	19.81	22.36	27.69	50	63.17	67.50	76.15
14	21.06	23.68	29.14	60	74.40	79.08	88.38
15	22.31	25.00	30.58	70	85.53	90.53	100.42
16	23.54	26.30	32.00	80	96.58	101.88	112.33
17	24.77	27.59	33.41	90	107.56	113.14	124.12
18	25.99	28.87	34.81	100	118.50	124.34	135.81

Xi-kvadrat taqsimoti zichlik funksiyasi $\chi^2(k)$:

$$f_{\chi^2}(x) = \begin{cases} 0, & x \leq 0; \\ 2^{-k/2} \Gamma^{-1}(k/2) x^{(k-2)/2} e^{-x/2}, & x > 0. \end{cases}$$

5-jadval

$F_p(k_1, k_2)$ - Fisher taqsimoti kvantili k_1, k_2 - ozodlik darajalari; p - kvantil tartibi $p=0.95$

$k_2 \setminus k_1$	4	6	12	24	30	40	60	120	∞
1	224	234	245	249	250	251	252	253	254
2	19.2	19.3	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	9.1	8.9	8.7	8.6	8.6	8.6	8.6	8.5	8.5
4	6.4	6.2	5.9	5.8	5.7	5.7	5.7	5.7	5.6
5	5.2	5.0	4.7	4.5	4.5	4.5	4.4	4.4	4.4
6	4.5	4.3	4.0	3.8	3.8	3.8	3.7	3.7	3.7
7	4.1	3.9	3.6	3.4	3.4	3.3	3.3	3.3	3.2
8	3.8	3.6	3.3	3.1	3.1	3.0	3.0	3.0	2.9
9	3.6	3.4	3.1	2.9	2.9	2.8	2.8	2.7	2.7
10	3.5	3.2	2.9	2.7	2.7	2.7	2.6	2.6	2.5
11	3.4	3.1	2.8	2.6	2.6	2.5	2.5	2.4	2.4
12	3.3	3.0	2.7	2.5	2.5	2.4	2.4	2.3	2.3
13	3.2	2.9	2.6	2.4	2.4	2.3	2.3	2.3	2.2
14	3.1	2.8	2.5	2.3	2.3	2.3	2.2	2.2	2.1
15	3.1	2.8	2.5	2.3	2.2	2.2	2.2	2.1	2.1
16	3.0	2.7	2.4	2.2	2.2	2.2	2.1	2.1	2.0
17	3.0	2.7	2.4	2.2	2.1	2.1	2.1	2.0	2.0
18	2.9	2.7	2.3	2.1	2.1	2.1	2.1	2.0	1.9
19	2.9	2.6	2.3	2.1	2.1	2.0	2.0	1.9	1.9
20	2.9	2.6	2.3	2.1	2.0	2.0	1.9	1.9	1.8
22	2.8	2.5	2.2	2.0	2.0	1.9	1.9	1.8	1.8
24	2.8	2.5	2.2	2.0	1.9	1.9	1.8	1.8	1.7
26	2.7	2.5	2.1	1.9	1.9	1.9	1.8	1.7	1.7
28	2.7	2.4	2.1	1.9	1.9	1.8	1.8	1.7	1.7
30	2.7	2.4	2.1	1.9	1.8	1.8	1.7	1.7	1.6
40	2.6	2.3	2.0	1.8	1.7	1.7	1.6	1.6	1.5
60	2.5	2.3	1.9	1.7	1.6	1.6	1.5	1.5	1.4
120	2.4	2.2	1.8	1.6	1.6	1.5	1.4	1.4	1.3
∞	2.4	2.1	1.8	1.5	1.5	1.4	1.3	1.2	1.0

6-jadval Tekis taqsimlangan tasodifiy sonlar

98 52 01 77 67 14 90 56 86 07 22 10 94 05 58 60 97 09 34 33 50 50 07 39 98 11 80
50 54 31 39 80 82 77 32 50 72 56 82 48 29 40 52 42 01 52 77 56 78 51 83 45 29 96
34 06 28 89 80 83 13 74 67 00 78 18 47 54 06 10 68 71 17 78 17 88 68 54 02 00 86
50 75 84 01 36 76 66 79 51 90 36 47 64 93 29 60 91 10 62 99 59 46 73 48 87 51 76
49 69 91 82 60 89 28 93 78 56 13 68 23 47 83 41 13 65 48 11 76 74 17 46 85 09 50
58 04 77 69 74 73 03 95 71 86 40 21 81 65 44 80 12 43 56 35 17 72 70 80 15 45 31
82 23 74 21 11 57 82 53 14 38 55 37 63 74 35 09 98 17 77 40 27 72 14 43 23 60 02
10 45 52 16 42 37 96 28 60 26 55 69 91 62 68 03 66 25 22 91 48 36 93 68 72 03 76
62 11 39 90 94 40 05 64 18 09 89 32 05 05 14 22 56 85 14 46 42 75 67 88 96 29 77
88 22 54 38 21 45 98 91 49 91 45 23 68 47 92 76 86 46 16 28 35 54 94 75 08 90 23
37 08 92 00 48 80 33 69 45 98 26 94 03 68 58 70 29 73 41 35 53 14 03 33 40 42 05
08 23 41 44 10 48 19 49 85 15 74 79 54 32 97 92 65 75 57 60 04 08 81 22 22 20 64
13 12 55 07 37 42 11 10 00 20 40 12 86 07 46 97 96 64 48 94 39 28 70 72 53 15 63
60 64 93 29 16 50 53 44 84 40 21 95 25 63 43 65 17 70 82 07 20 73 17 90 61 19 69
04 46 26 45 74 77 74 51 92 43 37 29 65 39 45 95 93 42 58 26 05 27 15 47 44 52 66
95 27 07 99 53 59 36 78 38 48 82 39 61 01 18 33 21 15 94 66 94 55 72 85 73 67 89
75 43 87 54 62 24 44 31 91 19 04 25 92 92 92 74 59 73 42 48 11 62 13 97 34 40 87
21 16 86 84 87 67 03 07 11 20 59 25 70 14 66 70 23 52 37 83 17 73 20 88 98 37 68
93 59 14 16 26 25 22 96 63 05 52 28 25 62 04 49 35 24 94 75 24 63 38 24 45 86 25
10 25 61 96 27 93 35 65 33 71 24 72 00 54 99 76 54 64 05 18 81 59 96 11 96 38 96
54 69 28 23 91 23 28 72 95 29 35 96 31 53 07 26 89 80 93 54 33 35 13 54 62 77 97
45 00 24 90 10 33 93 33 59 80 80 83 91 45 42 72 68 42 83 60 94 97 00 13 02 12 48
92 78 56 52 01 06 46 05 88 52 36 01 39 09 22 86 77 28 14 40 77 93 91 08 36 47 70
61 74 29 41 32 17 90 05 97 87 37 92 52 41 05 56 70 70 07 86 74 31 71 57 85 39 41
18 38 69 23 46 14 06 20 11 74 52 04 15 95 66 00 00 18 74 39 24 23 97 11 89 63 35
19 56 54 14 30 01 75 87 53 79 40 41 92 15 85 66 67 43 68 06 84 96 28 52 07 45 15
51 49 38 19 47 60 72 46 43 66 79 45 43 59 04 79 00 33 20 82 66 95 41 94 86 43 19
94 36 16 81 08 51 34 88 88 15 53 01 54 03 54 56 05 01 45 11 76 98 08 62 48 26 45
24 02 84 04 44 99 90 88 96 39 09 47 34 07 35 44 13 18 80 33 18 51 62 32 41 94 15
09 49 89 43 54 85 81 88 69 54 19 94 37 54 87 30 43 80 95 10 04 06 96 38 27 07 74
20 15 12 33 87 25 01 62 52 98 94 62 46 11 71 79 75 24 91 40 71 96 12 82 96 69 86
10 25 91 74 85 22 05 39 00 38 75 95 79 18 63 33 25 37 98 14 50 65 71 31 01 02 46
74 05 45 56 14 27 77 93 89 19 36 74 02 94 39 02 77 55 73 22 70 97 79 01 71 19 52
52 75 80 21 80 81 45 17 48 54 17 84 58 11 80 99 33 71 43 05 33 51 29 69 56 12 71

JAVOBLAR

1 - §

- I. 1. $C=2/3$; $F(x) = \begin{cases} 0, & x < 1 \\ x^2/3, & x \in [0, 1], \\ 2x/3 - 1/3, & x \in (1, 2], \\ 1, & x > 2; \end{cases} \quad M\xi=11/9;$
- $D\xi=37/162$; $P(\xi > M\xi)=14/27$. 3. $C=3$; $F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - \frac{1}{(x+1)^3}, & x > 0; \end{cases} \quad M\xi = 1/2; D\xi=3/4; P(\xi > M\xi)=8/27.$
5. $C = \frac{1}{\pi}$; $M\xi=0$; $D\xi=1/2$. 7. $C = \frac{3}{\pi}$; $F(x) = \begin{cases} 0, & x < 0 \\ \frac{3}{\pi} \arctg x, & x \in [0, \sqrt{3}], \\ 1, & x > \sqrt{3}; \end{cases} \quad M\xi = \frac{3}{\pi} \ln 2; D\xi=1,092.$ 9. $C=1/2$;
- $F(x) = \begin{cases} 0, & x < 1/e, \\ \frac{1}{2} \ln x + \frac{1}{2}, & x \in [1/e, 1], \\ 1, & x > e; \end{cases} \quad M\xi = sh1, D\xi = 0,5(1 - e^{-2}).$
11. $C=1/2$; $F(x) = \begin{cases} 0, & x < -2 \\ \frac{1}{2}x - \frac{1}{8}\text{sign}(x)x^2, & x \in [-2, 2], \\ 1, & x > 2; \end{cases}$
- $M\xi=0$; $D\xi=2/3$; $P(\xi > M\xi)=1/2$. 13. $C=1$; $F(x) = \begin{cases} 0, & x < 0 \\ \sin x, & x \in [0, \pi/2], \\ 1, & x > \pi/2; \end{cases} \quad M\xi = \frac{\pi-2}{2}; D\xi = \pi - 3.$ 15. $C=1$;
- $M\xi=2$; $D\xi=2$. 17. $C=1/3$; $M\xi=4/3$; $D\xi=7/18$. 19. $C=4/3$; $M\xi=3/7$; $D\xi=0,0735$.
- II. 1. $\eta_1 \sim R(0, 1)$. 2. $f_{\eta_2}(x) = \begin{cases} \frac{\alpha}{x^2} e^{-\alpha(\frac{1}{x}-1)}, & x \in (0, 1), \\ 0, & x \notin (0, 1). \end{cases}$
3. $\eta_1 \sim N(2a + 1; 4\sigma^2)$. 4. $f_{\eta_2}(x) = \begin{cases} \frac{1}{x^2}, & x \in (0, 1), \\ 0, & x \notin (0, 1). \end{cases}$
5. $\eta_1 \sim R(0, 1)$. 6. $\eta_2 \sim N(1 - a; \sigma^2)$.
7. $f_{\eta_1}(x) = \begin{cases} \frac{\alpha}{2\sqrt{x}} e^{-\alpha\sqrt{x}}, & x > 0, \\ 0, & x \leq 0. \end{cases}$ 8. $f_{\eta_2}(x) = \frac{1}{\pi\sqrt{x(1-x)}}, \quad x \in (0, 1).$
9. $\eta_1 \sim \Gamma(1/2, 1/2)$. 10. $f_{\eta_2}(x) = \frac{1}{\pi} \frac{1}{1+x^2}, x \in R.$

$$11. f_{\eta_1}(x) = \begin{cases} \frac{\alpha}{2\sqrt{x-1}} e^{-\alpha\sqrt{x-1}}, & x > 1, \\ 0, & x \leq 1. \end{cases} \quad 12. f_{\eta_2}(x) = \frac{1}{\pi} \frac{1}{1+x^2}, x \in R.$$

$$13. \eta_1 \sim N(14; 9\sigma^2). \quad 14. f_{\eta_2}(x) = \begin{cases} 0, & x < 1, \\ 3/x^4, & x \geq 1. \end{cases}$$

$$15. \eta_1 \sim R(1/4, 3/4). \quad 16. f_{\eta_2}(x) = \alpha^2 \exp\{-\alpha(e^{\alpha x} - x)\}, x \in R.$$

$$17. \eta_1 \sim R(-\frac{\pi}{2}, \frac{\pi}{2}). \quad 18. f_{\eta_2}(x) = \begin{cases} \frac{1}{\sqrt{2\pi x}} e^{-x/2}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

$$19. f_{\eta_1}(x) = \begin{cases} \frac{2a}{\sqrt{x-1}}, & x \in (2, 5), \\ 0, & x \notin (2, 5). \end{cases}$$

$$20. f_{\eta_2}(x) = \begin{cases} \frac{3(1-x)^2}{2x^4}, & x > 1/2, \\ 0, & x \leq 1/2. \end{cases}$$

$$\text{III. } 1. f(x) = n \left(\frac{b-x}{b-1} \right)^{n-1} \frac{1}{b-1}, x \in (1, b). \quad 3. \zeta \sim E(5/4).$$

$$5. \zeta \sim N(2, 5). \quad 7. F(x) = 1 - e^{-\lambda_1 x} - e^{-\lambda_2 x} + e^{-(\lambda_1 + \lambda_2)x}, x > 0.$$

$$9. \zeta \sim N(a_1 + a_2 - 5, 5). \quad 11. f(x) = \begin{cases} 0, & x \leq 0, \\ 1 - e^{-x}, & 0 < x \leq 1, \\ (e-1)e^{-x}, & x > 1. \end{cases}$$

$$13. \zeta \sim \pi(\lambda_1 + \lambda_2). \quad 17. \zeta \sim Bi(n_1 + n_2, p). \quad 19. \zeta \sim E\left(\sum_{i=1}^n \lambda_i\right).$$

$$\text{V. a) } 2. -9/13. \quad 4. \sqrt{15}/2\sqrt{7}. \quad 6. 0. \quad 8. 0. \quad 10. -2/3. \quad 12. -2/13.$$

$$14. -45/411. \quad 16. 0. \quad 18. 0. \quad 20. -5/139.$$

$$\text{b) } 1. \frac{2(9\pi-32)}{9\pi^2-64} \approx -0,3. \quad 3. \frac{2(9\pi-32)}{9\pi^2-64} \approx -0,3. \quad 7. 0. \quad 9. -0,5. \quad 11. 0.$$

$$13. 0. \quad 15. \frac{\sqrt{35}}{\sqrt{111}} \approx 0,562. \quad 17. \frac{\sqrt{35}}{2\sqrt{57}} \approx 0,392. \quad 19. \frac{9\sqrt{5}}{8\sqrt{19}} \approx 0,577.$$

$$\text{VI. } 2. \text{ bo'ysunadi. } \quad 4. \text{ yo'q. } \quad 6. \alpha < 0. \quad 8. \text{ bo'ysunadi. } \quad 10.$$

$$\text{yo'q. } \quad 12. \text{ bo'ysunadi. } \quad 16. \text{ bo'ysunadi. } \quad 18. \text{ bo'ysunadi. } \quad 20.$$

$$\alpha > 0\text{-ixtiyoriy.}$$

2 - §

$$1. M\bar{x} = \frac{\theta_1 + \theta_2}{2}, D\bar{x} = \frac{(\theta_2 - \theta_1)^2}{12n}. \quad 3. MX_{(1)} = \frac{\theta}{n+1}, DX_{(1)} = \frac{n\theta^2}{(n+1)^2(n+2)}.$$

$$5. M\bar{x} = \theta + 1, D\bar{x} = \frac{1}{n}. \quad 7. M\bar{x}^2 = \theta^2, D\bar{x}^2 = \frac{2(n-1)}{n^2}\theta^4.$$

$$9. MF_n(x) = F(x), DF_n(x) = \frac{1}{n}F(x)(1-F(x)).$$

3 - §

2. tegishli. 4. yo'q. 6. yo'q. 8. tegishli. 10. yo'q. 12. tegishli. 14. tegishli. 16. yo'q. 18. tegishli.

4 - §

1. $(X_{(1)}, X_{(n)})$; 3. $(X_{(1)}, \sum_{i=1}^n X_i)$; 7. $\sum_{i=1}^n X_i$; 9. $\sum_{i=1}^n X_i$; 11. $X^{(n)} = (X_1, \dots, X_n)$; 13. $\sum_{i=1}^n X_i$; 15. $X_{(1)}$; 17. $(X_{(1)}, \sum_{i=1}^n X_i)$; 19. $X^{(n)} = (X_1, \dots, X_n)$;

6 - §

1. siljimagan, asosli baho; 3. a) siljigan, asosli baho; b) siljimagan, asosli baho; 7. siljimagan, asosli baho; 9. siljimagan, asosli baho; 13. siljigan, asosli baho; 15. siljigan, asosli baho; 17. siljigan, asosli baho; 19. siljigan, asosli baho.

8 - §

1. $(\frac{S^2}{\bar{x}}, \frac{\bar{x}}{S})$; 3. (\bar{x}, S^2) ; 5. $(e^{\bar{x}-1/S}, S)$; 9. $\bar{x}/(1-\bar{x})$; 11. $\bar{x}/2-2$;
13. $\frac{1}{2}e^{\bar{x}-1}$; 17. $\frac{3\bar{x}}{2}$; 19. $\sqrt[k]{x^k/\Gamma(1+\frac{k}{3})}$.

9 - §

3. (\bar{x}, S^2) ; 6. $(-X_{(1)}, X_{(n)})$; 8. $(X_{(1)}+1, X_{(n)}-1)$; 10. $X_{(1)}$;
11. $(1/(\ln x - \ln X_{(1)}), X_{(1)})$; 12. $(X_{(1)}, \bar{x} - X_{(1)})$; 14. \bar{x}^2 ;
16. $-\frac{1}{\ln \ln x}$; 17. $2\bar{x}$; 20. $X_{(n)}$.

10 - §

1. $\frac{\bar{x}n\sigma^2+a}{1+n\sigma^2}$; 3. $(1+e^{n/2-n\bar{x}})^{-1}$; 5. $ae^{na}/(e^{na}-1)-1/n$, bu yerda $a = \min(1, X_{(1)})$; 7. $\frac{n+1}{n} \max(X_{(n)}, 1)$; 9. $\frac{n+1}{1+\bar{x}n}$; 11. $\max(X_{(n)}, 1) \frac{\beta+2n}{\beta+2n-1}$;
13. $\frac{2n-1}{2n-2} \frac{X_{(n)}^{2n-1}-X_{(n)}}{X_{(n)}^{2n-1}-1}$; 15. $\frac{\bar{x}n+\lambda}{n+1+\lambda}$; 17. $\frac{3^{n\bar{x}}+4 \cdot 2^{n\bar{x}}+9}{4(3n\bar{x}+2 \cdot 2^{n\bar{x}}+3)}$; 19. $\frac{\bar{x}n+1}{n+1}$.

11 - §

8. a.n. baho. 10. a.n. baho. 12. a.n. baho. 14. a.n. baho. 16. a.n. baho. 18. a.n. baho. 20. a.n. baho.

12 - §

1. $D_{\theta}S^2 = \frac{2\theta^4}{n-1}$; $D_{\theta}S_2^2 = \frac{2\theta^4}{n}$; 3. $M_{\theta}(T_1 - \theta)^2 = M_{\theta}(T_2 - \theta)^2 = \frac{2\theta^2}{(n+1)(n+2)}$ 5. $M_{\theta}(X_{(1)} - \theta)^2 < M_{\theta}(\bar{x} - 1/2 - \theta)^2$;
7. $M_{\theta}(X_{(1)} - \frac{1}{n} - \theta)^2 = \frac{1}{n^2}$, $M_{\theta}(\bar{x} - 1 - \theta)^2 = \frac{1}{n}$.

13 - §

2. χ_2^2 - xi-kvadrat; 4. χ_n^2 - xi-kvadrat; 10. $N(0, 2\sigma/n)$; 12. $N(0, 1)$;
14. χ_{n-1}^2 - xi-kvadrat; 16. $Bi(n, p)$; 18. $f(x) = \frac{n(n-1)x(b-x)^{n-2}}{b^n}$, $x \in [0, b]$; 20. $E(1)$.

14 - §

1. $(\bar{x} - \frac{\sigma}{\sqrt{n}}t_{\frac{1+\gamma}{2}}; \bar{x} + \frac{\sigma}{\sqrt{n}}t_{\frac{1+\gamma}{2}})$;
3. $(\bar{x} - \frac{S^2}{\sqrt{n}}t_{n-1, \frac{1+\gamma}{2}}; \bar{x} + \frac{S^2}{\sqrt{n}}t_{n-1, \frac{1+\gamma}{2}})$; 5. $(X_{(n)}; X_{(n)}/\sqrt[3]{1-\gamma})$;
7. $(\bar{x})^{-1} \left(1 - \frac{t_{\frac{1+\gamma}{2}}}{\sqrt{n}}\right)$; $(\bar{x})^{-1} \left(1 + \frac{t_{\frac{1+\gamma}{2}}}{\sqrt{n}}\right)$);
9. $(X_{(1)} + \ln(1-\gamma)/n; X_{(1)})$; 11. $(X_{(1)} - 1 + \sqrt[3]{1-\gamma}; X_{(1)})$;
13. $(0; -(\ln(1-\gamma))/X_1)$.

Testlarning javoblari

- | | | | | | | | |
|-------|--------|--------|--------|--------|--------|--------|--------|
| 1. C. | 6. B. | 11. B. | 16. C. | 21. B. | 26. C. | 31. D. | 36. E. |
| 2. A. | 7. E. | 12. B. | 17. D. | 22. C. | 27. A. | 37. B. | 32. A. |
| 3. A. | 8. E. | 13. D. | 18. E. | 23. A. | 28. B. | 33. A. | 38. D. |
| 4. A. | 9. C. | 14. B. | 19. E. | 24. E. | 29. E. | 34. C. | 39. D. |
| 5. C. | 10. E. | 15. A. | 20. B. | 25. C. | 30. B. | 35. B. | 40. D. |
-
- | | |
|--------|--------|
| 41. B. | 46. C. |
| 42. B. | 47. A. |
| 43. A. | 48. E. |
| 44. C. | 49. D. |
| 45. C. | 50. A. |

BELGILASHLAR VA QISQARTMALAR RO'YXATI

t.m.	tasodifiy miqdor
t.f.	taqsimot funksiyasi
t.e.q.	tekis eng quvvatli
a.n.	asimptotik normal
KSQ	katta sonlar qonuni
MUB	momentlar usuli bahosi
HMO'UB	haqiqatga maksimal o'xshashlik usuli bahosi
$P(A)$	A hodisaning ehtimolligi
$M\xi$	ξ t.m. matematik kutilmasi
$D\xi$	ξ t.m. dispersiyasi
$I(A)$	A hodisa indikatorini
$X^{(n)} = (X_1, \dots, X_n)$	statistik tanlanma
$X_{(k)}$	k - tartiblangan statistika
\xrightarrow{P}	ehtimollik bo'yicha yaqinlashish
\Rightarrow	taqsimot bo'yicha yaqinlashish
$\xi \sim N(a; \sigma^2)$	ξ t.m. $(a; \sigma^2)$ paramertli normal taqsimotga ega
\triangleright	yechimning boshlanishi
\triangleleft	yechimning tugashi
\perp	bog'liqsizlik
$f(x; \theta)$	absolyut uzluksiz t.m. uchun zichlik funksiyasi, diskret t.m. uchun ehtimollik
$F(x; \theta)$	t.f.
N_f	taqsimot bardori (nositel)
θ	noma'lum parametr
Θ	parametrik fazo

BA'ZI MUHIM TAQSIMOTLAR OILASI

1	<p>Normal taqsimot</p> $N(\theta_1, \theta_2^2)$	$f(x, \theta) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2^2}}$ $N_f = \{x : -\infty < x < +\infty\}$ $\theta = (\theta_1, \theta_2) \in \Theta = (-\infty; +\infty) \times (0; +\infty)$ $M\xi = \theta_1, D\xi = \theta_2^2$
2	<p>Tekis taqsimot</p> $R(\theta_1, \theta_2)$	$f(x, \theta) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 < x < \theta_2 \\ 0, & \text{aks holda} \end{cases}$ $N_f = \{x : \theta_1 < x < \theta_2\}$ $\theta = (\theta_1, \theta_2) \in \Theta; -\infty < \theta_1 < \theta_2 < +\infty$ $M\xi = \frac{\theta_1 + \theta_2}{2}, D\xi = \frac{(\theta_2 - \theta_1)^2}{12}$
3	<p>Ko'rsatkichli taqsimot</p> $E(\theta)$	$f(x, \theta) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$ $N_f = \{x : x > 0\}$ $\theta \in \Theta = (0; +\infty)$ $M\xi = 1/\theta, D\xi = 1/\theta^2$
4	<p>Gamma taqsimot</p> $\Gamma(\theta_1, \theta_2)$	$f(x, \theta) = \begin{cases} \frac{x^{\theta_2-1} e^{-x/\theta_1}}{\Gamma(\theta_2)\theta_1^{\theta_2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ $N_f = \{x : x > 0\}$ $\theta = (\theta_1, \theta_2) \in \Theta = (0; +\infty) \times (0; +\infty)$ $M\xi = \theta_1\theta_2, D\xi = \theta_1^2\theta_2$
5	<p>Lognormal taqsimot</p> $\text{Log}N(\theta_1, \theta_2^2)$	$f(x, \theta) = \begin{cases} \frac{1}{\sqrt{2\pi\theta_2}x} e^{-\frac{(\ln x - \theta_1)^2}{2\theta_2^2}}, & x > 0 \\ 0, & x \leq 0 \end{cases}$ $N_f = \{x : x > 0\}$

		$\theta = (\theta_1, \theta_2) \in \Theta = (-\infty; +\infty) \times (0; +\infty)$ $M\xi = e^{\theta_1 + \theta_2^2/2}, M\xi^2 = e^{2\theta_1 + 2\theta_2^2}$
6	Veybull taqsimoti $W(\theta_1, \theta_2)$	$f(x, \theta) = \begin{cases} \frac{\theta_1}{\theta_2} \left(\frac{x}{\theta_2}\right)^{\theta_1-1} e^{-\left(\frac{x}{\theta_2}\right)^{\theta_1}}, & x > 0, \\ 0, & x \leq 0 \end{cases}$ $N_f = \{x : x > 0\}$ $\theta = (\theta_1, \theta_2) \in \Theta = (0; +\infty) \times (0; +\infty)$ $M\xi = \theta_2 \Gamma\left(1 + \frac{1}{\theta_1}\right), M\xi^2 = \theta_2^2 \Gamma\left(1 + \frac{1}{\theta_1}\right)$
7	Pareto taqsimoti $P(\theta_1, \theta_2)$	$f(x, \theta) = \begin{cases} \theta_2 e^{-\theta_2(x - \ln \theta_1)}, & x > \ln \theta_1 \\ 0, & x \leq \ln \theta_1 \end{cases}$ $N_f = \{x : x > \ln \theta_1\}$ $\theta = (\theta_1, \theta_2) \in \Theta = (0; +\infty) \times (0; +\infty)$ $M\xi = \ln \theta_1 + 1/\theta_2, D\xi = 1/\theta_2^2$
8	Koshi taqsimoti $K(\theta_1, \theta_2)$	$f(x, \theta) = \frac{1}{\pi \theta_2} \frac{1}{1 + \left(\frac{x - \theta_1}{\theta_2}\right)^2}$ $N_f = \{x : -\infty < x < +\infty\}$ $\theta = (\theta_1, \theta_2) \in \Theta = (-\infty; +\infty) \times (0; +\infty)$ $M\xi \text{ mavjud emas}$
9	Puasson taqsimoti $\pi(\theta)$	$f(x, \theta) = \frac{e^{-\theta} \theta^x}{x!}$ $N_f = \{x : x = 0, 1, 2, \dots\}$ $\theta \in \Theta = (0; +\infty)$ $M\xi = D\xi = \theta$

10	Geometrik taqsimot $Ge(\theta)$	$f(x, \theta) = \theta(1 - \theta)^{x-1}$ $N_f = \{x : x = 1, 2, \dots\}$ $\theta \in \Theta = (0; 1)$ $M\xi = 1/\theta, \quad D\xi = \theta/(1 - \theta)$
11	Binomial taqsimot $Bi(n; \theta)$	$f(x, \theta) = C_n^x \theta^x (1 - \theta)^{n-x}$ $N_f = \{x : x = 0, 1, 2, \dots, n\}$ $\theta \in \Theta = (0; 1)$ $M\xi = n\theta, \quad D\xi = n\theta(1 - \theta)$
12	Teskari binomial taqsimot $\overline{Bi}(r; \theta)$	$f(x, \theta) = C_{r+x-1}^x \theta^x (1 - \theta)^r$ $N_f = \{x : x = 0, 1, 2, \dots\}$ $\theta \in \Theta = (0; 1)$ $M\xi = r\theta/(1 - \theta),$ $D\xi = r\theta/(1 - \theta)^2$

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